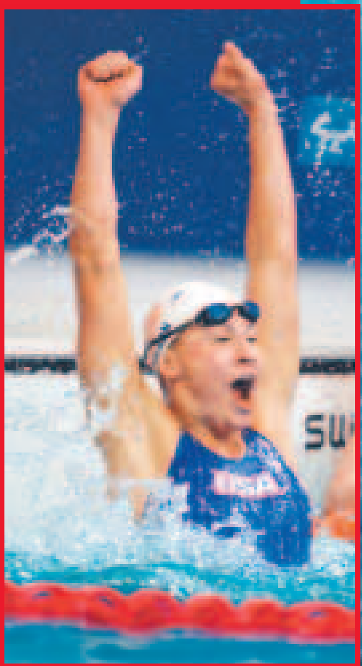
UNIT 2

Linear Functions

Many real-world situations such as Olympic race times can be represented using functions. In this unit, you will learn about linear functions and equations.



CONTENTS

WebQuest Internet Project

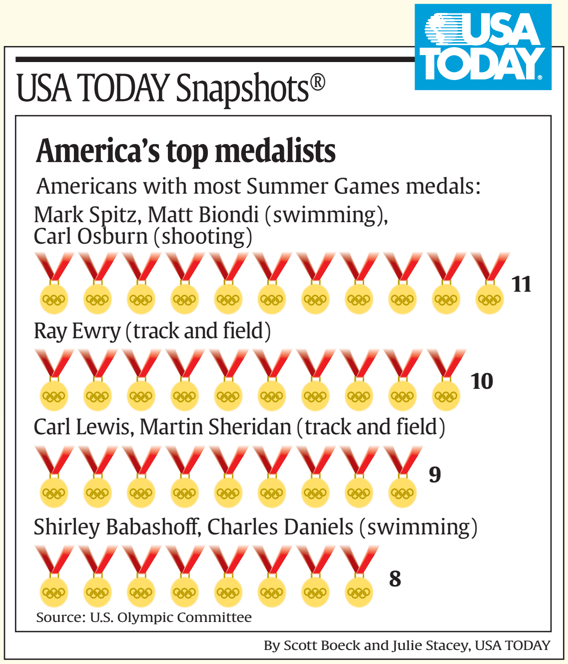
The Spirit of the Games

The first Olympic Games featured only one event— a foot race. The 2004 Games will include thousands of competitors in about 300 events. In this project, you will explore how linear functions can be illustrated by the Olympics.

Log on to www.algebra1.com/webquest. Begin your WebQuest by reading the Task.

Then continue working on your WebQuest as you study Unit 2.

Lesson	4-6	5-7	6-6	7-1
Page	230	304	357	373



Chapter 4

Graphing Relations and Functions

What You'll Learn

- **Lessons 4-1, 4-4, and 4-5** Graph ordered pairs, relations, and equations.
- **Lesson 4-2** Transform figures on a coordinate plane.
- **Lesson 4-3** Find the inverse of a relation.
- **Lesson 4-6** Determine whether a relation is a function.
- **Lessons 4-7 and 4-8** Look for patterns and write formulas for sequences.

Key Vocabulary

- coordinate plane (p. 192)
- transformation (p. 197)
- inverse (p. 206)
- function (p. 226)
- arithmetic sequence (p. 233)

Why It's Important

The concept of a function is used throughout higher mathematics, from algebra to calculus. A function is a rule or a formula. You can use a function to describe real-world situations like converting between currencies. For example, if you are in Mexico, you can calculate that an item that costs 100 pesos is equivalent to about 11 U.S. dollars. *You will learn how to convert different currencies in Lesson 4-4.*

Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 4.

For Lesson 4-1 — Graph Real Numbers

Graph each set of numbers. *(For review, see Lesson 2-1.)*

1. $\{1, 3, 5, 7\}$ **2.** $\{-3, 0, 1, 4\}$ **3.** $\{-8, -5, -2, 1\}$ **4.** $\left\{\frac{1}{2}, 1, 1\frac{1}{2}, 2\right\}$

For Lesson 4-2 — Distributive Property

Rewrite each expression using the Distributive Property. *(For review, see Lesson 1-5.)*

5. $3(7 - t)$ **6.** $-4(w + 2)$ **7.** $-5(3b - 2)$ **8.** $\frac{1}{2}(2z + 4)$

For Lessons 4-4 and 4-5 — Solve Equations for a Specific Variable

Solve each equation for y. *(For review, see Lesson 3-8.)*

9. $2x + y = 1$ **10.** $x = 8 - y$ **11.** $6x - 3y = 12$

12. $2x + 3y = 9$ **13.** $9 - \frac{1}{2}y = 4x$ **14.** $\frac{y + 5}{3} = x + 2$

For Lesson 4-6 — Evaluate Expressions

Evaluate each expression if $a = -1$, $b = 4$, and $c = -3$. *(For review, see Lesson 2-3.)*

15. $a + b - c$ **16.** $2c - b$ **17.** $c - 3a$

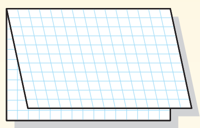
18. $3a - 6b - 2c$ **19.** $8a + \frac{1}{2}b - 3c$ **20.** $6a + 8b + \frac{2}{3}c$

FOLDABLES™ Study Organizer

Graphing Relations and Functions Make this Foldable to help you organize your notes. Begin with four sheets of grid paper.

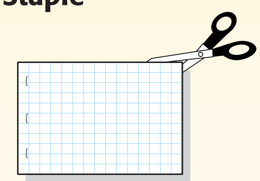
Step 1 Fold

Fold each sheet of grid paper in half from top to bottom.



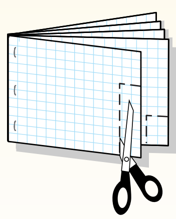
Step 2 Cut and Staple

Cut along each fold. Staple the eight half-sheets together to form a booklet.



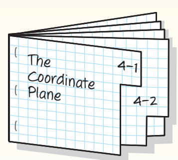
Step 3 Cut Tabs into Margin

The top tab is 4 lines wide, the next tab is 8 lines wide, and so on.



Step 4 Label

Label each of the tabs with a lesson number.



Reading and Writing As you read and study the chapter, use each page to write notes and to graph examples.

4-1 The Coordinate Plane

What You'll Learn

- Locate points on the coordinate plane.
- Graph points on a coordinate plane.

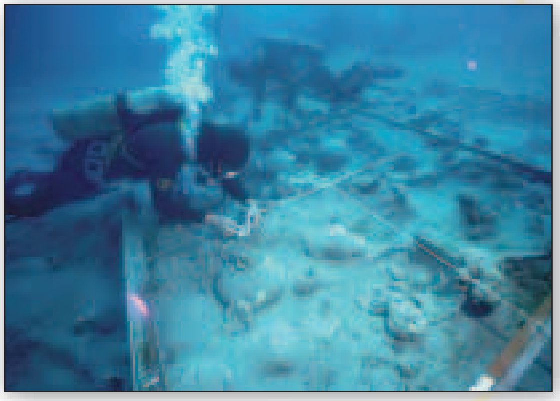
Vocabulary

- coordinate plane
- quadrant
- graph

How do archaeologists use coordinate systems?

Underwater archaeologists use a grid system to map excavation sites of sunken ships. The grid is used as a point of reference on the ocean floor.

The coordinate system is also used to record the location of objects they find. Knowing the position of each object helps archaeologists reconstruct how the ship sank and where to find other artifacts.



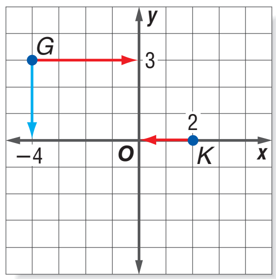
IDENTIFY POINTS In mathematics, points are located in reference to the x-axis and y-axis on a coordinate system or **coordinate plane**.

Example 1 Name an Ordered Pair

Write the ordered pair for each point.

a. point G

- Begin at point G.
- Follow along a vertical line through the point to find the x-coordinate on the x-axis. The x-coordinate is -4.
- Follow along a horizontal line through the point to find the y-coordinate on the y-axis. The y-coordinate is 3.
- So, the ordered pair for point G is $(-4, 3)$. This can also be written as $G(-4, 3)$.



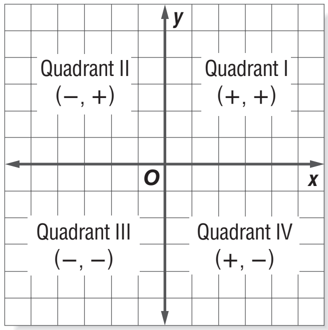
Unless marked otherwise, you can assume that each division on the axes represents 1 unit.

b. point K

- Begin at point K.
- K is on the x-axis 2 units from the origin. The x-coordinate is 2.
- The y-coordinate is 0 because point K is on the x-axis.
- Thus, the ordered pair for K is $(2, 0)$ or $K(2, 0)$.

CONTENTS

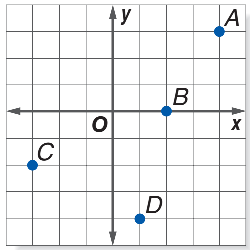
The x-axis and y-axis separate the coordinate plane into four regions, called **quadrants**. Notice which quadrants contain positive and negative x-coordinates and which quadrants contain positive and negative y-coordinates. The axes are not located in any of the quadrants.



Example 2 *Identify Quadrants*

Write ordered pairs for points A, B, C, and D. Name the quadrant in which each point is located.

Use a table to help find the coordinates of each point.



Point	x-Coordinate	y-Coordinate	Ordered Pair	Quadrant
A	4	3	(4, 3)	I
B	2	0	(2, 0)	none
C	−3	−2	(−3, −2)	III
D	1	−4	(1, −4)	IV

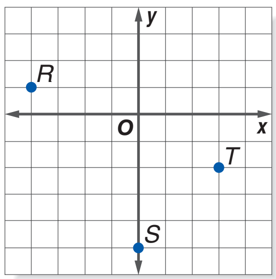
GRAPH POINTS To **graph** an ordered pair means to draw a dot at the point on the coordinate plane that corresponds to the ordered pair. This is sometimes called *plotting a point*. When graphing an ordered pair, start at the origin. The x-coordinate indicates how many units to move right (positive) or left (negative). The y-coordinate indicates how many units to move up (positive) or down (negative).

Example 3 *Graph Points*

Plot each point on a coordinate plane.

a. $R(-4, 1)$

- Start at the origin.
- Move left 4 units since the x-coordinate is -4.
- Move up 1 unit since the y-coordinate is 1.
- Draw a dot and label it R.



b. $S(0, -5)$

- Start at the origin.
- Since the x-coordinate is 0, the point will be located on the y-axis.
- Move down 5 units.
- Draw a dot and label it S.

c. $T(3, -2)$

- Start at the origin.
- Move right 3 units and down 2 units.
- Draw a dot and label it T.

Example 4 Use a Coordinate System

GEOGRAPHY **Latitude and longitude lines form a system of coordinates to designate locations on Earth. Latitude lines run east and west and are the first coordinate of the ordered pairs. Longitude lines run north and south and are the second coordinate of the ordered pairs.**

a. Name the city at (40°, 105°).
Locate the latitude line at 40°. Follow the line until it intersects with the longitude line at 105°. The city is Denver.

b. Estimate the latitude and longitude of Washington, D.C.
Locate Washington, D.C., on the map. It is close to 40° latitude and 75° longitude. There are 5° between each line, so a good estimate is 39° for the latitude and 77° for the longitude.

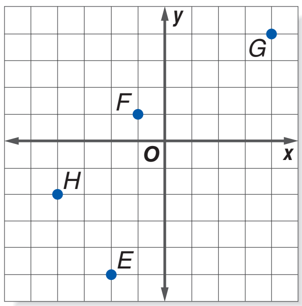
Check for Understanding

Concept Check

1. **Draw** a coordinate plane. Label the origin, x-axis, y-axis, and the quadrants.
2. **Explain** why $(-1, 4)$ does not name the same point as $(4, -1)$.
3. **OPEN ENDED** Give the coordinates of a point for each quadrant in the coordinate plane.

Guided Practice

Write the ordered pair for each point shown at the right. Name the quadrant in which the point is located.



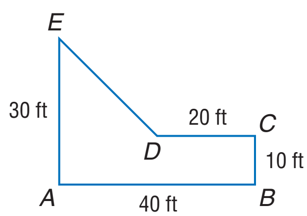
4. E
5. F
6. G
7. H

Plot each point on a coordinate plane.

8. $J(2, 5)$
9. $K(-1, 4)$
10. $L(0, -3)$
11. $M(-2, -2)$

Application

12. **ARCHITECTURE** Chun Wei has sketched the southern view of a building. If A is located on a coordinate system at $(-40, 10)$, locate the coordinates of the other vertices.



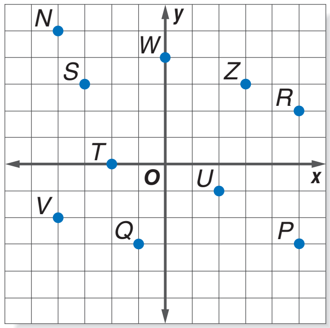
Practice and Apply

Homework Help

For Exercises	See Examples
13–24, 39	1, 2
25–36	3
37, 38, 40–43	4

Extra Practice
See page 828.

Write the ordered pair for each point shown at the right. Name the quadrant in which the point is located.



13. N **14.** P
15. Q **16.** R
17. S **18.** T
19. U **20.** V
21. W **22.** Z

23. Write the ordered pair that describes a point 12 units down from and 7 units to the right of the origin.

24. Write the ordered pair for a point that is 9 units to the left of the origin and lies on the x-axis.

Plot each point on a coordinate plane.

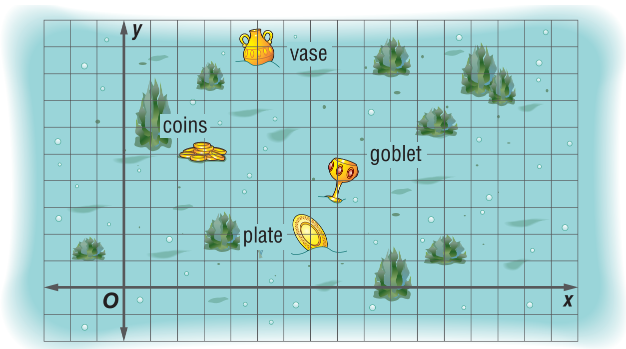
25. $A(3, 5)$ **26.** $B(-2, 2)$ **27.** $C(4, -2)$ **28.** $D(0, -1)$
29. $E(-2, 5)$ **30.** $F(-3, -4)$ **31.** $G(4, 4)$ **32.** $H(-4, 4)$
33. $I(3, 1)$ **34.** $J(-1, -3)$ **35.** $K(-4, 0)$ **36.** $L(2, -4)$

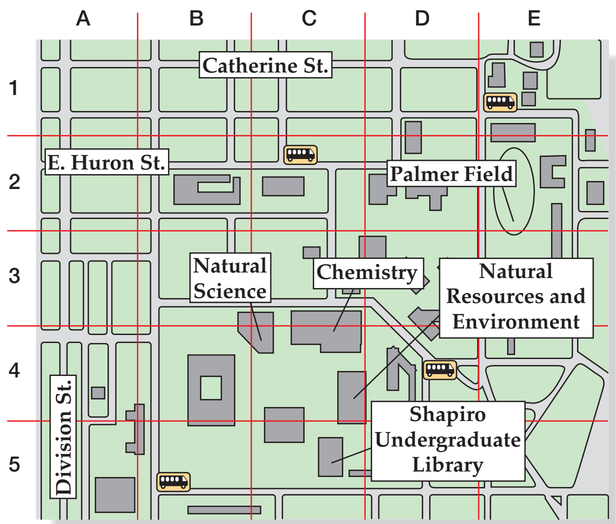
GEOGRAPHY **For Exercises 37 and 38, use the map on page 194.**

37. Name two cities that have approximately the same latitude.

38. Name two cities that have approximately the same longitude.

39. ARCHAEOLOGY The diagram at the right shows the positions of artifacts found on the ocean floor. Write the coordinates of the location for each object: coins, plate, goblet, and vase.





MAPS **For Exercises 40–43, use the map of the University of Michigan at the left.**
On many maps, letters and numbers are used to define a region or sector. For example, Palmer Field is located in sector E2.

40. In what sector is the Undergraduate Library?

41. In what sector are most of the science buildings?

42. Which street goes from sector (A, 2) to (D, 2)?

43. Name the sectors that have bus stops.

44. CRITICAL THINKING Describe the possible locations, in terms of quadrants or axes, for the graph of (x, y) given each condition.

a. $xy > 0$ **b.** $xy < 0$ **c.** $xy = 0$

45. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

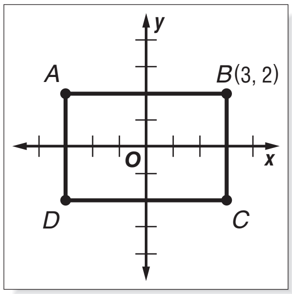
How do archaeologists use coordinate systems?

Include the following in your answer:

- an explanation of how dividing an excavation site into sectors can be helpful in excavating a site, and
- a reason why recording the exact location of an artifact is important.

Standardized Test Practice

For Exercises 46 and 47, refer to the figure at the right.



46. *ABCD* is a rectangle with its center at the origin. If the coordinates of vertex *B* are (3, 2), what are the coordinates of vertex *A*?

(A) $(-3, -2)$ (B) $(3, -2)$
(C) $(-3, 2)$ (D) $(3, 2)$

47. What is the length of $\overline{AD}$?

(A) 6 units (B) 4 units
(C) 5 units (D) 3 units

Extending the Lesson

The midpoint of a line segment is the point that lies exactly halfway between the two endpoints. The midpoint of a line segment whose endpoints are at (a, b) and (c, d) is at $\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$. Find the midpoint of each line segment whose endpoints are given.

48. $(7, 1)$ and $(-3, 1)$ 49. $(5, -2)$ and $(9, -8)$ 50. $(-4, 4)$ and $(4, -4)$

Maintain Your Skills

Mixed Review

51. **AIRPLANES** At 1:30 P.M., an airplane leaves Tucson for Baltimore, a distance of 2240 miles. The plane flies at 280 miles per hour. A second airplane leaves Tucson at 2:15 P.M. and is scheduled to land in Baltimore 15 minutes before the first airplane. At what rate must the second airplane travel to arrive on schedule? *(Lesson 3-9)*

Solve each equation or formula for the variable specified. *(Lesson 3-8)*

52. $3x + b = 2x + 5$ for x 53. $10c = 2(2d + 3c)$ for d

54. $6w - 3h = b$ for h 55. $\frac{3(a-t)}{4} = 2t$ for t

Find each square root. Round to the nearest hundredth if necessary. *(Lesson 2-7)*

56. $-\sqrt{81}$ 57. $\sqrt{63}$ 58. $\sqrt{180}$ 59. $-\sqrt{256}$

Evaluate each expression. *(Lesson 2-1)*

60. $52 + |18 - 7|$ 61. $|81 - 47| + 17$ 62. $42 - |60 - 74|$

63. $36 - |15 - 21|$ 64. $|10 - 16 + 27|$ 65. $|38 - 65 - 21|$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Rewrite each expression using the Distributive Property. Then simplify.** *(To review the **Distributive Property**, see Lesson 1-5.)*

66. $4(x + y)$ 67. $-1(x + 3)$ 68. $3(1 - 6y)$

69. $-3(2x - 5)$ 70. $\frac{1}{3}(2x + 6y)$ 71. $\frac{1}{4}(5x - 2y)$

4-2 Transformations on the Coordinate Plane

What You'll Learn

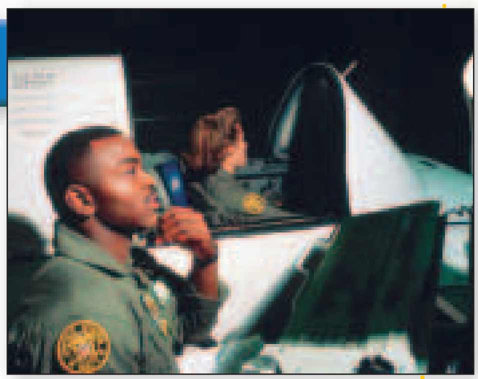
- Transform figures by using reflections, translations, dilations, and rotations.
- Transform figures on a coordinate plane by using reflections, translations, dilations, and rotations.

Vocabulary

- transformation
- preimage
- image
- reflection
- translation
- dilation
- rotation

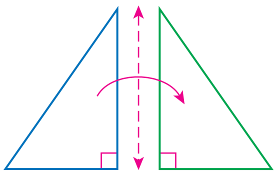
How are transformations used in computer graphics?

Computer programs can create movements that mimic real-life situations. A new CD-ROM-based flight simulator replicates an actual flight experience so closely that the U.S. Navy is using it for all of their student aviators. The movements of the on-screen graphics are accomplished by using mathematical transformations.

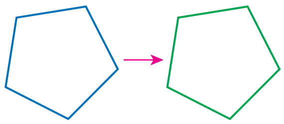


TRANSFORM FIGURES **Transformations** are movements of geometric figures. The **preimage** is the position of the figure before the transformation, and the **image** is the position of the figure after the transformation.

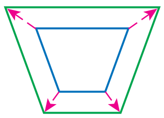
reflection
a figure is flipped over a line



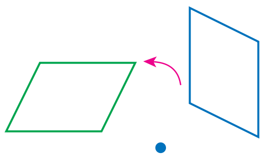
translation
a figure is slid in any direction



dilation
a figure is enlarged or reduced



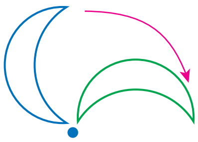
rotation
a figure is turned around a point



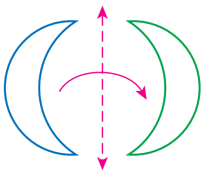
Example 1 Identify Transformations

Identify each transformation as a *reflection, translation, dilation,* or *rotation*.

