Linear equations are used to model a variety of real-world situations. The concept of slope allows you to analyze how a quantity changes over time. You can use a linear equation to model the cost of the space program. The United States began its exploration of space in January, 1958, when it launched its first satellite into orbit. In the 1970s, NASA developed the space shuttle to reduce costs by inventing the first reusable spacecraft.

You will use a linear equation to model the cost of the space program in Lesson 5-7.
Prerequisite Skills  
To be successful in this chapter, you’ll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 5.

For Lesson 5-1

Simplify Fractions

(For review, see pages 798 and 799.)

1. \(\frac{2}{10}\)
2. \(\frac{8}{12}\)
3. \(\frac{2}{-8}\)
4. \(\frac{-4}{8}\)
5. \(\frac{-5}{-15}\)
6. \(\frac{-7}{-28}\)
7. \(\frac{9}{3}\)
8. \(\frac{18}{12}\)

For Lesson 5-2

Evaluate Expressions

(For review, see Lesson 1-2.)

9. \(a = 6, b = 5, c = 8, d = 4\)
10. \(a = 5, b = -1, c = 2, d = -1\)
11. \(a = -2, b = 1, c = 4, d = 0\)
12. \(a = 8, b = -2, c = -1, d = 1\)
13. \(a = -3, b = -3, c = 4, d = 7\)
14. \(a = \frac{1}{2}, b = \frac{3}{2}, c = 7, d = 9\)

For Lessons 5-3 through 5-7

Identify Points on a Coordinate Plane

Write the ordered pair for each point.

(For review, see Lesson 4-1.)

15. \(J\)
16. \(K\)
17. \(L\)
18. \(M\)
19. \(N\)
20. \(P\)

Writing Linear Equations  
Make this Foldable to help you organize your notes. Begin with four sheets of grid paper.

**Fold and Cut**

Fold each sheet of grid paper in half along the width. Then cut along the crease.

**Cut Tabs**

Cut seven lines from the bottom