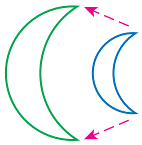
a.



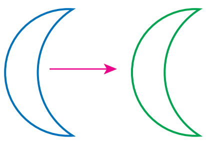
b.



c.



d.



a. The figure has been turned around a point. This is a rotation.

b. The figure has been flipped over a line. This is a reflection.

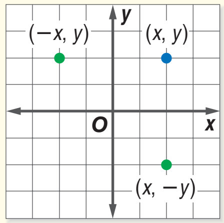
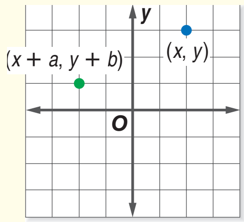
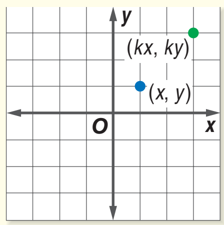
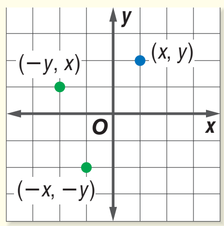
c. The figure has been increased in size. This is a dilation.

d. The figure has been shifted horizontally to the right. This is a translation.

TRANSFORM FIGURES ON THE COORDINATE PLANE

You can perform transformations on a coordinate plane by changing the coordinates of the points on a figure. The points on the translated figure are indicated by the prime symbol ′ to distinguish them from the original points.

Key Concept — *Transformations on the Coordinate Plane*

Name	Words	Symbols	Model
Reflection	To reflect a point over the x-axis, multiply the y-coordinate by -1. To reflect a point over the y-axis, multiply the x-coordinate by -1.	reflection over x-axis: $(x, y) \rightarrow (x, -y)$ reflection over y-axis: $(x, y) \rightarrow (-x, y)$	
Translation	To translate a point by an ordered pair (a, b), add a to the x-coordinate and b to the y-coordinate.	$(x, y) \rightarrow (x + a, y + b)$	
Dilation	To dilate a figure by a scale factor k, multiply both coordinates by k. If $k > 1$, the figure is enlarged. If $0 < k < 1$, the figure is reduced.	$(x, y) \rightarrow (kx, ky)$	
Rotation	To rotate a figure 90° *counterclockwise* about the origin, switch the coordinates of each point and then multiply the new first coordinate by -1. To rotate a figure 180° about the origin, multiply both coordinates of each point by -1.	90° rotation: $(x, y) \rightarrow (-y, x)$ 180° rotation: $(x, y) \rightarrow (-x, -y)$	

Study Tip

Reading Math
The *vertices* of a polygon are the endpoints of the angles.

Example 2 *Reflection*

A parallelogram has vertices $A(-4, 3)$, $B(1, 3)$, $C(0, 1)$, and $D(-5, 1)$.

a. Parallelogram $ABCD$ is reflected over the x-axis. Find the coordinates of the vertices of the image.

To reflect the figure over the x-axis, multiply each y-coordinate by -1.

$$(x, y) \rightarrow (x, -y) \qquad (x, y) \rightarrow (x, -y)$$
$$A(-4, 3) \rightarrow A'(-4, -3) \qquad C(0, 1) \rightarrow C'(0, -1)$$
$$B(1, 3) \rightarrow B'(1, -3) \qquad D(-5, 1) \rightarrow D'(-5, -1)$$

The coordinates of the vertices of the image are $A'(-4, -3)$, $B'(1, -3)$, $C'(0, -1)$, and $D'(-5, -1)$.

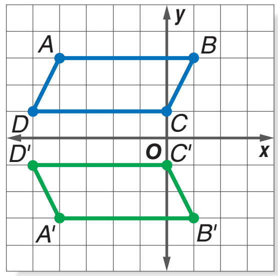
Study Tip

Reading Math
Parallelogram $ABCD$ and its image $A'B'C'D'$ are said to be **symmetric** about the x-axis. The x-axis is called the **line of symmetry**.

b. Graph parallelogram $ABCD$ and its image $A'B'C'D'$.

Graph each vertex of the parallelogram $ABCD$. Connect the points.

Graph each vertex of the reflected image $A'B'C'D'$. Connect the points.



Example 3 Translation

Triangle ABC has vertices $A(-2, 3)$, $B(4, 0)$, and $C(2, -5)$.

a. Find the coordinates of the vertices of the image if it is translated 3 units to the left and 2 units down.

To translate the triangle 3 units to the left, add -3 to the x-coordinate of each vertex. To translate the triangle 2 units down, add -2 to the y-coordinate of each vertex.

$$(x, y) \rightarrow (x - 3, y - 2)$$

$$A(-2, 3) \rightarrow A'(-2 - 3, 3 - 2) \rightarrow A'(-5, 1)$$

$$B(4, 0) \rightarrow B'(4 - 3, 0 - 2) \rightarrow B'(1, -2)$$

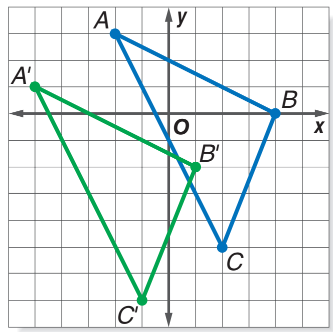
$$C(2, -5) \rightarrow C'(2 - 3, -5 - 2) \rightarrow C'(-1, -7)$$

The coordinates of the vertices of the image are $A'(-5, 1)$, $B'(1, -2)$, and $C'(-1, -7)$.

b. Graph triangle ABC and its image.

The preimage is $\triangle ABC$.

The translated image is $\triangle A'B'C'$.



Example 4 Dilation

A trapezoid has vertices $L(-4, 1)$, $M(1, 4)$, $N(7, 0)$, and $P(-3, -6)$.

a. Find the coordinates of the dilated trapezoid $L'M'N'P'$ if the scale factor is $\frac{3}{4}$.

To dilate the figure multiply the coordinates of each vertex by $\frac{3}{4}$.

$$(x, y) \rightarrow \left(\frac{3}{4}x, \frac{3}{4}y\right)$$

$$L(-4, 1) \rightarrow L'\left(\frac{3}{4} \cdot (-4), \frac{3}{4} \cdot 1\right) \rightarrow L'\left(-3, \frac{3}{4}\right)$$

$$M(1, 4) \rightarrow M'\left(\frac{3}{4} \cdot 1, \frac{3}{4} \cdot 4\right) \rightarrow M'\left(\frac{3}{4}, 3\right)$$

$$N(7, 0) \rightarrow N'\left(\frac{3}{4} \cdot 7, \frac{3}{4} \cdot 0\right) \rightarrow N'\left(5\frac{1}{4}, 0\right)$$

$$P(-3, -6) \rightarrow P'\left(\frac{3}{4} \cdot (-3), \frac{3}{4} \cdot (-6)\right) \rightarrow P'\left(-2\frac{1}{4}, -4\frac{1}{2}\right)$$

The coordinates of the vertices of the image are $L'\left(-3, \frac{3}{4}\right)$, $M'\left(\frac{3}{4}, 3\right)$, $N'\left(5\frac{1}{4}, 0\right)$, and $P'\left(-2\frac{1}{4}, -4\frac{1}{2}\right)$.

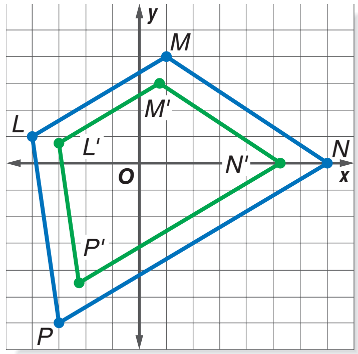
(continued on the next page)

b. Graph the preimage and its image.

The preimage is trapezoid *LMNP*.

The image is trapezoid $L'M'N'P'$.

Notice that the image has sides that are three-fourths the length of the sides of the original figure.



Example 5 Rotation

Triangle *XYZ* has vertices *X*(1, 5), *Y*(5, 2), and *Z*(−1, 2).

a. Find the coordinates of the image of $\triangle XYZ$ after it is rotated 90° counterclockwise about the origin.

To find the coordinates of the vertices after a 90° rotation, switch the coordinates of each point and then multiply the new first coordinate by −1.

$(x, y) \rightarrow (-y, x)$

$X(1, 5) \rightarrow X'(-5, 1)$

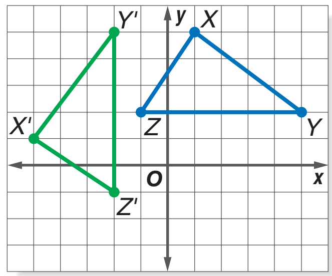
$Y(5, 2) \rightarrow Y'(-2, 5)$

$Z(-1, 2) \rightarrow Z'(-2, -1)$

b. Graph the preimage and its image.

The image is $\triangle XYZ$.

The rotated image is $\triangle X'Y'Z'$.



Check for Understanding

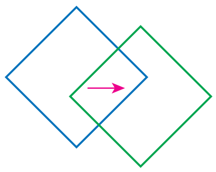
Concept Check

1. **Compare and contrast** the size, shape, and orientation of a preimage and an image for each type of transformation.
2. **OPEN ENDED** Draw a figure on the coordinate plane. Then show a dilation of the object that is an enlargement and a dilation of the object that is a reduction.

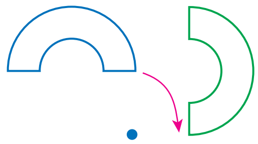
Guided Practice

Identify each transformation as a *reflection, translation, dilation,* or *rotation*.

3.



4.



Find the coordinates of the vertices of each figure after the given transformation is performed. Then graph the preimage and its image.

5. triangle *PQR* with *P*(1, 2), *Q*(4, 4), and *R*(2, −3) reflected over the *x*-axis
6. quadrilateral *ABCD* with *A*(4, 2), *B*(4, −2), *C*(−1, −3), and *D*(−3, 2) translated 3 units up
7. parallelogram *EFGH* with *E*(−1, 4), *F*(5, −1), *G*(2, −4), and *H*(−4, 1) dilated by a scale factor of 2
8. triangle *JKL* with *J*(0, 0), *K*(−2, −5), and *L*(−4, 5) rotated 90° counterclockwise about the origin

Application **NAVIGATION** **For Exercises 9 and 10, use the following information.** A ship was heading on a chartered route when it was blown off course by a storm. The ship is now ten miles west and seven miles south of its original destination.

9. Using a coordinate grid, make a drawing to show the original destination *A* and the current position *B* of the ship.

10. Using coordinates (x, y) to represent the original destination of the ship, write an ordered pair to show its current location.

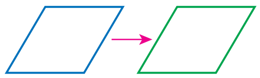
Practice and Apply

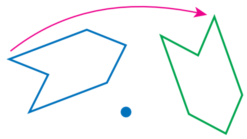
Homework Help

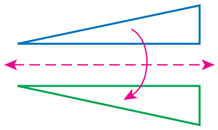
For Exercises	See Examples
11–16, 37, 38	1
17–36	2–5

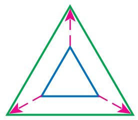
Extra Practice
See page 828.

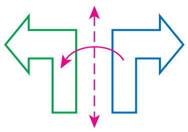
Identify each transformation as a *reflection, translation, dilation,* or *rotation.*

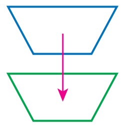
11. 

12. 

13. 

14. 

15. 

16. 

For Exercises 17–26, complete parts a and b.

a. **Find the coordinates of the vertices of each figure after the given transformation is performed.**

b. **Graph the preimage and its image.**

17. triangle *RST* with $R(2, 0)$, $S(-2, -3)$, and $T(-2, 3)$ reflected over the y-axis

18. trapezoid *ABCD* with $A(2, 3)$, $B(5, 3)$, $C(6, 1)$, and $D(-2, 1)$ reflected over the x-axis

19. quadrilateral *RSTU* with $R(-6, 3)$, $S(-4, 2)$, $T(-1, 5)$, and $U(-3, 7)$ translated 8 units right

20. parallelogram *MNOP* with $M(-6, 0)$, $N(-4, 3)$, $O(-1, 3)$, and $P(-3, 0)$ translated 3 units right and 2 units down

21. trapezoid *JKLM* with $J(-4, 2)$, $K(-2, 4)$, $L(4, 4)$, and $M(-4, -4)$ dilated by a scale factor of $\frac{1}{2}$

22. square *ABCD* with $A(-2, 1)$, $B(2, 2)$, $C(3, -2)$, and $D(-1, -3)$ dilated by a scale factor of 3

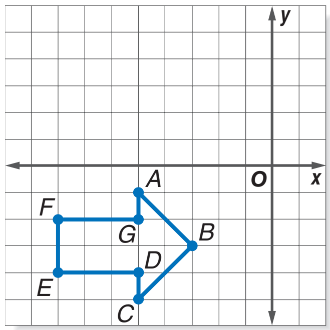
23. triangle *FGH* with $F(-3, 2)$, $G(2, 5)$, and $H(6, 3)$ rotated 180° about the origin

24. quadrilateral *TUVW* with $T(-4, 2)$, $U(-2, 4)$, $V(0, 2)$, and $W(-2, -4)$ rotated 90° counterclockwise about the origin

25. parallelogram *WXYZ* with $W(-1, 2)$, $X(3, 2)$, $Y(0, -4)$, and $Z(-4, -4)$ reflected over the y-axis, then rotated 180° about the origin

26. pentagon *PQRST* with $P(0, 5)$, $Q(3, 4)$, $R(2, 1)$, $S(-2, 1)$, and $T(-3, 4)$ reflected over the x-axis, then translated 2 units left and 1 unit up

ANIMATION **For Exercises 27–29, use the diagram at the right.**
An animator places an arrow representing an airplane on a coordinate grid. She wants to move the arrow 2 units right and then reflect it across the x-axis.



27. Write the coordinates for the vertices of the arrow.

28. Find the coordinates of the final position of the arrow.

29. Graph the image.

30. Trapezoid $JKLM$ with $J(-6, 0)$, $K(-1, 5)$, $L(-1, 1)$, and $M(-3, -1)$ is translated to $J'K'L'M'$ with $J'(-3, -2)$, $K'(2, 3)$, $L'(2, -1)$, $M'(0, -3)$. Describe this translation.

31. Triangle QRS with vertices $Q(-2, 6)$, $R(8, 0)$, and $S(6, 4)$ is dilated. If the image $Q'R'S'$ has vertices $Q'(-1, 3)$, $R'(4, 0)$, and $S'(3, 2)$, what is the scale factor?

32. Describe the transformation of parallelogram $WXYZ$ with $W(-5, 3)$, $X(-2, 5)$, $Y(0, 3)$, and $Z(-3, 1)$ if the coordinates of its image are $W'(5, 3)$, $X'(2, 5)$, $Y'(0, 3)$, and $Z'(3, 1)$.

33. Describe the transformation of triangle XYZ with $X(2, -1)$, $Y(-5, 3)$, and $Z(4, 0)$ if the coordinates of its image are $X'(1, 2)$, $Y'(-3, -5)$, and $Z'(0, 4)$.

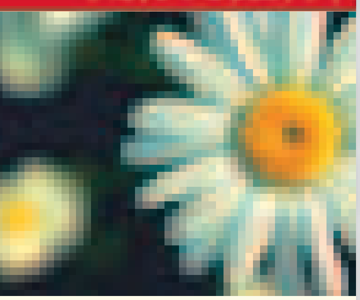
DIGITAL PHOTOGRAPHY **For Exercises 34–36, use the following information.**
Soto wants to enlarge a digital photograph that is 1800 pixels wide and 1600 pixels high (1800×1600) by a scale factor of $2\frac{1}{2}$.

34. What will be the dimensions of the new digital photograph?

35. Use a coordinate grid to draw a picture representing the 1800×1600 digital photograph. Place one corner of the photograph at the origin and write the coordinates of the other three vertices.

36. Draw the enlarged photograph and write its coordinates.

More About...

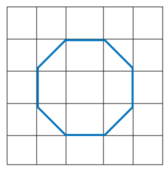


Digital Photography

Digital photographs contain hundreds of thousands or millions of tiny squares called pixels.

Source: www.shortcourses.com

ART **For Exercises 37 and 38, use the following information.**
On grid paper, draw an octagon like the one shown.



37. Reflect the octagon over each of its sides. Describe the pattern that results.

38. Could this same pattern be drawn using any of the other transformations? If so, which kind?

39. **CRITICAL THINKING** Make a conjecture about the coordinates of a point (x, y) that has been rotated 90° *clockwise* about the origin.

40. **CRITICAL THINKING** Determine whether the following statement is *sometimes*, *always*, or *never* true.

A reflection over the x-axis followed by a reflection over the y-axis gives the same result as a rotation of 180°.

41. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How are transformations used in computer graphics?

Include the following in your answer:

- examples of movements that could be simulated by transformations, and
- types of other industries that might use transformations in computer graphics to simulate movement.

Standardized Test Practice

42. The coordinates of the vertices of quadrilateral $QRST$ are $Q(-2, 4)$, $R(3, 7)$, $S(4, -2)$, and $T(-5, -3)$. If the quadrilateral is moved up 3 units and right 1 unit, which point below has the correct coordinates?

(A) $Q'(1, 5)$ (B) $R'(4, 4)$ (C) $S'(5, 1)$ (D) $T'(-6, 0)$

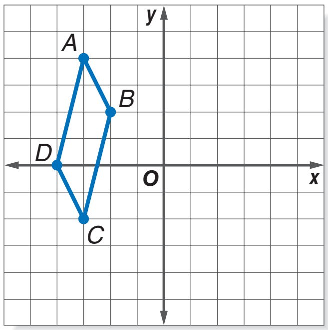
43. x is $\frac{2}{3}$ of y and y is $\frac{1}{4}$ of z. If $x = 14$, then $z =$

(A) 48. (B) 72. (C) 84. (D) 96.

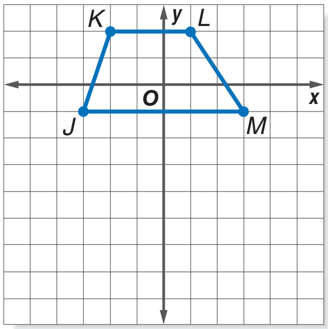
Extending the Lesson

Graph the image of each figure after a reflection over the graph of the given equation. Find the coordinates of the vertices.

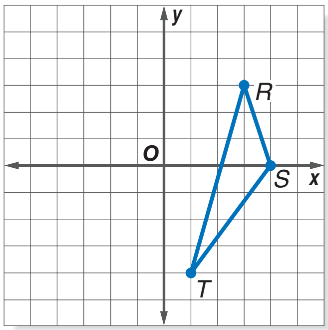
44. $x = 0$



45. $y = -3$



46. $y = x$



Maintain Your Skills

Mixed Review

Plot each point on a coordinate plane. *(Lesson 4-1)*

47. $A(2, -1)$
48. $B(-4, 0)$
49. $C(1, 5)$
50. $D(-1, -1)$
51. $E(-2, 3)$
52. $F(4, -3)$

53. **CHEMISTRY** Jamaal needs a 25% solution of nitric acid. He has 20 milliliters of a 30% solution. How many milliliters of a 15% solution should he add to obtain the required 25% solution? *(Lesson 3-9)*

Two dice are rolled and their sum is recorded. Find each probability. *(Lesson 2-6)*

54. P(sum is less than 9)
55. P(sum is greater than 10)
56. P(sum is less than 7)
57. P(sum is greater than 4)

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Write a set of ordered pairs that represents the data in the table.** *(To review **ordered pairs**, see Lesson 1-8.)*

58.

Number of toppings	1	2	3	4	5	6
Cost of large pizza ($)	9.95	11.45	12.95	14.45	15.95	17.45

59.

Time (minutes)	0	5	10	15	20	25	30
Temperature of boiled water as it cools (°C)	100	90	81	73	66	60	55

Graphing Calculator Investigation

A Preview of Lesson 4-3

Graphs of Relations

You can represent a relation as a graph using a TI-83 Plus graphing calculator.

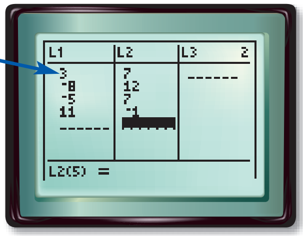
Graph the relation {(3, 7), (−8, 12), (−5, 7), (11, −1)}.

Step 1 *Enter the data.*

- Enter the x-coordinates in L1 and the y-coordinates in L2.

KEYSTROKES: STAT ENTER 3 ENTER −8 ENTER −5 ENTER 11 ENTER ▶ 7 ENTER 12 ENTER 7 ENTER −1 ENTER

The first ordered pair is (3, 7).



Step 2 *Format the graph.*

- Turn on the statistical plot.

KEYSTROKES: 2nd [STAT PLOT] ENTER ENTER

- Select the scatter plot, L1 as the Xlist and L2 as the Ylist.

KEYSTROKES: ▼ ENTER ▼ 2nd L1 ENTER 2nd L2 ENTER

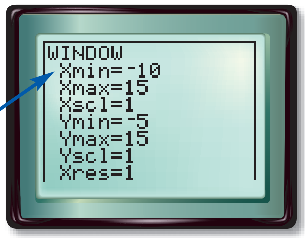
Step 3 *Choose the viewing window.*

- Be sure you can see all of the points.

[−10, 15] scl: 1 by [−5, 15] scl: 1

KEYSTROKES: WINDOW −10 ENTER 15 ENTER 1 ENTER −5 ENTER 15 ENTER 1

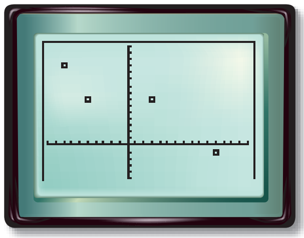
The x-axis will go from −10 to 15 with a tick mark at every unit.



Step 4 *Graph the relation.*

- Display the graph.

KEYSTROKES: GRAPH



[−10, 15] scl: 1 by [−5, 15] scl: 1

Exercises

Graph each relation. Sketch the result.

1. {(10, 10), (0, −6), (4, 7), (5, −2)}

2. {(−4, 1), (3, −5), (4, 5), (−5, 1)}

3. {(12, 15), (10, −16), (11, 7), (−14, −19)}

4. {(45, 10), (23, 18), (22, 26), (35, 26)}

5. MAKE A CONJECTURE How are the values of the domain and range used to determine the scale of the viewing window?

4-3 Relations

What You'll Learn

- Represent relations as sets of ordered pairs, tables, mappings, and graphs.
- Find the inverse of a relation.

How can relations be used to represent baseball statistics?

Ken Griffey, Jr.'s, batting statistics for home runs and strikeouts can be represented as a set of ordered pairs. The number of home runs are the first coordinates, and the number of strikeouts are the second coordinates.

You can plot the ordered pairs on a graph to look for patterns.

Ken Griffey, Jr.		
Year	Home Runs	Strikeouts
1994	40	73
1995	17	53
1996	49	104
1997	56	121
1998	56	121
1999	48	108
2000	40	117
2001	22	72

Vocabulary

- mapping
- inverse

REPRESENT RELATIONS Recall that a *relation* is a set of ordered pairs. A relation can be represented by a set of ordered pairs, a table, a graph, or a **mapping**. A mapping illustrates how each element of the domain is paired with an element in the range. Study the different representations of the same relation below.

Study Tip

Look Back

To review **relations**, see Lesson 1-8.

Ordered Pairs

(1, 2)

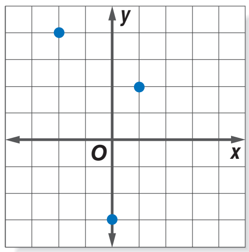
(−2, 4)

(0, −3)

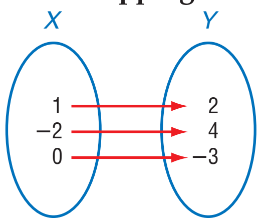
Table

x	y
1	2
−2	4
0	−3

Graph



Mapping



Example 1 Represent a Relation

a. **Express the relation {(3, 2), (−1, 4), (0, −3), (−3, 4), (−2, −2)} as a table, a graph, and a mapping.**

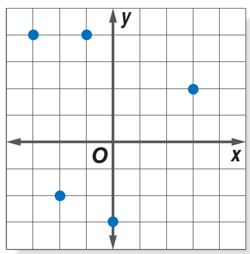
Table

List the set of x-coordinates in the first column and the corresponding y-coordinates in the second column.

x	y
3	2
−1	4
0	−3
−3	4
−2	−2

Graph

Graph each ordered pair on a coordinate plane.



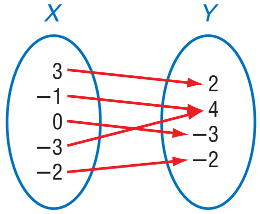
(continued on the next page)

Study Tip

Domain and Range
When writing the elements of the domain and range, if a value is repeated, you need to list it only once.

Mapping

List the x values in set X and the y values in set Y. Draw an arrow from each x value in X to the corresponding y value in Y.



b. Determine the domain and range.
The domain for this relation is $\{-3, -2, -1, 0, 3\}$.
The range is $\{-3, -2, 2, 4\}$.

When graphing relations that represent real-life situations, you may need to select values for the x- or y-axis that do not begin with 0 and do not have units of 1.

Example 2 Use a Relation

BALD EAGLES **In 1990, New York purchased 12,000 acres for the protection of bald eagles. The table shows the number of eagles observed in New York during the annual mid-winter bald eagle survey from 1993 to 2000.**

More About. . .

Bald Eagles
The bald eagle is not really bald. Its name comes from the Old English meaning of bald, "having white feathers on the head."
Source: *Webster's Dictionary*

Bald Eagle Survey

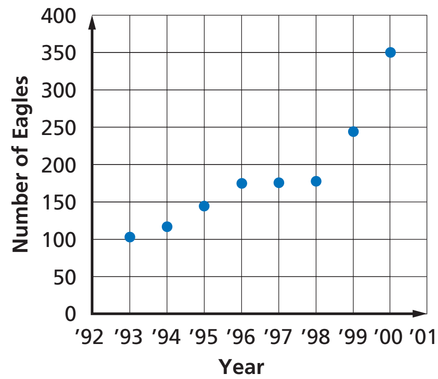
Year	1993	1994	1995	1996	1997	1998	1999	2000
Number of Eagles	102	116	144	174	175	177	244	350

Source: New York Department of Environmental Conservation

a. Determine the domain and range of the relation.
The domain is {1993, 1994, 1995, 1996, 1997, 1998, 1999, 2000}.
The range is {102, 116, 144, 174, 175, 177, 244, 350}.

b. Graph the data.

- The values of the x-axis need to go from 1993 to 2000. It is not practical to begin the scale at 0. Begin at 1992 and extend to 2001 to include all of the data. The units can be 1 unit per grid square.
- The values on the y-axis need to go from 102 to 350. In this case, it is possible to begin the scale at 0. Begin at 0 and extend to 400. You can use units of 50.



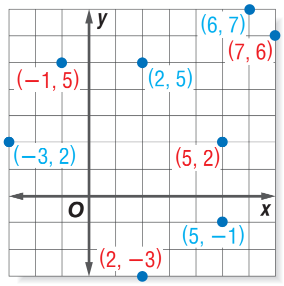
c. What conclusions might you make from the graph of the data?
The number of eagles has increased each year. This may be due to the efforts of those who are protecting the eagles in New York.

INVERSE RELATIONS The **inverse** of any relation is obtained by switching the coordinates in each ordered pair.

Key Concept *Inverse of a Relation*

Relation Q is the inverse of relation S if and only if for every ordered pair (a, b) in S, there is an ordered pair (b, a) in Q.

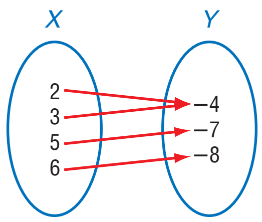
Relation	Inverse
(2, 5)	(5, 2)
(−3, 2)	(2, −3)
(6, 7)	(7, 6)
(5, −1)	(−1, 5)



Notice that the domain of a relation becomes the range of the inverse and the range of a relation becomes the domain of the inverse.

Example 3 Inverse Relation

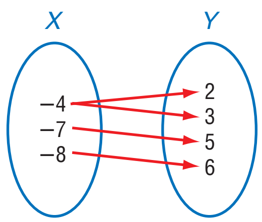
Express the relation shown in the mapping as a set of ordered pairs. Then write the inverse of the relation.



Relation Notice that both 2 and 3 in the domain are paired with −4 in the range.
{(2, −4), (3, −4), (5, −7), (6, −8)}

Inverse Exchange x and y in each ordered pair to write the inverse relation.
{(−4, 2), (−4, 3), (−7, 5), (−8, 6)}

The mapping of the inverse is shown at the right. Compare this to the mapping of the relation.



Algebra Activity

Relations and Inverses

- Graph the relation {(3, 4), (−2, 5), (−4, −3), (5, −6), (−1, 0), (0, 2)} on grid paper using a colored pencil. Connect the points in order using the same colored pencil.
- Use a different colored pencil to graph the inverse of the relation, connecting the points in order.
- Fold the grid paper through the origin so that the positive y-axis lies on top of the positive x-axis. Hold the paper up to a light so that you can see all of the points you graphed.

Analyze

1. What do you notice about the location of the points you graphed when you looked at the folded paper?

2. Unfold the paper. Describe the transformation of each point and its inverse.

3. What do you think are the ordered pairs that represent the points on the fold line? Describe these in terms of x and y.

Make a Conjecture

4. How could you graph the inverse of a function without writing ordered pairs first?

Check for Understanding

Concept Check

1. **Describe** the different ways a relation can be represented.
2. **OPEN ENDED** Give an example of a set of ordered pairs that has five elements in its domain and four elements in its range.
3. **State** the relationship between the domain and range of a relation and the domain and range of its inverse.

Guided Practice

Express each relation as a table, a graph, and a mapping. Then determine the domain and range.

4. $\{(5, -2), (8, 3), (-7, 1)\}$
5. $\{(6, 4), (3, -3), (-1, 9), (5, -3)\}$
6. $\{(7, 1), (3, 0), (-2, 5)\}$
7. $\{(-4, 8), (-1, 9), (-4, 7), (6, 9)\}$

Express the relation shown in each table, mapping, or graph as a set of ordered pairs. Then write the inverse of the relation.

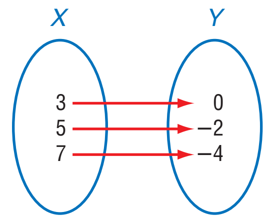
8.

x	y
3	−2
−6	7
4	3
−6	5

9.

x	y
−4	9
2	5
−2	−2
11	12

10.



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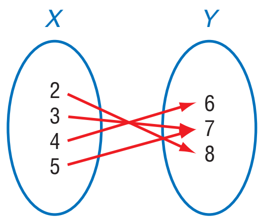

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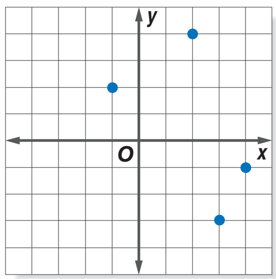

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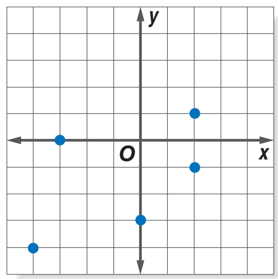
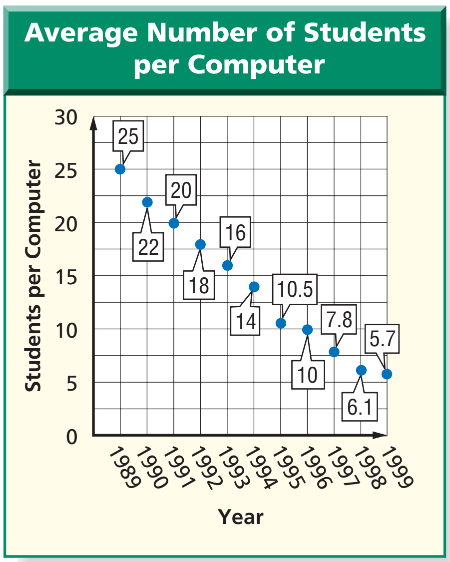


Application

TECHNOLOGY **For Exercises 14–17, use the graph of the average number of students per computer in U.S. public schools.**

14. Name three ordered pairs from the graph.
15. Determine the domain of the relation.
16. What are the least value and the greatest value in the range?
17. What conclusions can you make from the graph of the data?



Source: Quality Education Data

Online Research **Data Update** What is the average number of students per computer in your state? Visit www.algebra1.com/data_update to learn more.

Practice and Apply

Homework Help

For Exercises	See Examples
18–25	1
26–37	3
38–48	2

Extra Practice
See page 829.

Express each relation as a table, a graph, and a mapping. Then determine the domain and range.

18. {(4, 3), (1, −7), (1, 3), (2, 9)}

19. {(5, 2), (−5, 0), (6, 4), (2, 7)}

20. {(0, 0), (6, −1), (5, 6), (4, 2)}

21. {(3, 8), (3, 7), (2, −9), (1, −9)}

22. {(4, −2), (3, 4), (1, −2), (6, 4)}

23. {(0, 2), (−5, 1), (0, 6), (−1, 9)}

24. {(3, 4), (4, 3), (2, 2), (5, −4), (−4, 5)}

25. {(7, 6), (3, 4), (4, 5), (−2, 6), (−3, 2)}

Express the relation shown in each table, mapping, or graph as a set of ordered pairs. Then write the inverse of the relation.

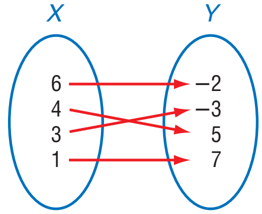
26.

x	y
1	2
3	4
5	6
7	8

27.

x	y
0	3
−5	2
4	7
−3	2

28.



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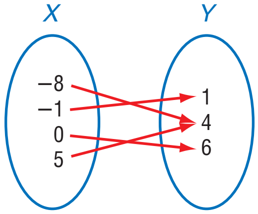

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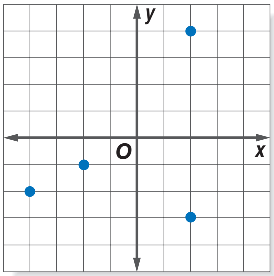

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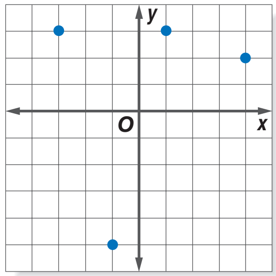


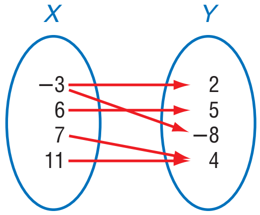
32.

x	y
0	0
4	7
8	10.5
12	13
16	14.5

33.

x	y
1	16.50
1.75	28.30
2.5	49.10
3.25	87.60
4	103.40

34.



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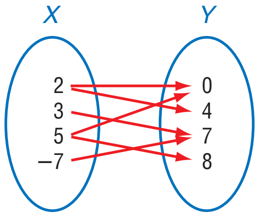

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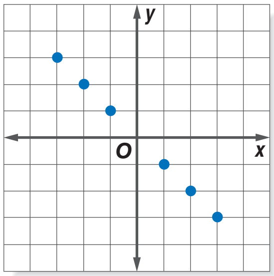

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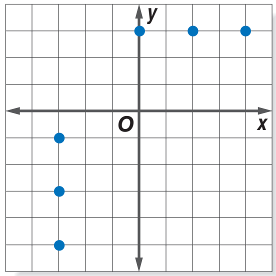


COOKING **For Exercises 38–40, use the table that shows the boiling point of water at various altitudes.** Many recipes have different cooking times for high altitudes. This is due to the fact that water boils at a lower temperature in higher altitudes.

38. Graph the relation.

39. Write the inverse as a set of ordered pairs.

40. How could you estimate your altitude by finding the boiling point of water at your location?

Altitude (feet)	Boiling Point of Water (°F)
0	212.0
1000	210.2
2000	208.4
3000	206.5
5000	201.9
10,000	193.7

Source: Stevens Institute of Technology

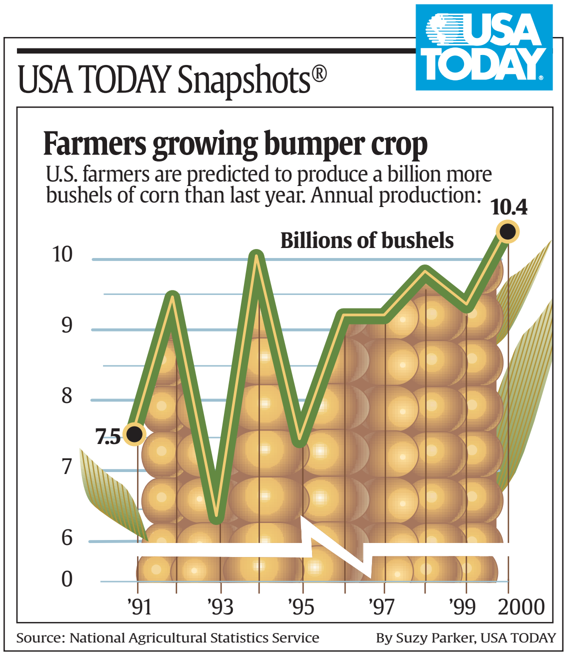
www.algebra1.com/self_check_quiz

FOOD **For Exercises 41–43, use the graph that shows the annual production of corn from 1991–2000.**

41. Estimate the domain and range of the relation.

42. Which year had the lowest production? the highest?

43. Describe any pattern you see.



HEALTH **For Exercises 44–48, use the following information.**
A person's muscle weight is about 2 pounds of muscle for each 5 pounds of body weight.

44. Make a table to show the relation between body and muscle weight for people weighing 100, 105, 110, 115, 120, 125, and 130 pounds.

45. What are the domain and range?

46. Graph the relation.

47. What are the domain and range of the inverse?

48. Graph the inverse relation.

49. **CRITICAL THINKING** Find a counterexample to disprove the following.
The domain of relation F contains the same elements as the range of relation G. The range of relation F contains the same elements as the domain of relation G. Therefore, relation G must be the inverse of relation F.

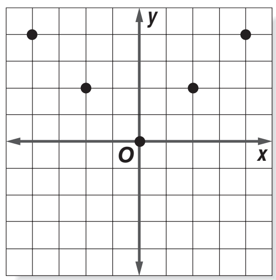
50. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can relations be used to represent baseball statistics?

Include the following in your answer:
- a graph of the relation of the number of Ken Griffey, Jr.'s, home runs and his strikeouts, and
- an explanation of any relationship between the number of home runs hit and the number of strikeouts.

Standardized Test Practice

For Exercises 51 and 52, use the graph at the right.



51. State the domain and range of the relation.

(A) D = {0, 2, 4}; R = {−4, −2, 0, 2, 4}

(B) D = {−4, −2, 0, 2, 4}; R = {0, 2, 4}

(C) D = {0, 2, 4}; R = {−4, −2, 0}

(D) D = {−4, −2, 0, 2, 4}; R = {−4, −2, 0, 2, 4}

52. **SHORT RESPONSE** Graph the inverse of the relation.

Graphing Calculator

For Exercises 53–56, use a graphing calculator.
a. Graph each relation.
b. State the WINDOW settings that you used.
c. Write the coordinates of the inverse. Then graph the inverse.
d. Name the quadrant in which each point of the relation and its inverse lies.

53. {(0, 10), (2, −8), (6, 6), (9, −4)}

54. {(−1, 18), (−2, 23), (−3, 28), (−4, 33)}

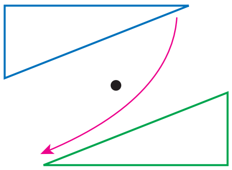
55. {(35, 12), (48, 25), (60, 52)}

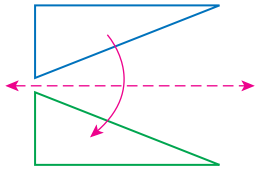
56. {(−92, −77), (−93, 200), (19, −50)}

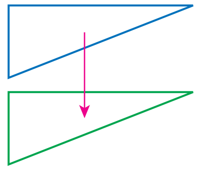
Maintain Your Skills

Mixed Review

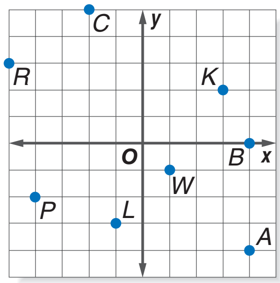
Identify each transformation as a *reflection*, *translation*, *dilation*, or *rotation*. *(Lesson 4-2)*

57. 

58. 

59. 

Write the ordered pair for each point shown at the right. Name the quadrant in which the point is located. *(Lesson 4-1)*



60. A **61.** K

62. L **63.** W

64. B **65.** P

66. R **67.** C

68. HOURLY PAY Dominique earns \$9.75 per hour. Her employer is increasing her hourly rate to \$10.15 per hour. What is the percent of increase in her salary? *(Lesson 3-7)*

Simplify each expression. *(Lesson 2-4)*

69. $72 \div 9$ **70.** $105 \div 15$

71. $3 \div \frac{1}{3}$ **72.** $16 \div \frac{1}{4}$

73. $\frac{54n + 78}{6}$ **74.** $\frac{98x - 35y}{7}$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find the solution set for each equation if the replacement set is {3, 4, 5, 6, 7, 8}.** *(To review **solution sets**, see Lesson 1-3.)*

75. $a + 15 = 20$ **76.** $r - 6 = 2$

77. $9 = 5n - 6$ **78.** $3 + 8w = 35$

79. $\frac{g}{3} + 15 = 17$ **80.** $\frac{m}{5} + \frac{3}{5} = 2$

Practice Quiz 1 — Lessons 4-1 through 4-3

Plot each point on a coordinate plane. *(Lesson 4-1)*

1. $Q(2, 3)$ **2.** $R(-4, -4)$ **3.** $S(5, -1)$ **4.** $T(-1, 3)$

Find the coordinates of the vertices of each figure after the given transformation is performed. Then graph the preimage and its image. *(Lesson 4-2)*

5. triangle ABC with $A(4, 8)$, $B(7, 5)$, and $C(2, -1)$ reflected over the x-axis

6. quadrilateral $WXYZ$ with $W(1, 0)$, $X(2, 3)$, $Y(4, 1)$, and $Z(3, -3)$ translated 5 units to the left and 4 units down

State the domain, range, and inverse of each relation. *(Lesson 4-3)*

7. {(1, 3), (4, 6), (2, 3), (1, 5)} **8.** {(−2, 6), (0, 3), (4, 2), (8, −5)}

9. {(11, 5), (15, 3), (−8, 22), (11, 31)} **10.** {(−5, 8), (−1, 0), (−1, 4), (2, 7), (6, 3)}

4-4 Equations as Relations

What You'll Learn

- Use an equation to determine the range for a given domain.
- Graph the solution set for a given domain.

Vocabulary

- equation in two variables
- solution

Why are equations of relations important in traveling?

During the summer, Eric will be taking a trip to England. He has saved \$500 for his trip, and he wants to find how much that will be worth in British pounds sterling. The exchange rate today is 1 dollar $= 0.69$ pound. Eric can use the equation $p = 0.69d$ to convert dollars d to pounds p.

SOLVE EQUATIONS The equation $p = 0.69d$ is an example of an **equation in two variables**. A **solution** of an equation in two variables is an ordered pair that results in a true statement when substituted into the equation.

Example 1 Solve Using a Replacement Set

Find the solution set for $y = 2x + 3$, given the replacement set $\{(-2, -1), (-1, 3), (0, 4), (3, 9)\}$.

Make a table. Substitute each ordered pair into the equation.

x	y	$y = 2x + 3$	True or False?
−2	−1	$-1 = 2(-2) + 3$ $-1 = -1$	true ✓
−1	3	$3 = 2(-1) + 3$ $3 = 1$	false
0	4	$4 = 2(0) + 3$ $4 = 3$	false
3	9	$9 = 2(3) + 3$ $9 = 9$	true ✓

The ordered pairs $(-2, -1)$ and $(3, 9)$ result in true statements. The solution set is $\{(-2, -1), (3, 9)\}$.

Since the solutions of an equation in two variables are ordered pairs, the equation describes a relation. So, in an equation involving x and y, the set of x values is the domain, and the corresponding set of y values is the range.

Study Tip

Variables
Unless the variables are chosen to represent real quantities, when variables other than x and y are used in an equation, assume that the letter that comes first in the alphabet is the domain.

Example 2 Solve Using a Given Domain

Solve $b = a + 5$ if the domain is $\{-3, -1, 0, 2, 4\}$.

Make a table. The values of a come from the domain. Substitute each value of a into the equation to determine the values of b in the range.

a	$a + 5$	b	(a, b)
−3	$-3 + 5$	2	(−3, 2)
−1	$-1 + 5$	4	(−1, 4)
0	$0 + 5$	5	(0, 5)
2	$2 + 5$	7	(2, 7)
4	$4 + 5$	9	(4, 9)

The solution set is $\{(-3, 2), (-1, 4), (0, 5), (2, 7), (4, 9)\}$.

GRAPH SOLUTION SETS You can graph the ordered pairs in the solution set for an equation in two variables. The domain contains values represented by the *independent variable*. The range contains the corresponding value represented by the *dependent variable*.

Study Tip

Look Back
To review **independent and dependent variables**, see Lesson 1-8.

Example 3 Solve and Graph the Solution Set

Solve $4x + 2y = 10$ if the domain is $\{-1, 0, 2, 4\}$. Graph the solution set.

First solve the equation for y in terms of x. This makes creating a table of values easier.

$$4x + 2y = 10 \quad \text{Original equation}$$

$$4x + 2y - 4x = 10 - 4x \quad \text{Subtract } 4x \text{ from each side.}$$

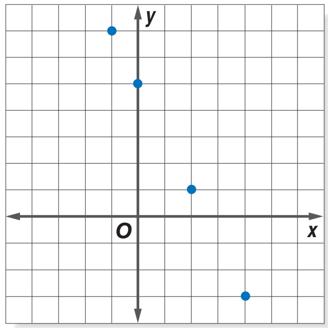
$$2y = 10 - 4x \quad \text{Simplify.}$$

$$\frac{2y}{2} = \frac{10 - 4x}{2} \quad \text{Divide each side by 2.}$$

$$y = 5 - 2x \quad \text{Simplify.}$$

Substitute each value of x from the domain to determine the corresponding values of y in the range.

x	$5 - 2x$	y	(x, y)
−1	5 − 2(−1)	7	(−1, 7)
0	5 − 2(0)	5	(0, 5)
2	5 − 2(2)	1	(2, 1)
4	5 − 2(4)	−3	(4, −3)

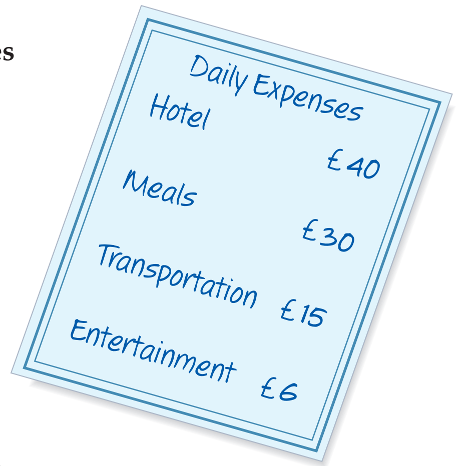


Graph the solution set $\{(-1, 7), (0, 5), (2, 1), (4, -3)\}$.

When you solve an equation for a given variable, that variable becomes the dependent variable. That is, its value depends upon the domain values chosen for the other variable.

Example 4 Solve for a Dependent Variable

Refer to the application at the beginning of the lesson. Eric has made a list of the expenses he plans to incur while in England. Use the conversion rate to find the equivalent U.S. dollars for these amounts given in pounds (£) and graph the ordered pairs.



Explore In the equation $p = 0.69d$, d represents U.S. dollars and p represents British pounds. However, we are given values in pounds and want to find values in dollars. Solve the equation for d since the values of d depend on the given values of p.

$$p = 0.69d \quad \text{Original equation}$$

$$\frac{p}{0.69} = \frac{0.69d}{0.69} \quad \text{Divide each side by 0.69.}$$

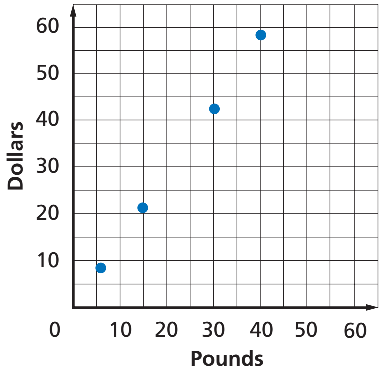
$$1.45p = d \quad \text{Simplify and round to the nearest hundredth.}$$

(continued on the next page)

Plan The values of p, {40, 30, 15, 6}, are the domain. Use the equation $d = 1.45p$ to find the values for the range.

Solve Make a table of values. Substitute each value of p from the domain to determine the corresponding values of d. Round to the nearest dollar.

p	$1.45p$	d	(p, d)
40	1.45(40)	58.00	(40, 58)
30	1.45(30)	43.50	(30, 44)
15	1.45(15)	21.75	(15, 22)
6	1.45(6)	8.70	(6, 9)



Graph the ordered pairs. Notice that the values for the independent variable p are graphed along the horizontal axis, and the values for dependent variable d are graphed along the vertical axis.

Examine Look at the values in the range. The cost in dollars is higher than the cost in pounds. Do the results make sense?

Expense	Pounds	Dollars
Hotel	40	58
Meals	30	43
Entertainment	15	22
Transportation	6	9

Check for Understanding

Concept Check

1. **Describe** how to find the domain of an equation if you are given the range.
2. **OPEN ENDED** Give an example of an equation in two variables and state two solutions for your equation.
3. **FIND THE ERROR** Malena says that (5, 1) is a solution of $y = 2x + 3$. Bryan says it is not a solution.

Malena	Bryan
$y = 2x + 3$	$y = 2x + 3$
$5 = 2(1) + 3$	$1 = 2(5) + 3$
$5 = 5$	$1 \neq 13$

Who is correct? Explain your reasoning.

Guided Practice

Find the solution set for each equation, given the replacement set.

4. $y = 3x + 4$; {(−1, 1), (2, 10), (3, 12), (7, 1)}
5. $2x - 5y = 1$; {(−7, −3), (7, 3), (2, 1), (−2, −1)}

Solve each equation if the domain is {−3, −1, 0, 2}.

6. $y = 2x - 1$
7. $y = 4 - x$
8. $2y + 2x = 12$
9. $3x + 2y = 13$

Solve each equation for the given domain. Graph the solution set.

10. $y = 3x$ for x = {−3, −2, −1, 0, 1, 2, 3}
11. $2y = x + 2$ for x = {−4, −2, 0, 2, 4}

Application

JEWELRY **For Exercises 12 and 13, use the following information.**
Since pure gold is very soft, other metals are often added to it to make an alloy that is stronger and more durable. The relative amount of gold in a piece of jewelry is measured in karats. The formula for the relationship is $g = \frac{25k}{6}$, where k is the number of karats and g is the percent of gold in the jewelry.

12. Find the percent of gold if the domain is {10, 14, 18, 24}. Make a table of values and graph the function.

13. How many karats are in a ring that is 50% gold?

Practice and Apply

Homework Help

For Exercises	See Examples
14–19	1
20–31	2
32–39	3
40–45	4

Extra Practice
See page 829.

Find the solution set for each equation, given the replacement set.

14. $y = 4x + 1$; {(2, −1), (1, 5), (9, 2), (0, 1)}

15. $y = 8 - 3x$; {(4, −4), (8, 0), (2, 2), (3, 3)}

16. $x - 3y = -7$; {(−1, 2), (2, −1), (2, 4), (2, 3)}

17. $2x + 2y = 6$; {(3, 0), (2, 1), (−2, −1), (4, −1)}

18. $3x - 8y = -4$; {(0, 0.5), (4, 1), (2, 0.75), (2, 4)}

19. $2y + 4x = 8$; {(0, 2), (−3, 0.5), (0.25, 3.5), (1, 2)}

Solve each equation if the domain is {−2, −1, 1, 3, 4}.

20. $y = 4 - 5x$

21. $y = 2x + 3$

22. $x = y + 4$

23. $x = 7 - y$

24. $6x - 3y = 18$

25. $6x - y = -3$

26. $8x + 4y = 12$

27. $2x - 2y = 0$

28. $5x - 10y = 20$

29. $3x + 2y = 14$

30. $x + \frac{1}{2}y = 8$

31. $2x - \frac{1}{3}y = 4$

Solve each equation for the given domain. Graph the solution set.

32. $y = 2x + 3$ for $x = \{-3, -2, -1, 1, 2, 3\}$

33. $y = 3x - 1$ for $x = \{-5, -2, 1, 3, 4\}$

34. $3x - 2y = 5$ for $x = \{-3, -1, 2, 4, 5\}$

35. $5x + 4y = 8$ for $x = \{-4, -1, 0, 2, 4, 6\}$

36. $\frac{1}{2}x + y = 2$ for $x = \{-4, -1, 1, 4, 7, 8\}$

37. $y = \frac{1}{4}x - 3$ for $x = \{-4, -2, 0, 2, 4, 6\}$

38. The domain for $3x + y = 8$ is {−1, 2, 5, 8}. Find the range.

39. The range for $2y - x = 6$ is {−4, −3, 1, 6, 7}. Find the domain.

TRAVEL **For Exercises 40 and 41, use the following information.**
Heinrich and his brother live in Germany. They are taking a trip to the United States. They are unfamiliar with the Fahrenheit scale, so they would like to convert U.S. temperatures to Celsius. The equation $F = 1.8C + 32$ relates the temperature in degrees Celsius C to degrees Fahrenheit F.

City	Temperature (°F)
New York	34
Chicago	23
San Francisco	55
Miami	72
Washington, D.C.	40

40. Solve the equation for C.

41. Find the temperatures in degrees Celsius for each city.

GEOMETRY **For Exercises 42–44, use the following information.**
The equation for the perimeter of a rectangle is $P = 2\ell + 2w$. Suppose the perimeter of rectangle $ABCD$ is 24 centimeters.

42. Solve the equation for ℓ.

43. State the independent and dependent variables.

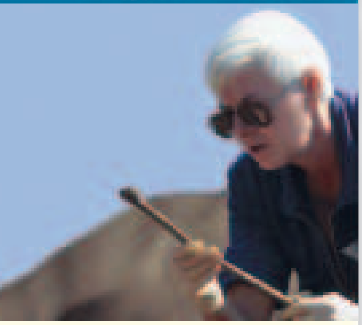
44. Choose five values for w and find the corresponding values of ℓ.

45. **ANTHROPOLOGY** When the remains of ancient people are discovered, usually only a few bones are found. Anthropologists can determine a person's height by using a formula that relates the length of the tibia T (shin bone) to the person's height H, both measured in centimeters. The formula for males is $H = 81.7 + 2.4T$ and for females is $H = 72.6 + 2.5T$. Copy and complete the tables below. Then graph each set of ordered pairs.

Male		
Length of Tibia (cm)	Height (cm)	(T, H)
30.5		
34.8		
36.3		
37.9		

Female		
Length of Tibia (cm)	Height (cm)	(T, H)
30.5		
34.8		
36.3		
37.9		

Career Choices



Forensic Anthropologist

Forensic anthropologists assist police investigations. They can determine the age and stature of a victim by examining dental wear on the teeth and measuring certain bones.

Online Research
For information about a career as a forensic anthropologist, visit: www.algebra1.com/careers

46. **RESEARCH** Choose a country that you would like to visit. Use the Internet or other reference to find the cost of various services such as hotels, meals, and transportation. Use the currency exchange rate to determine how much money in U.S. dollars you will need on your trip.

47. **CRITICAL THINKING** Find the domain values of each relation if the range is $\{0, 16, 36\}$.

a. $y = x^2$ **b.** $y = |4x| - 16$ **c.** $y = |4x - 16|$

48. **CRITICAL THINKING** Select five values for the domain and find the range of $y = x + 4$. Then look at the range and domain of the inverse relation. Make a conjecture about the equation that represents the inverse relation.

49. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

Why are equations of relations important in traveling?

Include the following in your answer:

- an example of how you would keep track of how much you were spending in pounds and the equivalent amount in dollars, and
- an explanation of your spending power if the currency exchange rate is 0.90 pound compared to one U.S. dollar or 1.04 pounds compared to one dollar.

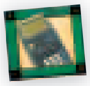
Standardized Test Practice

50. If $3x - y = 18$ and $y = 3$, then $x =$

(A) 4. (B) 5. (C) 6. (D) 7.

51. If the perimeter of a rectangle is 14 units and the area is 12 square units, what are the dimensions of the rectangle?

(A) 2×6 (B) 3×3
(C) 3×4 (D) 1×12



Graphing Calculator

TABLE FEATURE You can enter selected x values in the TABLE feature of a graphing calculator, and it will calculate the corresponding y values for a given equation. To do this, enter an equation into the Y= list. Go to TBLSET and highlight Ask under the Independent variable. Now you can use the TABLE function to enter any domain value and the corresponding range value will appear in the second column.

Use a graphing calculator to find the solution set for the given equation and domain.

52. $y = 3x - 4$; $x = \{-11, 15, 23, 44\}$

53. $y = -6.5x + 42$; $x = \{-8, -5, 0, 3, 7, 12\}$

54. $y = 3x + 12$ for $x = \{0.4, 0.6, 1.8, 2.2, 3.1\}$

55. $y = 1.4x - 0.76$ for $x = \{-2.5, -1.75, 0, 1.25, 3.33\}$

Maintain Your Skills

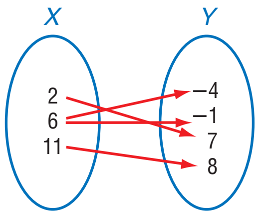
Mixed Review

Express the relation shown in each table, mapping, or graph as a set of ordered pairs. Then write the inverse of the relation. *(Lesson 4-3)*

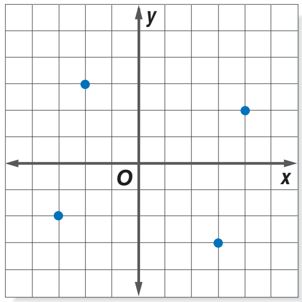
56.

x	y
4	9
3	−2
1	5
−4	2

57.



58.



Find the coordinates of the vertices of each figure after the given transformation is performed. Then graph the preimage and its image. *(Lesson 4-2)*

59. triangle XYZ with $X(-6, 4)$, $Y(-5, 0)$, and $Z(3, 3)$ reflected over the y-axis

60. quadrilateral $QRST$ with $Q(2, 2)$, $R(3, -3)$, $S(-1, -4)$ and $T(-4, -3)$ rotated 90° counterclockwise about the origin

Use cross products to determine whether each pair of ratios forms a proportion. Write *yes* or *no*. *(Lesson 3-6)*

61. $\frac{6}{15}, \frac{18}{45}$

62. $\frac{11}{12}, \frac{33}{34}$

63. $\frac{8}{22}, \frac{20}{55}$

64. $\frac{6}{8}, \frac{3}{4}$

65. $\frac{3}{5}, \frac{9}{25}$

66. $\frac{26}{35}, \frac{12}{15}$

Identify the hypothesis and conclusion of each statement. *(Lesson 1-7)*

67. If it is hot, then we will go swimming.

68. If you do your chores, then you get an allowance.

69. If $3n - 7 = 17$, then $n = 8$.

70. If $a > b$ and $b > c$, then $a > c$.

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Solve each equation.** *(To review **solving equations**, see Lesson 3-4.)*

71. $a + 15 = 20$

72. $r - 9 = 12$

73. $-4 = 5n + 6$

74. $3 - 8w = 35$

75. $\frac{g}{4} + 2 = 5$

76. $\frac{m}{5} + \frac{3}{5} = 2$

4-5 Graphing Linear Equations

What You'll Learn

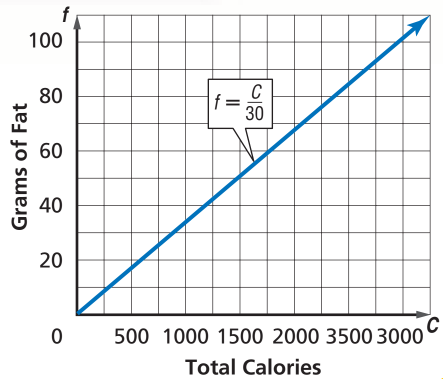
- Determine whether an equation is linear.
- Graph linear equations.

Vocabulary

- linear equation
- standard form
- *x*-intercept
- *y*-intercept

How can linear equations be used in nutrition?

Nutritionists recommend that no more than 30% of a person's daily caloric intake come from fat. Each gram of fat contains nine Calories. To determine the most grams of fat f you should have, find the total number of Calories C you consume each day and use the equation $f = 0.3\left(\frac{C}{9}\right)$ or $f = \left(\frac{C}{30}\right)$. The graph of this equation shows the maximum number of grams of fat you can consume based on the total number of Calories consumed.



IDENTIFY LINEAR EQUATIONS A **linear equation** is the equation of a line. Linear equations can often be written in the form $Ax + By = C$. This is called the **standard form** of a linear equation.

Key Concept — *Standard Form of a Linear Equation*

The standard form of a linear equation is

$$Ax + By = C,$$

where $A \geq 0$, A and B are not both zero, and A, B, and C are integers whose greatest common factor is 1.

Example 1 *Identify Linear Equations*

Determine whether each equation is a linear equation. If so, write the equation in standard form.

a. $y = 5 - 2x$

First rewrite the equation so that both variables are on the same side of the equation.

$y = 5 - 2x$	Original equation
$y + 2x = 5 - 2x + 2x$	Add $2x$ to each side.
$2x + y = 5$	Simplify.

The equation is now in standard form where $A = 2$, $B = 1$, and $C = 5$. This is a linear equation.

b. $2xy - 5y = 6$

Since the term $2xy$ has two variables, the equation cannot be written in the form $Ax + By = C$. Therefore, this is not a linear equation.

c. $3x + 9y = 15$

Since the GCF of 3, 9, and 15 is not 1, the equation is not written in standard form. Divide each side by the GCF.

$$3x + 9y = 15 \quad \text{Original equation}$$

$$3(x + 3y) = 15 \quad \text{Factor the GCF.}$$

$$\frac{3(x + 3y)}{3} = \frac{15}{3} \quad \text{Divide each side by 3.}$$

$$x + 3y = 5 \quad \text{Simplify.}$$

The equation is now in standard form where $A = 1$, $B = 3$, and $C = 5$.

d. $\frac{1}{3}y = -1$

To write the equation with integer coefficients, multiply each term by 3.

$$\frac{1}{3}y = -1 \quad \text{Original equation}$$

$$3\left(\frac{1}{3}\right)y = 3(-1) \quad \text{Multiply each side of the equation by 3.}$$

$$y = -3 \quad \text{Simplify.}$$

The equation $y = -3$ can be written as $0x + y = -3$. Therefore, it is a linear equation in standard form where $A = 0$, $B = 1$, and $C = -3$.

GRAPH LINEAR EQUATIONS

The graph of a linear equation is a continuous line. It extends beyond the endpoints in each direction and represents all the solutions of the linear equation. Also, every ordered pair on this line satisfies the equation.

Example 2 Graph by Making a Table

Graph $x + 2y = 6$.

In order to find values for y more easily, solve the equation for y.

$$x + 2y = 6 \quad \text{Original equation}$$

$$x + 2y - x = 6 - x \quad \text{Subtract } x \text{ from each side.}$$

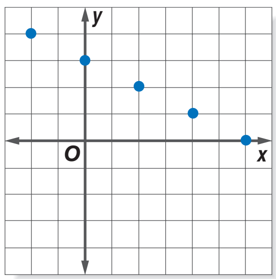
$$2y = 6 - x \quad \text{Simplify.}$$

$$\frac{2y}{2} = \frac{6 - x}{2} \quad \text{Divide each side by 2.}$$

$$y = 3 - \frac{1}{2}x \quad \text{Simplify.}$$

Select five values for the domain and make a table. Then graph the ordered pairs.

x	$3 - \frac{1}{2}x$	y	(x, y)
−2	$3 - \frac{1}{2}(-2)$	4	(−2, 4)
0	$3 - \frac{1}{2}(0)$	3	(0, 3)
2	$3 - \frac{1}{2}(2)$	2	(2, 2)
4	$3 - \frac{1}{2}(4)$	1	(4, 1)
6	$3 - \frac{1}{2}(6)$	0	(6, 0)

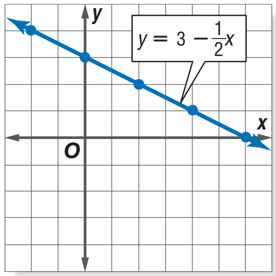


(continued on the next page)

Study Tip

Graphing Equations
When you graph an equation, use arrows at both ends to show that the graph continues. You should also label the graph with the equation.

When you graph the ordered pairs, a pattern begins to form. The domain of $y = 3 - \frac{1}{2}x$ is the set of all real numbers, so there are an infinite number of solutions of the equation. Draw a line through the points. This line represents all of the solutions of $y = 3 - \frac{1}{2}x$.



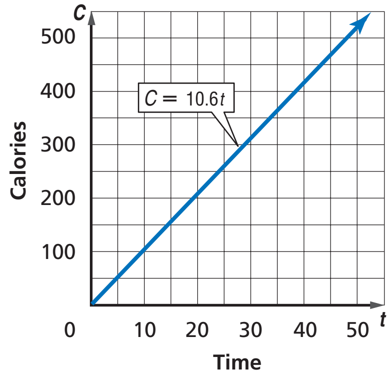
Example 3 Use the Graph of a Linear Equation

PHYSICAL FITNESS **Carlos swims every day. He burns approximately 10.6 Calories per minute when swimming laps.**

a. Graph the equation $C = 10.6t$, where C represents the number of Calories burned and t represents the time in minutes spent swimming.

Select five values for t and make a table. Graph the ordered pairs and connect them to draw a line.

t	$10.6t$	C	(t, C)
10	10.6(10)	106	(10, 106)
15	10.6(15)	159	(15, 159)
20	10.6(20)	212	(20, 212)
30	10.6(30)	318	(30, 318)



More About...

Physical Fitness
In a triathlon competition, athletes swim 1.5 kilometers, bicycle 40 kilometers, and run 10 kilometers.
Source: www.usatriathlon.org

b. Suppose Carlos wanted to burn 350 Calories. Approximately how long should he swim?

Since any point on the line is a solution of the equation, use the graph to estimate the value of the x-coordinate in the ordered pair that contains 350 as the y-coordinate. The ordered pair (33, 350) appears to be on the line so Carlos should swim for 33 minutes to burn 350 Calories. *Check this solution algebraically by substituting (33, 350) into the original equation.*

Since two points determine a line, a simple method of graphing a linear equation is to find the points where the graph crosses the x-axis and the y-axis. The x-coordinate of the point at which it crosses the x-axis is the **x-intercept**, and the y-coordinate of the point at which the graph crosses the y-axis is called the **y-intercept**.

Example 4 Graph Using Intercepts

Determine the x-intercept and y-intercept of $3x + 2y = 9$. Then graph the equation.

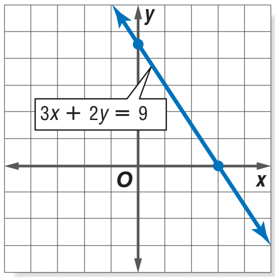
To find the x-intercept, let $y = 0$.

$3x + 2y = 9$	Original equation
$3x + 2(0) = 9$	Replace y with 0.
$3x = 9$	Divide each side by 3.
$x = 3$	

To find the y-intercept, let $x = 0$.

$3x + 2y = 9$	Original equation
$3(0) + 2y = 9$	Replace x with 0.
$2y = 9$	Divide each side by 2.
$y = 4.5$	

The x-intercept is 3, so the graph intersects the x-axis at (3, 0). The y-intercept is 4.5, so the graph intersects the y-axis at (0, 4.5). Plot these points. Then draw a line that connects them.



Check for Understanding

Concept Check

1. **Explain** how the graph of $y = 2x + 1$ for the domain {1, 2, 3, 4} differs from the graph of $y = 2x + 1$ for the domain of all real numbers.
2. **OPEN ENDED** Give an example of a linear equation in the form $Ax + By = C$ for each of the following conditions.
   a. $A = 0$ b. $B = 0$ c. $C = 0$
3. **Explain** how to graph an equation using the x- and y-intercepts.

Guided Practice

Determine whether each equation is a linear equation. If so, write the equation in standard form.

4. $x + y^2 = 25$
5. $3y + 2 = 0$
6. $\frac{3}{5}x - \frac{2}{5}y = 5$
7. $x + \frac{1}{y} = 7$

Graph each equation.

8. $x = 3$
9. $x - y = 0$
10. $y = 2x + 8$
11. $y = -3 - x$
12. $x + 4y = 10$
13. $4x + 3y = 12$

Application

TAXI FARE **For Exercises 14 and 15, use the following information.**
A taxi company charges a fare of \$2.25 plus \$0.75 per mile traveled. The cost of the fare c can be described by the equation $c = 0.75m + 2.25$, where m is the number of miles traveled.

14. Graph the equation.
15. If you need to travel 18 miles, how much will the taxi fare cost?

Practice and Apply

Homework Help

For Exercises	See Examples
16–25	1
26–45	2, 4
46–56	3

Extra Practice
See page 829.

Determine whether each equation is a linear equation. If so, write the equation in standard form.

16. $3x = 5y$
17. $6 - y = 2x$
18. $6xy + 3x = 4$
19. $y + 5 = 0$
20. $7y = 2x + 5x$
21. $y = 4x^2 - 1$
22. $\frac{3}{x} + \frac{4}{y} = 2$
23. $\frac{x}{2} = 10 + \frac{2y}{3}$
24. $7n - 8m = 4 - 2m$
25. $3a + b - 2 = b$

Graph each equation.

26. $y = -1$
27. $y = 2x$
28. $y = 5 - x$
29. $y = 2x - 8$
30. $y = 4 - 3x$
31. $y = x - 6$
32. $x = 3y$
33. $x = 4y - 6$
34. $x - y = -3$
35. $x + 3y = 9$
36. $4x + 6y = 8$
37. $3x - 2y = 15$

www.algebra1.com/self_check_quiz

Graph each equation.

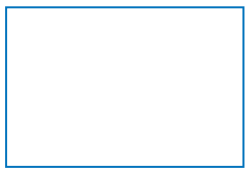
38. $1.5x + y = 4$ **39.** $2.5x + 5y = 75$ **40.** $\frac{1}{2}x + y = 4$

41. $x - \frac{2}{3}y = 1$ **42.** $\frac{4x}{3} = \frac{3y}{4} + 1$ **43.** $y + \frac{1}{3} = \frac{1}{4}x - 3$

44. Find the x- and y-intercept of the graph of $4x - 7y = 14$.

45. Write an equation in standard form of the line with an x-intercept of 3 and a y-intercept of 5.

GEOMETRY **For Exercises 46–48, refer to the figure.** The perimeter P of a rectangle is given by $2\ell + 2w = P$, where ℓ is the length of the rectangle and w is the width.



46. If the perimeter of the rectangle is 30 inches, write an equation for the perimeter in standard form.

47. What are the x- and y-intercepts of the graph of the equation?

48. Graph the equation.

METEOROLOGY **For Exercises 49–51, use the following information.** As a thunderstorm approaches, you see lightning as it occurs, but you hear the accompanying sound of thunder a short time afterward. The distance d in miles that sound travels in t seconds is given by the equation $d = 0.21t$.

49. Make a table of values.

50. Graph the equation.

51. Estimate how long it will take to hear the thunder from a storm 3 miles away.

BIOLOGY **For Exercises 52 and 53, use the following information.** The amount of blood in the body can be predicted by the equation $y = 0.07w$, where y is the number of pints of blood and w is the weight of a person in pounds.

52. Graph the equation.

53. Predict the weight of a person whose body holds 12 pints of blood.

OCEANOGRAPHY **For Exercises 54–56, use the information at left and below.** Under water, pressure increases 4.3 pounds per square inch (psi) for every 10 feet you descend. This can be expressed by the equation $p = 0.43d + 14.7$, where p is the pressure in pounds per square inch and d is the depth in feet.

54. Graph the equation.

55. Divers cannot work at depths below about 400 feet. What is the pressure at this depth?

56. How many times as great is the pressure at 400 feet as the pressure at sea level?

More About. . .

Oceanography

How heavy is air? The atmospheric pressure is a measure of the weight of air. At sea level, air pressure is 14.7 pounds per square inch.

Source: www.brittanica.com

57. **CRITICAL THINKING** Explain how you can determine whether a point at (x, y) is *above*, *below*, or *on* the line given by $2x - y = 8$ without graphing it. Give an example of each.

58. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How can linear equations be used in nutrition?

Include the following in your answer:

- an explanation of how you could use the Nutrition Information labels on packages to limit your fat intake, and
- an equation you could use to find how many grams of protein you should have each day if you wanted 10% of your diet to consist of protein. (*Hint*: Protein contains 4 Calories per gram.)

Standardized Test Practice

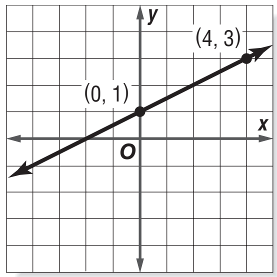
59. Which point lies on the line given by $y = 3x - 5$?

Ⓐ $(1, -2)$ Ⓑ $(0, 5)$ Ⓒ $(1, 2)$ Ⓓ $(4, 3)$

60. In the graph at the right, $(0, 1)$ and $(4, 3)$ lie on the line. Which ordered pair also lies on the line?

Ⓐ $(1, 1)$ Ⓑ $(2, 2)$

Ⓒ $(3, 3)$ Ⓓ $(4, 4)$



Maintain Your Skills

Mixed Review

Solve each equation if the domain is $\{-3, -1, 2, 5, 8\}$. *(Lesson 4-4)*

61. $y = x - 5$
62. $y = 2x + 1$
63. $3x + y = 12$
64. $2x - y = -3$
65. $3x - \frac{1}{2}y = 6$
66. $-2x + \frac{1}{3}y = 4$

Express each relation as a table, a graph, and a mapping. Then determine the domain and range. *(Lesson 4-3)*

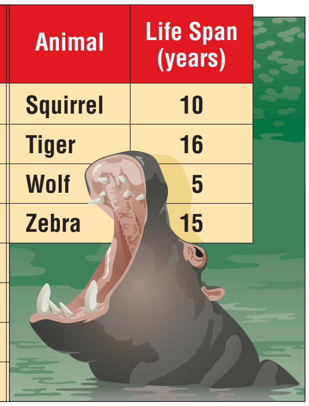
67. $\{(3, 5), (-4, -1), (-3, 2), (3, 1)\}$
68. $\{(4, 0), (2, -3), (-1, -3), (4, 4)\}$
69. $\{(1, 4), (3, 0), (-1, -1), (3, 5)\}$
70. $\{(4, 5), (2, 5), (4, -1), (3, 2)\}$

Solve each equation. Then check your solution. *(Lesson 3-5)*

71. $2(x - 2) = 3x - (4x - 5)$
72. $3a + 8 = 2a - 4$
73. $3n - 12 = 5n - 20$
74. $6(x + 3) = 3x$

ANIMALS **For Exercises 75–78, use the table below that shows the average life spans of 20 different animals.** *(Lesson 2-5)*

Animal	Life Span (years)	Animal	Life Span (years)	Animal	Life Span (years)
Baboon	20	Lion	15	Squirrel	10
Camel	12	Monkey	15	Tiger	16
Cow	15	Mouse	3	Wolf	5
Elephant	40	Opossum	1	Zebra	15
Fox	7	Pig	10		
Gorilla	20	Rabbit	5		
Hippopotamus	25	Sea Lion	12		
Kangaroo	7	Sheep	12		



75. Make a line plot of the average life spans of the animals in the table.
76. How many animals live between 7 and 16 years?
77. Which number occurred most frequently?
78. How many animals live at least 20 years?

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Evaluate each expression.**

*(To review **evaluating expressions**, see Lesson 1-2.)*

79. $19 + 5 \cdot 4$
80. $(25 - 4) \div (2^2 - 1^3)$
81. $12 \div 4 + 15 \cdot 3$
82. $12(19 - 15) - 3 \cdot 8$
83. $6(4^3 + 2^2)$
84. $7[4^3 - 2(4 + 3)] \div 7 + 2$

Graphing Calculator

A Follow-Up of Lesson 4-5

Graphing Linear Equations

The power of a graphing calculator is the ability to graph different types of equations accurately and quickly. Often linear equations are graphed in the standard viewing window. The **standard viewing window** is [−10, 10] by [−10, 10] with a scale of 1 on both axes. To quickly choose the standard viewing window on a TI-83 Plus, press ZOOM 6.

Example 1

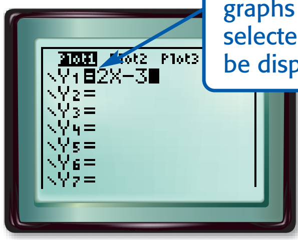
Graph $2x - y = 3$ on a TI-83 Plus graphing calculator.

Step 1 *Enter the equation in the Y= list.*

- The Y= list shows the equation or equations that you will graph.
- Equations must be entered with the y isolated on one side of the equation. Solve the equation for y, then enter it into the calculator.

$2x - y = 3$	Original equation
$2x - y - 2x = 3 - 2x$	Subtract $2x$ from each side.
$-y = -2x + 3$	Simplify.
$y = 2x - 3$	Multiply each side by -1.

KEYSTROKES: Y= 2 X,T,θ,n − 3

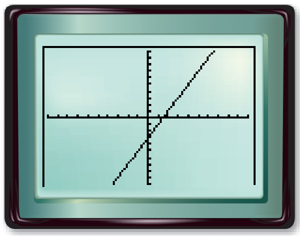


The equals sign appears shaded for graphs that are selected to be displayed.

Step 2 *Graph the equation in the standard viewing window.*

Graph the selected equations.

KEYSTROKES: ZOOM 6



[−10, 10] scl: 1 by [−10, 10] scl: 1

Notice that the graph of $2x - y = 3$ above is a complete graph because all of these points are visible.

Sometimes a complete graph is not displayed using the standard viewing window. A **complete graph** includes all of the important characteristics of the graph on the screen. These include the origin, and the x- and y-intercepts.

When a complete graph is not displayed using the standard viewing window, you will need to change the viewing window to accommodate these important features. You can use what you have learned about intercepts to help you choose an appropriate viewing window.

Investigation

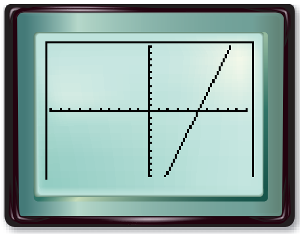
Example 2

Graph $y = 3x - 15$ on a graphing calculator.

Step 1 *Enter the equation in the Y= list and graph in the standard viewing window.*

Clear the previous equation from the Y= list. Then enter the new equation and graph.

KEYSTROKES: [Y=] [CLEAR] 3 [X,T,θ,n] [–] 15 [ZOOM] 6



[−10, 10] scl: 1 by [−10, 10] scl: 1

Step 2 *Modify the viewing window and graph again.*

The origin and the x-intercept are displayed in the standard viewing window. But notice that the y-intercept is outside of the viewing window. Find the y-intercept.

$y = 3x - 15$ Original equation

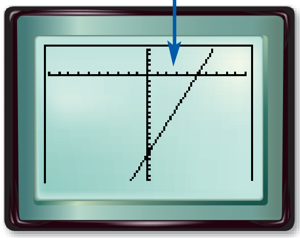
$y = 3(0) - 15$ Replace x with 0.

$y = -15$ Simplify.

Since the y-intercept is -15, choose a viewing window that includes a number less than -15. The window $[-10, 10]$ by $[-20, 5]$ with a scale of 1 on each axis is a good choice.

KEYSTROKES: [WINDOW] −10 [ENTER] 10 [ENTER] 1 [ENTER] −20 [ENTER] 5 [ENTER] 1 [GRAPH]

This window allows the complete graph, including the y-intercept, to be displayed.



[−10, 10] scl: 1 by [−20, 5] scl: 1

Exercises

Use a graphing calculator to graph each equation in the standard viewing window. Sketch the result.

1. $y = x + 2$ **2.** $y = 4x + 5$ **3.** $y = 6 - 5x$

4. $2x + y = 6$ **5.** $x + y = -2$ **6.** $x - 4y = 8$

Graph each linear equation in the standard viewing window. Determine whether the graph is complete. If the graph is not complete, choose a viewing window that will show a complete graph and graph the equation again.

7. $y = 5x + 9$ **8.** $y = 10x - 6$ **9.** $y = 3x - 18$

10. $3x - y = 12$ **11.** $4x + 2y = 21$ **12.** $3x + 5y = -45$

For Exercises 13–15, consider the linear equation $y = 2x + b$.

13. Choose several different positive and negative values for b. Graph each equation in the standard viewing window.

14. For which values of b is the complete graph in the standard viewing window?

15. How is the value of b related to the y-intercept of the graph of $y = 2x + b$?

www.algebra1.com/other_calculator_keystrokes

4-6 Functions

What You'll Learn

- Determine whether a relation is a function.
- Find function values.

Vocabulary

- function
- vertical line test
- function notation

How are functions used in meteorology?

The table shows barometric pressures and temperatures recorded by the National Climatic Data Center over a three-day period.

Pressure (millibars)	1013	1006	997	995	995	1000	1006	1011	1016	1019
Temperature (°C)	3	4	10	13	8	4	1	–2	–6	–9

Notice that when the pressure is 995 and 1006 millibars, there is more than one value for the temperature.

IDENTIFY FUNCTIONS Recall that relations in which each element of the domain is paired with exactly one element of the range are called **functions**.

Study Tip

Functions

In a function, knowing the value of x tells you the value of y.

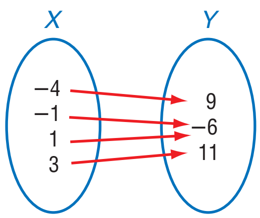
Key Concept — Function

A function is a relation in which each element of the domain is paired with *exactly* one element of the range.

Example 1 Identify Functions

Determine whether each relation is a function. Explain.

a.



This mapping represents a function since, for each element of the domain, there is only one corresponding element in the range. It does not matter if two elements of the domain are paired with the same element in the range.

b.

x	y
–3	6
2	5
3	1
2	4

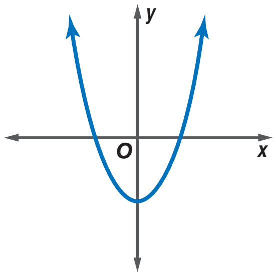
This table represents a relation that is not a function. The element 2 in the domain is paired with both 5 and 4 in the range. If you are given that x is 2, you cannot determine the value of y.

c. $\{(-2, 4), (1, 5), (3, 6), (5, 8), (7, 10)\}$

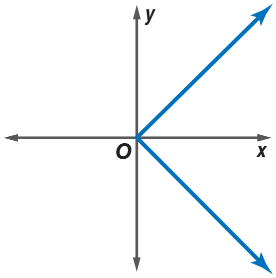
Since each element of the domain is paired with exactly one element of the range, this relation is a function. If you are given that x is -3, you can determine that the value of y is 6 since 6 is the only value of y that is paired with $x = 3$.

You can use the **vertical line test** to see if a graph represents a function. If no vertical line can be drawn so that it intersects the graph more than once, then the graph is a function. If a vertical line can be drawn so that it intersects the graph at two or more points, the relation is not a function.

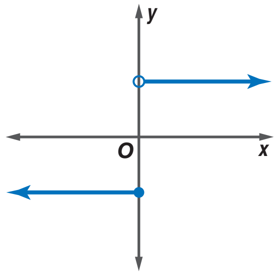
Function



Not a Function



Function

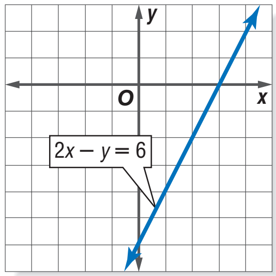


One way to perform the vertical line test is to use a pencil.

Example 2 Equations as Functions

Determine whether $2x - y = 6$ is a function.

Graph the equation using the x- and y-intercepts.



Since the equation is in the form $Ax + By = C$, the graph of the equation will be a line. Place your pencil at the left of the graph to represent a vertical line. Slowly move the pencil to the right across the graph.

For each value of x, this vertical line passes through no more than one point on the graph. Thus, the line represents a function.

FUNCTION VALUES Equations that are functions can be written in a form called **function notation**. For example, consider $y = 3x - 8$.

equation	function notation
$y = 3x - 8$	$f(x) = 3x - 8$

Study Tip

Reading Math
The symbol $f(x)$ is read *f of x*.

In a function, x represents the elements of the domain, and $f(x)$ represents the elements of the range. Suppose you want to find the value in the range that corresponds to the element 5 in the domain. This is written $f(5)$ and is read "f of 5." The value $f(5)$ is found by substituting 5 for x in the equation.

Example 3 Function Values

If $f(x) = 2x + 5$, find each value.

a. $f(-2)$

$f(-2) = 2(-2) + 5$ Replace x with -2.

$= -4 + 5$ Multiply.

$= 1$ Add.

b. $f(1) + 4$

$f(1) + 4 = [2(1) + 5] + 4$ Replace x with 1.

$= 7 + 4$ Simplify.

$= 11$ Add.

c. $f(x+3)$

$$\begin{aligned} f(x+3) &= 2(x+3)+5 && \text{Replace } x \text{ with } x+3. \\ &= 2x+6+5 && \text{Distributive Property} \\ &= 2x+11 && \text{Simplify.} \end{aligned}$$

The functions we have studied thus far have been linear functions. However, many functions are not linear. You can find the value of these functions in the same way.

Example 4 Nonlinear Function Values

If $h(z) = z^2 + 3z - 4$, find each value.

a. $h(-4)$

$$\begin{aligned} h(-4) &= (-4)^2 + 3(-4) - 4 && \text{Replace } z \text{ with } -4. \\ &= 16 - 12 - 4 && \text{Multiply.} \\ &= 0 && \text{Simplify.} \end{aligned}$$

b. $h(5a)$

$$\begin{aligned} h(5a) &= (5a)^2 + 3(5a) - 4 && \text{Replace } z \text{ with } 5a. \\ &= 25a^2 + 15a - 4 && \text{Simplify.} \end{aligned}$$

c. $2[h(g)]$

$$\begin{aligned} 2[h(g)] &= 2[(g)^2 + 3(g) - 4] && \text{Evaluate } h(g) \text{ by replacing } z \text{ with } g. \\ &= 2(g^2 + 3g - 4) && \text{Multiply the value of } h(g) \text{ by 2.} \\ &= 2g^2 + 6g - 8 && \text{Simplify.} \end{aligned}$$

Study Tip

Reading Math
Other letters such as g and h can be used to represent functions, for example, $g(x)$ or $h(z)$.

On some standardized tests, an arbitrary symbol may be used to represent a function.

Standardized Test Practice

Example 5 Nonstandard Function Notation

Multiple-Choice Test Item

If $\ll x \gg = x^2 - 4x + 2$, then $\ll 3 \gg =$

(A) -2. (B) -1. (C) 1. (D) 2.

Read the Test Item

The symbol $\ll x \gg$ is just a different notation for $f(x)$.

Solve the Test Item

Replace x with 3.

$$\begin{aligned} \ll x \gg &= x^2 - 4x + 2 && \text{Think: } \ll x \gg = f(x) \\ \ll 3 \gg &= (3)^2 - 4(3) + 2 && \text{Replace } x \text{ with 3.} \\ &= 9 - 12 + 2 \text{ or } -1 && \text{The answer is B.} \end{aligned}$$

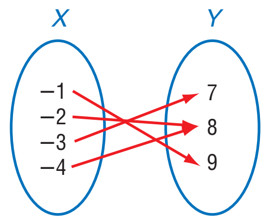
Check for Understanding

Concept Check

1. **Study** the following set of ordered pairs that describe a relation between x and y: $\{(1, -1), (-1, 2), (4, -3), (3, 2), (-2, 4), (3, -3)\}$. Is y a function of x? Is x a function of y? Explain your answer.
2. **OPEN ENDED** Define a function using nonstandard function notation.
3. **Find a counterexample** to disprove the following statement.
   All linear equations are functions.

Guided Practice

Determine whether each relation is a function.

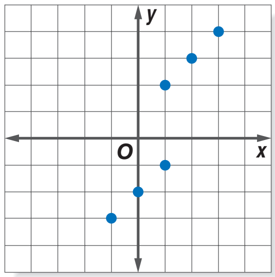
4. 

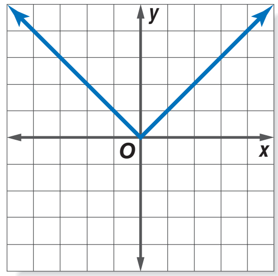
5.

x	y
−3	0
2	1
2	4
6	5

6. {(24, 1), (21, 4), (3, 22), (24, 5)}

7. $y = x + 3$

8. 

9. 

If $f(x) = 4x - 5$ and $g(x) = x^2 + 1$, find each value.

10. $f(2)$

11. $g(-1)$

12. $f(c)$

13. $g(t) - 4$

14. $f(3a^2)$

15. $f(x + 5)$

Standardized Test Practice

16. If $x^{**} = 2x - 1$, then $5^{**} - 2^{**} =$

Ⓐ 3. Ⓑ 4. Ⓒ 5. Ⓓ 6.

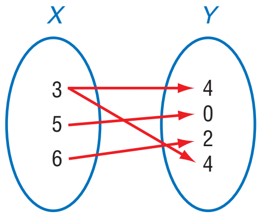
Practice and Apply

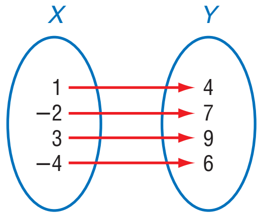
Homework Help

For Exercises	See Examples
17–31, 44	1, 2
32–43, 45–51	3–5

Extra Practice
See page 830.

Determine whether each relation is a function.

17. 

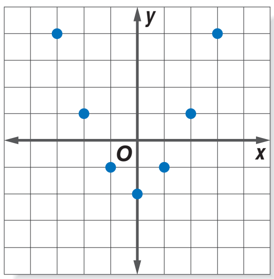
18. 

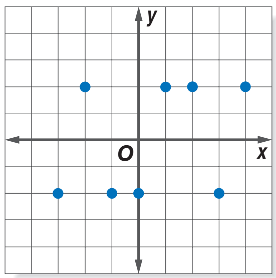
19.

x	y
2	7
4	9
5	5
8	−1

20.

x	y
−9	−5
−4	0
3	6
7	1
6	−5
3	2

21. 

22. 

23. $\{(5, -7), (6, -7), (-8, -1), (0, -1)\}$

24. $\{(4, 5), (3, -2), (-2, 5), (4, 7)\}$

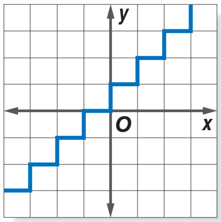
25. $y = -8$

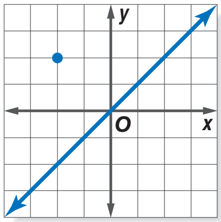
26. $x = 15$

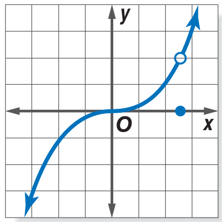
27. $y = 3x - 2$

28. $y = 3x + 2y$

Determine whether each relation is a function.

29.


30.


31.


If $f(x) = 3x + 7$ and $g(x) = x^2 - 2x$, find each value.

32. $f(3)$

33. $f(-2)$

34. $g(5)$

35. $g(0)$

36. $g(-3) + 1$

37. $f(8) - 5$

38. $g(2c)$

39. $f(a^2)$

40. $f(k + 2)$

41. $f(2m - 5)$

42. $3[g(x) + 4]$

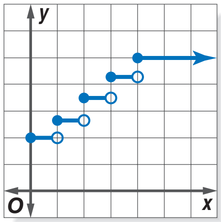
43. $2[f(x^2) - 5]$

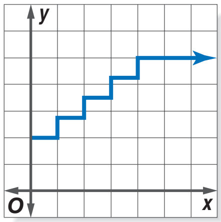
Study Tip

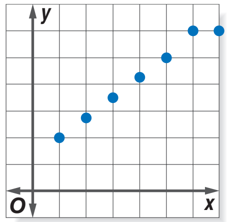
Reading Math

The graph shown in Part a of Exercise 44 is also called a *step function*.

44. PARKING The rates for a parking garage are $2.00 for the first hour, $2.75 for the second hour, $3.50 for the third hour, $4.25 for the fourth hour, and $5.00 for any time over four hours. Choose the graph that best represents the information and determine whether the graph represents a function. Explain.

a.


b.


c.


CLIMATE **For Exercises 45–48, use the following information.**

The temperature of the atmosphere decreases about 5°F for every 1000 feet increase in altitude. Thus, if the temperature at ground level is 77°F, the temperature at a given altitude is found by using the equation $t = 77 - 0.005h$, where h is the height in feet.

45. Write the equation in function notation.

46. Find $f(100)$, $f(200)$, and $f(1000)$.

47. Graph the equation.

48. Use the graph of the function to determine the temperature at 4000 feet.

WebQuest

A graph of the winning Olympic swimming times will help you determine whether the winning time is a function of the year. Visit www.algebra1.com/webquest to continue work on your WebQuest project.

EDUCATION **For Exercises 49–51, use the following information.**

The National Assessment of Educational Progress tests 4th, 8th, and 12th graders in the United States. The average math test scores for 17-year-olds can be represented as a function of the science scores by $f(s) = 0.8s + 72$, where $f(s)$ is the math score and s is the science score.

49. Graph this function.

50. What is the science score that corresponds to a math score of 308?

51. Krista scored 260 in science and 320 in math. How does her math score compare to the average score of other students who scored 260 in science? Explain your answer.

52. CRITICAL THINKING State whether the following is *sometimes*, *always*, or *never* true.

The inverse of a function is also a function.

53. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How are functions used in meteorology?

Include the following in your answer:

- a description of the relationship between pressure and temperature, and
- an explanation of whether the relation is a function.

Standardized Test Practice

54. If $f(x) = 20 - 2x$, find $f(7)$.

(A) 6 (B) 7 (C) 13 (D) 14

55. If $f(x) = 2x$, which of the following statements must be true?

I. $f(3x) = 3[f(x)]$

II. $f(x + 3) = f(x) + 3$

III. $f(x^2) = [f(x)]^2$

(A) I only (B) II only (C) I and II only (D) I, II, and III

Maintain Your Skills

Mixed Review

Graph each equation. *(Lesson 4-5)*

56. $y = x + 3$ 57. $y = 2x - 4$ 58. $2x + 5y = 10$

Find the solution set for each equation, given the replacement set. *(Lesson 4-4)*

59. $y = 5x - 3$; {(3, 12), (1, −2), (−2, −7), (−1, −8)}

60. $y = 2x + 6$; {(3, 0), (−1, 4), (6, 0), (5, −1)}

61. **RUNNING** Adam is training for an upcoming 26-mile marathon. He can run a 10K race (about 6.2 miles) in 45 minutes. If he runs the marathon at the same pace, how long will it take him to finish? *(Lesson 3-6)*

Name the property used in each equation. Then find the value of n. *(Lesson 1-4)*

62. $16 = n + 16$ 63. $3.5 + 6 = n + 6$ 64. $\frac{3}{5}n = \frac{3}{5}$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find each difference.**

*(To review **subtracting integers**, see Lesson 2-2.)*

65. $12 - 16$ 66. $-5 - (-8)$ 67. $16 - (-4)$

68. $-9 - 6$ 69. $\frac{3}{4} - \frac{1}{8}$ 70. $3\frac{1}{2} - \left(-1\frac{2}{3}\right)$

Practice Quiz 2 — Lessons 4-4 through 4-6

Solve each equation if the domain is {−3, −1, 0, 2, 4}. *(Lesson 4-4)*

1. $y = x + 5$ 2. $y = 3x + 4$ 3. $x + 2y = 8$

Graph each equation. *(Lesson 4-5)*

4. $y = x - 2$ 5. $3x + 2y = 6$

Determine whether each relation is a function. *(Lesson 4-6)*

6. {(3, 4), (5, 3), (−1, 4), (6, 2)} 7. {(−1, 4), (−2, 5), (7, 2), (3, 9), (−2, 1)}

If $f(x) = 3x + 5$, find each value. *(Lesson 4-6)*

8. $f(-4)$ 9. $f(2a)$ 10. $f(x + 2)$

Number Sequences

You can use a spreadsheet to generate number sequences and patterns. The simplest type of sequence is one in which the difference between successive terms is constant. This type of sequence is called an **arithmetic sequence**.

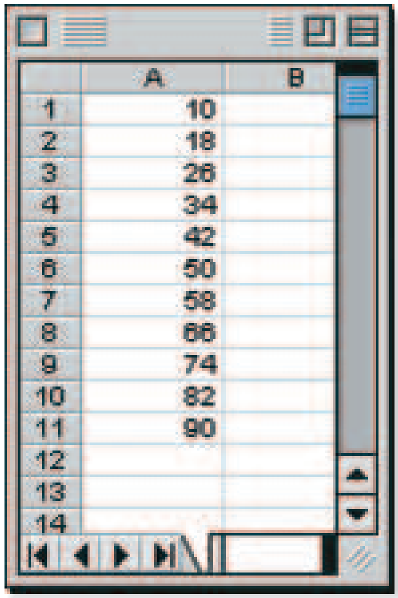
Example

Use a spreadsheet to generate a sequence of numbers from an initial value of 10 to 90 with a fixed interval of 8.

Step 1 Enter the initial value 10 in cell A1.

Step 2 Highlight the cells in column A. Under the Edit menu, choose the Fill option and then Series.

Step 3 A command box will appear on the screen asking for the Step value and the Stop value. The Step value is the fixed interval between each number, which in this case is 8. The Stop value is the last number in your sequence, 90. Enter these numbers and click OK. The column is filled with the numbers in the sequence from 10 to 90 at intervals of 8.



Exercises

For Exercises 1–5, use a sequence of numbers from 7 to 63 with a fixed interval of 4.

1. Use a spreadsheet to generate the sequence. Write the numbers in the sequence.

2. How many numbers are in the sequence?

MAKE A CONJECTURE Let a_n represent each number in a sequence if n is the position of the number in the sequence. For example, a_1 = the first number in the sequence, a_2 = the second number, a_3 = the third number, and so on.

3. Write a formula for a_2 in terms of a_1. Write similar formulas for a_3 and a_4 in terms of a_1.

4. Look for a pattern. Write an equation that can be used to find the nth term of a sequence.

5. Use the equation from Exercise 4 to find the 21st term in the sequence.

4-7 Arithmetic Sequences

What You'll Learn

- Recognize arithmetic sequences.
- Extend and write formulas for arithmetic sequences.

Vocabulary

- sequence
- terms
- arithmetic sequence
- common difference

How are arithmetic sequences used to solve problems in science?

A probe to measure air quality is attached to a hot-air balloon. The probe has an altitude of 6.3 feet after the first second, 14.5 feet after the next second, 22.7 feet after the third second, and so on. You can make a table and look for a pattern in the data.

Time (s)	1	2	3	4	5	6	7	8
Altitude (ft)	6.3	14.5	22.7	30.9	39.1	47.3	55.5	63.7

+ 8.2 + 8.2 + 8.2 + 8.2 + 8.2 + 8.2 + 8.2

RECOGNIZE ARITHMETIC SEQUENCES A **sequence** is a set of numbers in a specific order. The numbers in the sequence are called **terms**. If the difference between successive terms is constant, then it is called an **arithmetic sequence**. The difference between the terms is called the **common difference**.

terms

7 12 17 22 27

+ 5 + 5 + 5 + 5

common difference

Key Concept — *Arithmetic Sequence*

An arithmetic sequence is a numerical pattern that increases or decreases at a constant rate or value called the common difference.

Study Tip

Reading Math

The three dots after the last number in a sequence are called an *ellipsis*. The ellipsis indicates that there are more terms in the sequence that are not listed.

Example 1 *Identify Arithmetic Sequences*

Determine whether each sequence is arithmetic. Justify your answer.

a. 1, 2, 4, 8, …

1 2 4 8

+ 1 + 2 + 4

This is not an arithmetic sequence because the difference between terms is not constant.

b. $\frac{1}{2}, \frac{1}{4}, 0, -\frac{1}{4}, \ldots$

$\frac{1}{2}$ $\frac{1}{4}$ 0 $-\frac{1}{4}$

$-\frac{1}{4}$ $-\frac{1}{4}$ $-\frac{1}{4}$

This is an arithmetic sequence because the difference between terms is constant.

WRITE ARITHMETIC SEQUENCES

You can use the common difference of an arithmetic sequence to find the next term in the sequence.

Key Concept — *Writing Arithmetic Sequences*

- **Words** Each term of an arithmetic sequence after the first term can be found by adding the common difference to the preceding term.
- **Symbols** An arithmetic sequence can be found as follows

$$a_1, a_1 + d, a_2 + d, a_3 + d, \ldots,$$

where d is the common difference, a_1 is the first term, a_2 is the second term, and so on.

Example 2 *Extend a Sequence*

Find the next three terms of the arithmetic sequence 74, 67, 60, 53, …

Find the common difference by subtracting successive terms.

74 67 60 53 ? ? ?

−7 −7 −7 −7 −7 −7

The common difference is −7.

Add −7 to the last term of the sequence to get the next term in the sequence. Continue adding −7 until the next three terms are found.

53 46 39 32

−7 −7 −7

The next three terms are 46, 39, 32.

Each term in an arithmetic sequence can be expressed in terms of the first term a_1 and the common difference d.

Term	Symbol	In Terms of a_1 and d	Numbers
first term	a_1	a_1	8
second term	a_2	$a_1 + d$	$8 + 1(3) = 11$
third term	a_3	$a_1 + 2d$	$8 + 2(3) = 14$
fourth term	a_4	$a_1 + 3d$	$8 + 3(3) = 17$
⋮	⋮	⋮	⋮
nth term	a_n	$a_1 + (n - 1)d$	$8 + (n - 1)(3)$

Study Tip

Reading Math
The formula for the nth term of an arithmetic sequence is called a *recursive formula*. This means that each succeeding term is formulated from one or more of the previous terms.

The following formula generalizes this pattern and can be used to find any term in an arithmetic sequence.

Key Concept — *nth Term of an Arithmetic Sequence*

The nth term a_n of an arithmetic sequence with first term a_1 and common difference d is given by

$$a_n = a_1 + (n - 1)d,$$

where n is a positive integer.

Example 3 Find a Specific Term

Find the 14th term in the arithmetic sequence 9, 17, 25, 33, …

In this sequence, the first term, a_1, is 9. You want to find the 14th term, so $n = 14$. Find the common difference.

$$9 \xrightarrow{+8} 17 \xrightarrow{+8} 25 \xrightarrow{+8} 33$$

The common difference is 8.

Use the formula for the nth term of an arithmetic sequence.

$a_n = a_1 + (n - 1)d$	Formula for the *n*th term
$a_{14} = 9 + (14 - 1)8$	$a_1 = 9$, $n = 14$, $d = 8$
$a_{14} = 9 + 104$	Simplify.
$a_{14} = 113$	The 14th term in the sequence is 113.

Example 4 Write an Equation for a Sequence

Consider the arithmetic sequence 12, 23, 34, 45, …

a. Write an equation for the *n*th term of the sequence.

In this sequence, the first term, a_1, is 12. Find the common difference.

$$12 \xrightarrow{+11} 23 \xrightarrow{+11} 34 \xrightarrow{+11} 45$$

The common difference is 11.

Use the formula for the nth term to write an equation.

$a_n = a_1 + (n - 1)d$	Formula for *n*th term
$a_n = 12 + (n - 1)11$	$a_1 = 12$, $d = 11$
$a_n = 12 + 11n - 11$	Distributive Property
$a_n = 11n + 1$	Simplify.

CHECK For $n = 1$, $11(1) + 1 = 12$.
For $n = 2$, $11(2) + 1 = 23$.
For $n = 3$, $11(3) + 1 = 34$, and so on.

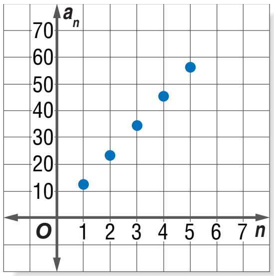
b. Find the 10th term in the sequence.

Replace n with 10 in the equation written in part **a**.

$a_n = 11n + 1$	Equation for the *n*th term
$a_{10} = 11(10) + 1$	Replace *n* with 10.
$a_{10} = 111$	Simplify.

c. Graph the first five terms of the sequence.

n	$11n + 1$	a_n	(n, a_n)
1	$11(1) + 1$	12	(1, 12)
2	$11(2) + 1$	23	(2, 23)
3	$11(3) + 1$	34	(3, 34)
4	$11(4) + 1$	45	(4, 45)
5	$11(5) + 1$	56	(5, 56)



Notice that the points fall on a line. The graph of an arithmetic sequence is linear.

Check for Understanding

Concept Check

1. **OPEN ENDED** Write an arithmetic sequence whose common difference is -10.

2. **Find** the common difference and the first term in the sequence defined by $a_n = 5n + 2$.

3. **FIND THE ERROR** Marisela and Richard are finding the common difference for the arithmetic sequence $-44, -32, -20, -8$.

Marisela	Richard
$-32 - (-44) = 12$	$-44 - (-32) = -12$
$-20 - (-32) = 12$	$-32 - (-20) = -12$
$-8 - (-20) = 12$	$-20 - (-8) = -12$

Who is correct? Explain your reasoning.

Guided Practice

Determine whether each sequence is an arithmetic sequence. If it is, state the common difference.

4. $24, 16, 8, 0, \ldots$
5. $3, 6, 12, 24, \ldots$

Find the next three terms of each arithmetic sequence.

6. $7, 14, 21, 28, \ldots$
7. $34, 29, 24, 19, \ldots$

Find the *n*th term of each arithmetic sequence described.

8. $a_1 = 3, d = 4, n = 8$
9. $a_1 = 10, d = -5, n = 21$
10. $23, 25, 27, 29, \ldots$ for $n = 12$
11. $-27, -19, -11, -3, \ldots$ for $n = 17$

Write an equation for the *n*th term of each arithmetic sequence. Then graph the first five terms of the sequence.

12. $6, 12, 18, 24, \ldots$
13. $12, 17, 22, 27, \ldots$

Application

14. **FITNESS** Latisha is beginning an exercise program that calls for 20 minutes of walking each day for the first week. Each week thereafter, she has to increase her walking by 7 minutes a day. Which week of her exercise program will be the first one in which she will walk over an hour a day?

Practice and Apply

Homework Help

For Exercises	See Examples
15–20, 43, 44	1
21–26	2
27–38, 45–49, 54, 55	3
39–42, 50–53	4

Extra Practice
See page 830.

Determine whether each sequence is an arithmetic sequence. If it is, state the common difference.

15. $7, 6, 5, 4, \ldots$
16. $10, 12, 15, 18, \ldots$
17. $9, 5, -1, -5, \ldots$
18. $-15, -11, -7, -3, \ldots$
19. $-0.3, 0.2, 0.7, 1.2, \ldots$
20. $2.1, 4.2, 8.4, 17.6, \ldots$

Find the next three terms of each arithmetic sequence.

21. $4, 7, 10, 13, \ldots$
22. $18, 24, 30, 36, \ldots$
23. $-66, -70, -74, -78, \ldots$
24. $-31, -22, -13, -4, \ldots$
25. $2\frac{1}{3}, 2\frac{2}{3}, 3, 3\frac{1}{3}, \ldots$
26. $\frac{7}{12}, 1\frac{1}{3}, 2\frac{1}{12}, 2\frac{5}{6}, \ldots$

Find the nth term of each arithmetic sequence described.

27. $a_1 = 5, d = 5, n = 25$

28. $a_1 = 8, d = 3, n = 16$

29. $a_1 = 52, d = 12, n = 102$

30. $a_1 = 34, d = 15, n = 200$

31. $a_1 = \frac{5}{8}, d = \frac{1}{8}, n = 22$

32. $a_1 = 1\frac{1}{2}, d = 2\frac{1}{4}, n = 39$

33. $-9, -7, -5, -3, \ldots$ for $n = 18$

34. $-7, -3, 1, 5, \ldots$ for $n = 35$

35. $0.5, 1, 1.5, 2, \ldots$ for $n = 50$

36. $5.3, 5.9, 6.5, 7.1, \ldots$ for $n = 12$

37. 200 is the __?__ th term of 24, 35, 46, 57, …

38. −34 is the __?__ th term of 30, 22, 14, 6, …

Write an equation for the nth term of each arithmetic sequence. Then graph the first five terms in the sequence.

39. $-3, -6, -9, -12, \ldots$

40. $8, 9, 10, 11, \ldots$

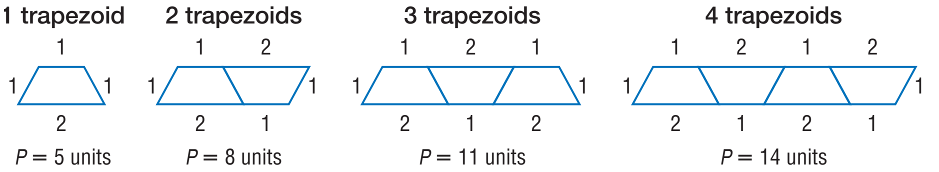
41. $2, 8, 14, 20, \ldots$

42. $-18, -16, -14, -12, \ldots$

43. Find the value of y that makes $y + 4, 6, y, \ldots$ an arithmetic sequence.

44. Find the value of y that makes $y + 8, 4y + 6, 3y, \ldots$ an arithmetic sequence.

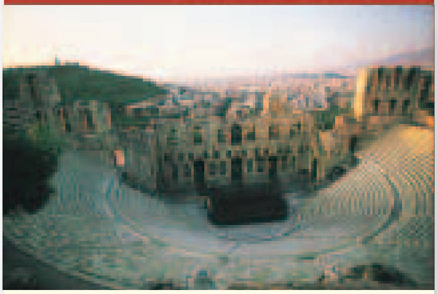
GEOMETRY **For Exercises 45 and 46, use the diagram below that shows the perimeter of the pattern consisting of trapezoids.**



45. Write a formula that can be used to find the perimeter of a pattern containing n trapezoids.

46. What is the perimeter of the pattern containing 12 trapezoids?

More About…



Theater

The open-air theaters of ancient Greece held about 20,000 people. They became the models for amphitheaters, Roman coliseums, and modern sports arenas.

Source: www.encarta.msn.com

THEATER **For Exercises 47–49, use the following information.**
The Coral Gables Actors' Playhouse has 76 seats in the last row of the orchestra section of the theater, 68 seats in the next row, 60 seats in the next row, and so on. There are 7 rows of seats in the section. On opening night, 368 tickets were sold for the orchestra section.

47. Write a formula to find the number of seats in any given row of the orchestra section of the theater.

48. How many seats are in the first row?

49. Was this section oversold?

PHYSICAL SCIENCE **For Exercises 50–53, use the following information.**
Taylor and Brooklyn are recording how far a ball rolls down a ramp during each second. The table below shows the data they have collected.

Time (s)	1	2	3	4	5	6
Distance traveled (cm)	9	13	17	21	25	29

50. Do the distances traveled by the ball form an arithmetic sequence? Justify your answer.

51. Write an equation for the sequence.

52. How far will the ball travel during the 35th second?

53. Graph the sequence.

GAMES **For Exercises 54 and 55, use the following information.**
Contestants on a game show win money by answering 10 questions. The value of each question increases by $1500.

54. If the first question is worth $2500, find the value of the 10th question.

55. If the contestant answers all ten questions correctly, how much money will he or she win?

56. **CRITICAL THINKING** Is $2x + 5, 4x + 5, 6x + 5, 8x + 5 \ldots$ an arithmetic sequence? Explain your answer.

57. **CRITICAL THINKING** Use an arithmetic sequence to find how many multiples of 7 are between 29 and 344.

58. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are arithmetic sequences used to solve problems in science?

Include the following in your answer:
- a formula for the arithmetic sequence that represents the altitude of the probe after each second, and
- an explanation of how you could use this information to predict the altitude of the probe after 15 seconds.

Standardized Test Practice

59. Luis puts $25 a week into a savings account from his part-time job. If he has $350 in savings now, how much will he have 12 weeks from now?

Ⓐ $600　Ⓑ $625　Ⓒ $650　Ⓓ $675

60. In an arithmetic sequence a_n, if $a_1 = 2$ and $a_4 = 11$, find a_{20}.

Ⓐ 40　Ⓑ 59　Ⓒ 78　Ⓓ 97

Maintain Your Skills

Mixed Review

If $f(x) = 3x - 2$ and $g(x) = x^2 - 5$, find each value. *(Lesson 4-6)*

61. $f(4)$　62. $g(-3)$　63. $2[f(6)]$

Determine whether each equation is a linear equation. If so, write the equation in standard form. *(Lesson 4-5)*

64. $x^2 + 3x - y = 8$　65. $y - 8 = 10 - x$　66. $2y = y + 2x - 3$

Translate each sentence into an algebraic equation. *(Lesson 3-1)*

67. Two hundred minus three times x is equal to nine.

68. The sum of twice r and three times s is identical to thirteen.

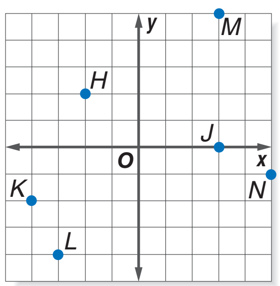
Find each product. *(Lesson 2-3)*

69. $7(-3)$　70. $-11 \cdot 15$　71. $-8(-1.5)$

72. $6\left(\frac{2}{3}\right)$　73. $\left(-\frac{5}{8}\right)\left(\frac{4}{7}\right)$　74. $5 \cdot 3\frac{1}{2}$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Write the ordered pair for each point shown at the right.**
*(To review **graphing points**, see Lesson 4-1.)*



75. H　76. J

77. K　78. L

79. M　80. N

Reading Mathematics

Reasoning Skills

Throughout your life, you have used reasoning skills, possibly without even knowing it. As a child, you used inductive reasoning to conclude that your hand would hurt if you touched the stove while it was hot. Now, you use inductive reasoning when you decide, after many trials, that one of the worst ways to prepare for an exam is by studying only an hour before you take it. **Inductive reasoning** is used to derive a general rule after observing many individual events.

Inductive reasoning involves . . .

- observing many examples
- looking for a pattern
- making a conjecture
- checking the conjecture
- discovering a likely conclusion

With **deductive reasoning**, you use a general rule to help you decide about a specific event. You come to a conclusion by accepting facts. There is no conjecturing involved. Read the two statements below.

1) If a person wants to play varsity sports, he or she must have a C average in academic classes.

2) Jolene is playing on the varsity tennis team.

If these two statements are accepted as facts, then the obvious conclusion is that Jolene has at least a C average in her academic classes. This is an example of deductive reasoning.

Reading to Learn

1. Explain the difference between *inductive* and *deductive* reasoning. Then give an example of each.

2. When Sherlock Holmes reaches a conclusion about a murderer's height because he knows the relationship between a man's height and the distance between his footprints, what kind of reasoning is he using? Explain.

3. When you examine a sequence of numbers and decide that it is an arithmetic sequence, what kind of reasoning are you using? Explain.

4. Once you have found the common difference for an arithmetic sequence, what kind of reasoning do you use to find the 100th term in the sequence?

5. **a.** Copy and complete the following table.

3^1	3^2	3^3	3^4	3^5	3^6	3^7	3^8	3^9
3	9	27						

b. Write the sequence of numbers representing the numbers in the ones place.

c. Find the number in the ones place for the value of 3^{100}. Explain your reasoning. State the type of reasoning that you used.

6. A sequence contains all numbers less than 50 that are divisible by 5. You conclude that 35 is in the sequence. Is this an example of inductive or deductive reasoning? Explain.

4-8 Writing Equations from Patterns

What You'll Learn

- Look for a pattern.
- Write an equation given some of the solutions.

Vocabulary

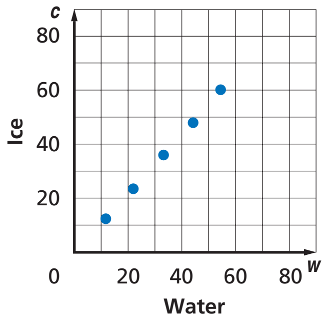
- look for a pattern
- inductive reasoning

Why is writing equations from patterns important in science?

Water is one of the few substances that expands when it freezes. The table shows different volumes of water and the corresponding volumes of ice.

Volume of Water (ft^3)	11	22	33	44	55
Volume of Ice (ft^3)	12	24	36	48	60

The relation in the table can be represented by a graph. Let w represent the volume of water, and let c represent the volume of ice. When the ordered pairs are graphed, they form a linear pattern. This pattern can be described by an equation.



Study Tip

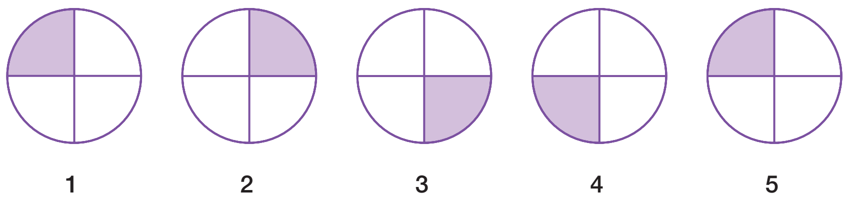
Look Back

To review **deductive reasoning**, see Lesson 1-7.

LOOK FOR PATTERNS A very useful problem-solving strategy is **look for a pattern**. When you make a conclusion based on a pattern of examples, you are using **inductive reasoning**. Recall that *deductive reasoning* uses facts, rules, or definitions to reach a conclusion.

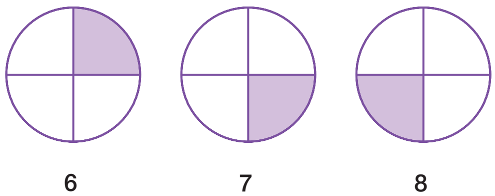
Example 1 Extend a Pattern

Study the pattern below.



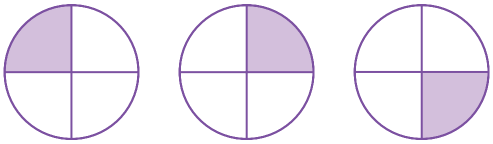
a. Draw the next three figures in the pattern.

The pattern consists of circles with one-fourth shaded. The section that is shaded is rotated in a clockwise direction. The next three figures are shown.



b. Draw the 27th circle in the pattern.

The pattern repeats every fourth design. Therefore designs 4, 8, 12, 16, and so on, will all be the same. Since 24 is the greatest number less than 27 that is a multiple of 4, the 25th circle in the pattern will be the same as the first circle.



25 26 27

Other sequences besides arithmetic sequences can follow a pattern.

Example 2 Patterns in a Sequence

Find the next three terms in the sequence 3, 6, 12, 24, … .

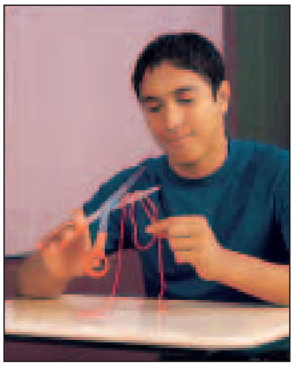
Study the pattern in the sequence.

$$3 \xrightarrow{+3} 6 \xrightarrow{+6} 12 \xrightarrow{+12} 24$$

You can use inductive reasoning to find the next term in a sequence. Notice the pattern 3, 6, 12, … The difference between each term doubles in each successive term. To find the next three terms in the sequence, continue doubling each successive difference. Add 24, 48, and 96.

$$3 \xrightarrow{+3} 6 \xrightarrow{+6} 12 \xrightarrow{+12} 24 \xrightarrow{+24} 48 \xrightarrow{+48} 96 \xrightarrow{+96} 192$$

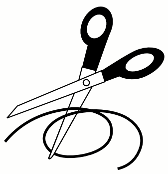
The next three terms are 48, 96, and 192.



Algebra Activity

Looking for Patterns

- You will need several pieces of string.
- Loop a piece of string around one of the cutting edges of the scissors and cut. How many pieces of string do you have as a result of this cut? Discard those pieces.
- Use another piece of string to make 2 loops around the scissors and cut. How many pieces of string result?
- Continue making loops and cutting until you see a pattern.



Analyze

1. Describe the pattern and write a sequence that describes the number of loops and the number of pieces of string.
2. Write an expression that you could use to find the number of pieces of string you would have if you made n loops.
3. How many pieces of string would you have if you made 20 loops?

WRITE EQUATIONS Sometimes a pattern can lead to a general rule. If the relationship between the domain and range of a relation is linear, the relationship can be described by a linear equation.

Example 3 Write an Equation from Data

FUEL ECONOMY **The table below shows the average amount of gas Rogelio's car uses depending on how many miles he drives.**

Gallons of gasoline	1	2	3	4	5
Miles driven	28	56	84	112	140

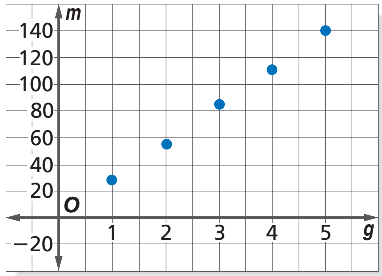
Fuel Economy

Not all cars use gasoline. Many alternative fuels are being used today in place of fossil fuels like oil.

Source: U.S. Department of Energy

a. Graph the data. What conclusion can you make about the relationship between the number of gallons used and the number of miles driven?

The graph shows a linear relationship between the number of gallons used g and the number of miles driven m.



b. Write an equation to describe this relationship.

Look at the relationship between the domain and range to find a pattern that can be described by an equation.

+ 1 + 1 + 1 + 1

Gallons of gasoline	1	2	3	4	5
Miles driven	28	56	84	112	140

+ 28 + 28 + 28 + 28

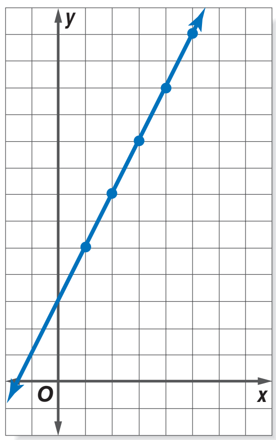
The difference of the values for g is 1, and the difference of the values for m is 28. This suggests that $m = 28g$. Check to see if this equation is correct by substituting values of g into the equation.

CHECK If $g = 1$, then $m = 28(1)$ or 28. ✓
If $g = 2$, then $m = 28(2)$ or 56. ✓
If $g = 3$, then $m = 28(3)$ or 84. ✓

The equation checks. Since this relation is also a function, we can write the equation as $f(g) = 28g$, where $f(g)$ represents the number of miles driven.

Example 4 Write an Equation with a Constant

Write an equation in function notation for the relation graphed at the right.



Make a table of ordered pairs for several points on the graph.

+ 1 + 1 + 1 + 1

x	1	2	3	4	5
y	5	7	9	11	13

+ 2 + 2 + 2 + 2

The difference of the x values is 1, and the difference of the y values is 2. The difference in y values is twice the difference of x values. This suggests that $y = 2x$. Check this equation.

CHECK If $x = 1$, then $y = 2(1)$ or 2. But the y value for $x = 1$ is 5. This is a difference of 3. Try some other values in the domain to see if the same difference occurs.

x	1	2	3	4	5
$2x$	2	4	6	8	10
y	5	7	9	11	13

y is always 3 more than $2x$.

This pattern suggests that 3 should be added to one side of the equation in order to correctly describe the relation. Check $y = 2x + 3$.

If $x = 2$, then $y = 2(2) + 3$ or 7.
If $x = 3$, then $y = 2(3) + 3$, or 9.

Thus, $y = 2x + 3$ correctly describes this relation. Since this relation is also a function, we can write the equation in function notation as $f(x) = 2x + 3$.

Check for Understanding

Concept Check

1. **Explain** how you can use inductive reasoning to write an equation from a pattern.
2. **OPEN ENDED** Write a sequence for which the first term is 4 and the second term is 8. Explain the pattern that you used.
3. **Explain** how you can determine whether an equation correctly represents a relation given in a table.

Guided Practice

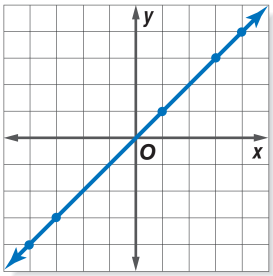
4. Find the next two items for the pattern. Then find the 16th figure in the pattern.

Find the next three terms in each sequence.

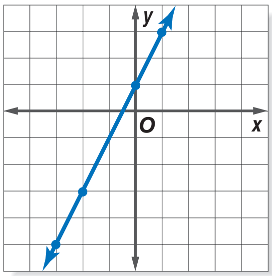
5. 1, 2, 4, 7, 11, …
6. 5, 9, 6, 10, 7, 11, …

Write an equation in function notation for each relation.

7.



8.



Application

GEOLOGY For Exercises 9–11, use the table below that shows the underground temperature of rocks at various depths below Earth's surface.

Depth (km)	1	2	3	4	5	6
Temperature (°C)	55	90	125	160	195	230

9. Graph the data.
10. Write an equation in function notation for the relation.
11. Find the temperature of a rock that is 10 kilometers below the surface.

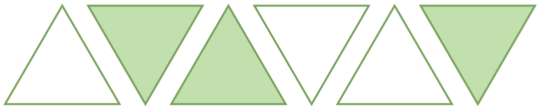
Practice and Apply

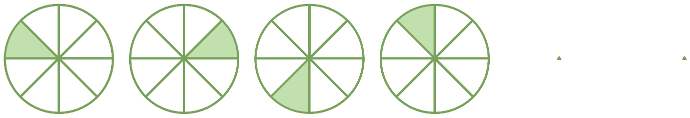
Homework Help

For Exercises	See Examples
12, 13, 26	1
14–19, 27, 28	2
20–25	4
29, 30	5

Extra Practice
See page 830.

Find the next two items for each pattern. Then find the 21st figure in the pattern.

12. 

13. 

Find the next three terms in each sequence.

14. 0, 2, 6, 12, 20, …

15. 9, 7, 10, 8, 11, 9, 12, …

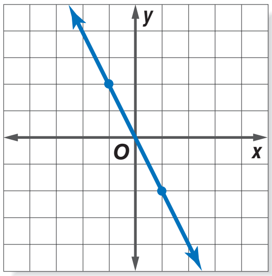
16. 1, 4, 9, 16, …

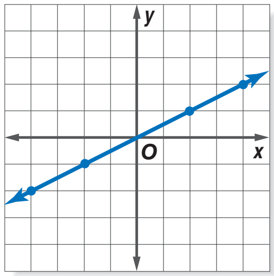
17. 0, 2, 5, 9, 14, 20, …

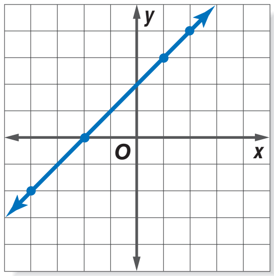
18. $a + 1, a + 2, a + 3, \ldots$

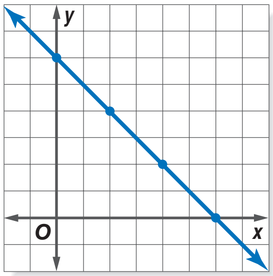
19. $x + 1, 2x + 1, 3x + 1, \ldots$

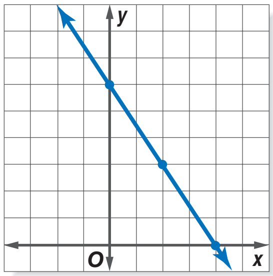
Write an equation in function notation for each relation.

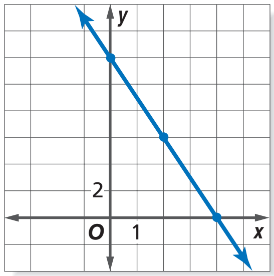
20. 

21. 

22. 

23. 

24. 

25. 

26. TRAVEL On an island cruise in Hawaii, each passenger is given a flower chain. A crew member hands out 3 red, 3 blue, and 3 green chains in that order. If this pattern is repeated, what color chain will the 50th person receive?

More About. . .

Number Theory

Fibonacci numbers occur in many areas of nature, including pine cones, shell spirals, flower petals, branching plants, and many fruits and vegetables.

NUMBER THEORY **For Exercises 27 and 28, use the following information.**
In 1201, Leonardo Fibonacci introduced his now famous pattern of numbers called the Fibonacci sequence.

$$1, 1, 2, 3, 5, 8, 13, \ldots$$

Notice the pattern in this sequence. After the second number, each number in the sequence is the sum of the two numbers that precede it. That is $2 = 1 + 1$, $3 = 2 + 1$, $5 = 3 + 2$, and so on.

27. Write the first 12 terms of the Fibonacci sequence.

28. Notice that every third term is divisible by 2. What do you notice about every fourth term? every fifth term?

FITNESS For Exercises 29 and 30, use the table below that shows the maximum heart rate to maintain, for different ages, during aerobic activities such as running, biking, or swimming.

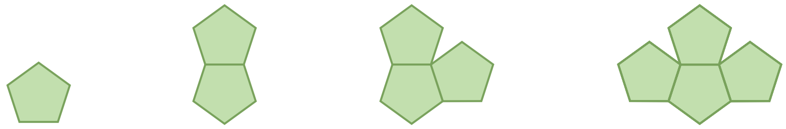
Age (yr)	20	30	40	50	60	70
Pulse rate (beats/min)	175	166	157	148	139	130

Source: Ontario Association of Sport and Exercise Sciences

29. Write an equation in function notation for the relation.

30. What would be the maximum heart rate to maintain in aerobic training for a 10-year old? an 80-year old?

CRITICAL THINKING For Exercises 31–33, use the following information. Suppose you arrange a number of regular pentagons so that only one side of each pentagon touches. Each side of each pentagon is 1 centimeter.



1 pentagon 2 pentagons 3 pentagons 4 pentagons

31. For each arrangement of pentagons, compute the perimeter.

32. Write an equation in function form to represent the perimeter $f(n)$ of n pentagons.

33. What is the perimeter if 24 pentagons are used?

34. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

Why is writing equations from patterns important in science?

Include the following in your answer:

- an explanation of the relationship between the volume of water and the volume of ice, and
- a reasonable estimate of the size of a container that had 99 cubic feet of water, if it was going to be frozen.

Standardized Test Practice

35. Find the next two terms in the sequence 3, 4, 6, 9, … .

Ⓐ 12, 15 Ⓑ 13, 18 Ⓒ 14, 19 Ⓓ 15, 21

36. After P pieces of candy are divided equally among 5 children, 4 pieces remain. How many would remain if $P + 4$ pieces of candy were divided equally among the 5 children?

Ⓐ 0 Ⓑ 1 Ⓒ 2 Ⓓ 3

Maintain Your Skills

Mixed Review **Find the next three terms of each arithmetic sequence.** *(Lesson 4-7)*

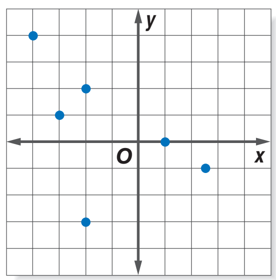
37. 1, 4, 7, 10, …

38. 9, 5, 1, −3, …

39. −25, −19, −13, −7, …

40. 22, 34, 46, 58, …

41. Determine whether the relation graphed at the right is a function. *(Lesson 4-6)*



42. **GEOGRAPHY** The world's tallest waterfall is Angel Falls in Venezuela at 3212 feet. It is 102 feet higher than Tulega Falls in South Africa. How high is Tulega Falls? *(Lesson 3-2)*

Chapter 4

Study Guide and Review

Vocabulary and Concept Check

arithmetic sequence (p. 233)	graph (p. 193)	preimage (p. 197)	terms (p. 233)
common difference (p. 233)	image (p. 197)	quadrant (p. 193)	transformation (p. 197)
coordinate plane (p. 192)	inductive reasoning (p. 240)	reflection (p. 197)	translation (p. 197)
dilation (p. 197)	inverse (p. 206)	rotation (p. 197)	vertical line test (p. 227)
equation in two variables (p. 212)	linear equation (p. 218)	sequence (p. 233)	*x*-intercept (p. 220)
function (p. 226)	look for a pattern (p. 240)	solution (p. 212)	*y*-intercept (p. 220)
function notation (p. 227)	mapping (p. 205)	standard form (p. 218)	

Choose the letter of the term that best matches each statement or phrase.

1. In the coordinate plane, the axes intersect at the __?__.
2. A(n) __?__ is a set of ordered pairs.
3. A(n) __?__ flips a figure over a line.
4. In a coordinate system, the __?__ is a horizontal number line.
5. In the ordered pair, $A(2, 7)$, 7 is the __?__.
6. The coordinate axes separate a plane into four __?__.
7. A(n) __?__ has a graph that is a nonvertical straight line.
8. In the relation $\{(4, -2), (0, 5), (6, 2), (-1, 8)\}$, the __?__ is $\{-1, 0, 4, 6\}$.
9. A(n) __?__ enlarges or reduces a figure.
10. In a coordinate system, the __?__ is a vertical number line.

a. domain
b. dilation
c. linear function
d. reflection
e. origin
f. quadrants
g. relation
h. *x*-axis
i. *y*-axis
j. *x*-coordinate
k. *y*-coordinate

Lesson-by-Lesson Review

4-1 The Coordinate Plane

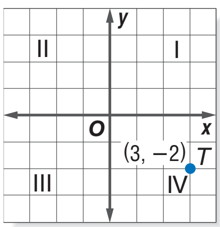
See pages 192–196.

Concept Summary

- The first number, or *x*-coordinate, of an ordered pair corresponds to the numbers on the *x*-axis.
- The second number, or *y*-coordinate, corresponds to the numbers on the *y*-axis.

Example

Plot $T(3, -2)$ on a coordinate plane. Name the quadrant in which the point is located.



$T(3, -2)$ is located in Quadrant IV.

Exercises **Plot each point on a coordinate plane.** *See Example 3 on page 193.*

11. $A(4, 2)$
12. $B(-1, 3)$
13. $C(0, -5)$
14. $D(-3, -2)$
15. $E(-4, 0)$
16. $F(2, -1)$

www.algebra1.com/vocabulary_review

4-2 Transformations on the Coordinate Plane

See pages 197–203.

Concept Summary

- A reflection is a flip.
- A translation is a slide.
- A dilation is a reduction or enlargement.
- A rotation is a turn.

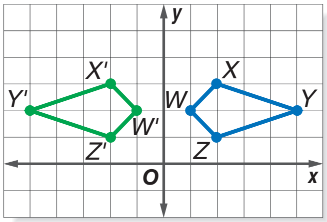
Example **A quadrilateral with vertices $W(1, 2)$, $X(2, 3)$, $Y(5, 2)$, and $Z(2, 1)$ is reflected over the y-axis. Find the coordinates of the vertices of the image. Then graph quadrilateral $WXYZ$ and its image $W'X'Y'Z'$.**

Multiply each x-coordinate by –1.

$W(1, 2) \rightarrow W'(-1, 2)$ $\quad$ $Y(5, 2) \rightarrow Y'(-5, 2)$

$X(2, 3) \rightarrow X'(-2, 3)$ $\quad$ $Z(2, 1) \rightarrow Z'(-2, 1)$

The coordinates of the image are $W'(-1, 2)$, $X'(-2, 3)$, $Y'(-5, 2)$, and $Z'(-2, 1)$.



Exercises **Find the coordinates of the vertices of each figure after the given transformation is performed. Then graph the preimage and its image.**

See Examples 2–5 on pages 198–200.

17. triangle ABC with $A(3, 3)$, $B(5, 4)$, and $C(4, -3)$ reflected over the x-axis
18. quadrilateral $PQRS$ with $P(-2, 4)$, $Q(0, 6)$, $R(3, 3)$, and $S(-1, -4)$ translated 3 units down
19. parallelogram $GHIJ$ with $G(2, 2)$, $H(6, 0)$, $I(6, 2)$, and $J(2, 4)$ dilated by a scale factor of $\frac{1}{2}$
20. trapezoid $MNOP$ with $M(2, 0)$, $N(4, 3)$, $O(6, 3)$, and $P(8, 0)$ rotated 90° counterclockwise about the origin

4-3 Relations

See pages 205–211.

Concept Summary

- A relation can be expressed as a set of ordered pairs, a table, a graph, or a mapping.

Example **Express the relation {(3, 2), (5, 3), (4, 3), (5, 2)} as a table, a graph, and a mapping.**

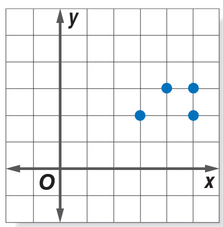
Table

List the set of x-coordinates and corresponding y-coordinates.

x	y
3	2
5	3
4	3
5	2

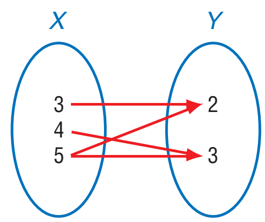
Graph

Graph each ordered pair on a coordinate plane.



Mapping

List the x and y values. Draw arrows to show the relation.



Exercises **Express each relation as a table, a graph, and a mapping. Then determine the domain and range.** *See Example 1 on page 205.*

21. $\{(-2, 6), (3, -2), (3, 0), (4, 6)\}$ **22.** $\{(-1, 0), (3, 0), (6, 2)\}$

23. $\{(3, 8), (9, 3), (-3, 8), (5, 3)\}$ **24.** $\{(2, 5), (-3, 1), (4, -2), (2, 3)\}$

4-4 Equations as Relations

See pages 212–217.

Concept Summary

- In an equation involving x and y, the set of x values is the domain, and the corresponding set of y values is the range.

Example **Solve $2x + y = 8$ if the domain is $\{3, 2, 1\}$. Graph the solution set.**

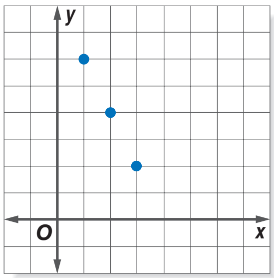
First solve the equation for y in terms of x.

$2x + y = 8$ Original equation

$y = 8 - 2x$ Subtract $2x$ from each side.

Substitute each value of x from the domain to determine the corresponding values of y in the range. Then graph the solution set $\{(3, 2), (2, 4), (1, 6)\}$.

x	$8 - 2x$	y	(x, y)
3	$8 - 2(3)$	2	(3, 2)
2	$8 - 2(2)$	4	(2, 4)
1	$8 - 2(1)$	6	(1, 6)



Exercises **Solve each equation if the domain is $\{-4, -2, 0, 2, 4\}$. Graph the solution set.** *See Example 3 on page 213.*

25. $y = x - 9$ **26.** $y = 4 - 2x$ **27.** $4x - y = -5$

28. $2x + y = 8$ **29.** $3x + 2y = 9$ **30.** $4x - 3y = 0$

4-5 Graphing Linear Equations

See pages 218–223.

Concept Summary

- Standard form: $Ax + By = C$, where $A \geq 0$ and A and B are not both zero
- To find the x-intercept, let $y = 0$. To find the y-intercept, let $x = 0$.

Example **Determine the x- and y-intercepts of $3x - y = 4$. Then graph the equation.**

To find the x-intercept, let $y = 0$.

$3x - y = 4$ Original equation

$3x - 0 = 4$ Replace y with 0.

$3x = 4$ Simplify.

$x = \frac{4}{3}$ Divide each side by 3.

To find the y-intercept, let $x = 0$.

$3x - y = 4$ Original equation

$3(0) - y = 4$ Replace x with 0.

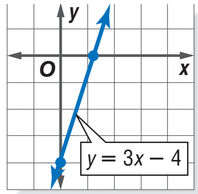
$-y = 4$ Simplify.

$y = -4$ Divide each side by -1.

The x-intercept is $\frac{4}{3}$, so the graph intersects the x-axis at $\left(\frac{4}{3}, 0\right)$.

The y-intercept is -4, so the graph intersects the y-axis at $(0, -4)$.

Plot these points, then draw a line that connects them.



Exercises **Graph each equation.** *See Examples 2 and 4 on pages 219 and 220.*

31. $y = -x + 2$ **32.** $x + 5y = 4$ **33.** $2x - 3y = 6$

34. $5x + 2y = 10$ **35.** $\frac{1}{2}x + \frac{1}{3}y = 3$ **36.** $y - \frac{1}{3} = \frac{1}{3}x + \frac{2}{3}$

4-6 Functions

See pages 226–231.

Concept Summary

- A relation is a function if each element of the domain is paired with exactly one element of the range.
- Substitute values for x to determine $f(x)$ for a specific value.

Examples

1 **Determine whether the relation $\{(0, -4), (1, -1), (2, 2), (6, 3)\}$ is a function.**

Since each element of the domain is paired with exactly one element of the range, the relation is a function.

2 **If $g(x) = 2x - 1$, find $g(-6)$.**

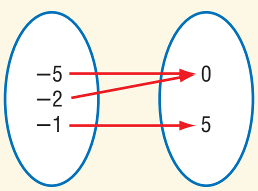
$g(-6) = 2(-6) - 1$ Replace x with -6.

$= -12 - 1$ Multiply.

$= -13$ Subtract.

Exercises **Determine whether each relation is a function.** *See Example 1 on page 226.*

37.



38.

x	y
5	3
1	4
−6	5
1	6
−2	7

39. $\{(2, 3), (-3, -4), (-1, 3)\}$

If $g(x) = x^2 - x + 1$, find each value. *See Examples 3 and 4 on pages 227 and 228.*

40. $g(2)$ **41.** $g(-1)$ **42.** $g\left(\frac{1}{2}\right)$

43. $g(5) - 3$ **44.** $g(a)$ **45.** $g(-2a)$

4-7 Arithmetic Sequences

See pages 233–238.

Concept Summary

- An arithmetic sequence is a numerical pattern that increases or decreases at a constant rate or value called the common difference.
- To find the next term in an arithmetic sequence, add the common difference to the last term.

Chapter 4 For More ...
• Extra Practice, see pages 828–830.
• Mixed Problem Solving, see page 856.

Example **Find the next three terms of the arithmetic sequence 10, 23, 36, 49, … .**

Find the common difference.

$$10 \xrightarrow{+13} 23 \xrightarrow{+13} 36 \xrightarrow{+13} 49$$

So, $d = 13$.

Add 13 to the last term of the sequence to get the next term. Continue adding 13 until the next three terms are found.

$$49 \xrightarrow{+13} 62 \xrightarrow{+13} 75 \xrightarrow{+13} 88$$

The next three terms are 62, 75, and 88.

Exercises **Find the next three terms of each arithmetic sequence.**
See Example 2 on page 234.

46. 9, 18, 27, 36, …
47. 6, 11, 16, 21, …
48. 10, 21, 32, 43, …
49. 14, 12, 10, 8, …
50. −3, −11, −19, −27, …
51. −35, −29, −23, −17, …

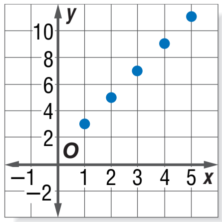
4-8 Writing Equations from Patterns

See pages 240–245.

Concept Summary

- Look for a pattern in data. If the relationship between the domain and range is linear, the relationship can be described by an equation.

Example **Write an equation in function notation for the relation graphed at the right.**



Make a table of ordered pairs for several points on the graph.

x	1	2	3	4	5
y	3	5	7	9	11

The difference in y values is twice the difference of x values. This suggests that $y = 2x$. However, $3 \neq 2(1)$. Compare the values of y to the values of $2x$.

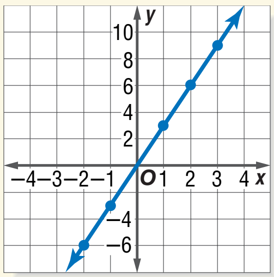
x	1	2	3	4	5
2x	2	4	6	8	10
y	3	5	7	9	11

y is always 3 more than $2x$.

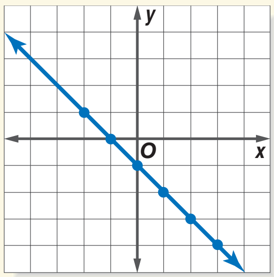
The difference between y and $2x$ is always 1. So the equation is $y = 2x + 1$. Since this relation is also a function, it can be written as $f(x) = 2x + 1$.

Exercises **Write an equation in function notation for each relation.**
See Example 4 on pages 242 and 243.

52.



53.



Chapter 4 Practice Test

Vocabulary and Concepts

Choose the letter that best matches each description.

1. a figure turned around a point
2. a figure slid horizontally, vertically, or both
3. a figure flipped over a line

a.	reflection
b.	rotation
c.	translation

Skills and Applications

4. Graph $K(0, -5)$, $M(3, -5)$, and $N(-2, -3)$.
5. Name the quadrant in which $P(25, 1)$ is located.

For Exercises 6 and 7, use the following information.
A parallelogram has vertices $H(-2, -2)$, $I(-4, -6)$, $J(-5, -5)$, and $K(-3, -1)$.

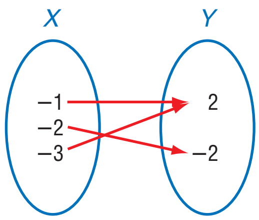
6. Reflect parallelogram $HIJK$ over the y-axis and graph its image.
7. Translate parallelogram $HIJK$ up 2 units and graph its image.

Express the relation shown in each table, mapping, or graph as a set of ordered pairs. Then write the inverse of the relation.

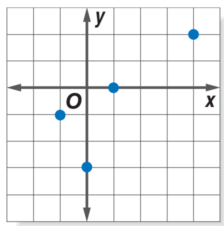
8.

x	$f(x)$
0	−1
2	4
4	5
6	10

9.



10.



Solve each equation if the domain is $\{-2, -1, 0, 2, 4\}$. Graph the solution set.

11. $y = -4x + 10$
12. $3x - y = 10$
13. $\frac{1}{2}x - y = 5$

Graph each equation.

14. $y = x + 2$
15. $x + 2y = -1$
16. $-3x = 5 - y$

Determine whether each relation is a function.

17. $\{(2, 4), (3, 2), (4, 6), (5, 4)\}$
18. $\{(3, 1), (2, 5), (4, 0), (3, -2)\}$
19. $8y = 7 + 3x$

If $f(x) = -2x + 5$ and $g(x) = x^2 - 4x + 1$, find each value.

20. $g(-2)$
21. $f\left(\frac{1}{2}\right)$
22. $g(3a) + 1$
23. $f(x + 2)$

Determine whether each sequence is an arithmetic sequence. If it is, state the common difference.

24. 16, 24, 32, 40, …
25. 99, 87, 76, 65, …
26. 5, 17, 29, 41, …

Find the next three terms in each sequence.

27. 5, −10, 15, −20, 25, …
28. 5, 5, 6, 8, 11, 15, …

29. **TEMPERATURE** The equation to convert Celsius temperature to Kelvin temperature is $K = C + 273$. Solve the equation for C. State the independent and dependent variables. Choose five values for K and their corresponding values for C.

30. **STANDARDIZED TEST PRACTICE** If $f(x) = 3x - 2$, find $f(8) - f(-5)$.

(A) 7 (B) 9 (C) 37 (D) 39

Chapter 4 Standardized Test Practice

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. The number of students in Highview School is currently 315. The school population is predicted to increase by 2% next year. According to the prediction, how many students will attend next year? (Prerequisite Skill)

(A) 317 (B) 321
(C) 378 (D) 630

2. In 2001, two women skied 1675 miles in 89 days across the land mass of Antarctica. They still had to ski 508 miles across the Ross Ice Shelf to reach McMurdo Station. About what percent of their total distance remained? (Prerequisite Skill)

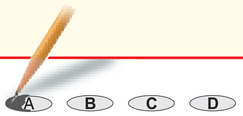
(A) 2% (B) 17%
(C) 23% (D) 30%

3. Only 2 out of 5 students surveyed said they eat five servings of fruits or vegetables daily. If there are 470 students in a school, how many would you predict eat five servings of fruits or vegetables daily? (Lesson 2-6)

(A) 94 (B) 188
(C) 235 (D) 282

4. Solve $13x = 2(5x + 3)$ for x. (Lesson 3-4)

(A) 0 (B) 2 (C) 3 (D) 4



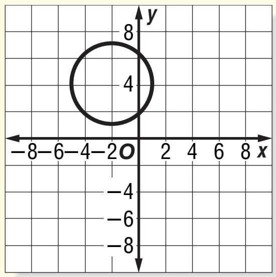
Test-Taking Tip

Questions 4 and 14

Some multiple-choice questions ask you to solve an equation or inequality. You can check your solution by replacing the variable in the equation or inequality with your answer. The answer choice that results in a true statement is the correct answer.

5. The circle shown below passes through points at (1, 4),(−2, 1), (−5, 4), and (−2, 7). Which point represents the center of the circle? (Lesson 4-1)

(A) (−2, −4)
(B) (−2, 4)
(C) (−4, 2)
(D) (4, −2)



6. Which value of x would cause the relation {(2, 5), (x, 8), (7, 10)} *not* to be a function? (Lesson 4-4)

(A) 1 (B) 2 (C) 5 (D) 8

7. Which ordered pair (x, y) is a solution of $3x + 4y = 12$? (Lesson 4-4)

(A) (−2, 4) (B) (0, −3)
(C) (1, 2) (D) (4, 0)

8. Which missing value for y would make this relation a linear relation? (Lesson 4-7)

(A) −2
(B) 0
(C) 1
(D) 2

x	y
1	−3
2	−1
3	?
4	3

9. Which equation describes the data in the table? (Lesson 4-8)

(A) $y = -2x + 1$
(B) $y = x + 1$
(C) $y = -x + 3$
(D) $y = x - 5$

x	y
−2	5
1	2
4	−1
6	−3

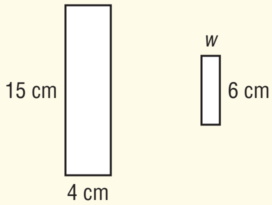
CONTENTS

Preparing for Standardized Tests
For test-taking strategies and more practice, see pages 867–884.

Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

10. The lengths of the corresponding sides of these two rectangles are proportional. What is the width w? (Lesson 2-6)



11. The PTA at Fletcher's school sold raffle tickets for a television set. Two thousand raffle tickets were sold. Fletcher's family bought 25 raffle tickets. What is the probability that his family will win the television? Express the answer as a percent. (Lesson 2-7)

12. The sum of three integers is 52. The second integer is 3 more than the first. The third integer is 1 more than twice the first. What are the integers? (Lessons 3-1 and 3-4)

13. Solve $5(x - 2) - 3(x + 4) = 10$ for x. (Lesson 3-4)

14. A CD player originally cost \$160. It is now on sale for \$120. What is the percent of decrease in its price? (Lesson 3-5)

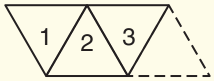
15. A swimming pool holds 1800 cubic feet of water. It is 6 feet deep and 20 feet long. How many feet wide is the pool? ($V = \ell wh$) (Lesson 3-8)

16. Write the ordered pair that describes a point 7 units up from and 3 units to the left of the origin. (Lesson 4-1)

17. A triangle that has vertices $D(1, 3)$, $E(7, 2)$, and $F(-3, 4)$ is reflected over the x-axis. Find the coordinates of the vertices of the image. (Lesson 4-2)

18. The range for $2x + y = -5$ is $\{1, -13, -5, -7\}$. Find the domain. (Lesson 4-4)

19. Garth used toothpicks to form a pattern of triangles as shown below. If he continues this pattern, what is the total number of toothpicks that he will use to form a pattern of 7 triangles? (Lessons 4-7 and 4-8)



Part 3 Extended Response

Record your answers on a sheet of paper. Show your work.

20. A car company lists the stopping distances of a car at different speeds. Does the table of values represent a function? Explain. (Lesson 4-6)

Speed (ft/s)	Minimum Stopping Distance (ft)
10	2
20	8
40	31
60	70
100	194

21. Latoya bought 48 one-foot-long sections of fencing. She plans to use the fencing to enclose a rectangular area for a garden. (Lesson 3-8)

a. Using ℓ for the length and w for the width of the garden, write an equation for its perimeter.

b. If the length ℓ in feet and width w in feet are positive integers, what is the greatest possible area of this garden?

c. If the length and width in feet are positive integers what is the least possible area of the garden?

d. How do the shapes of the gardens with the greatest and least areas compare?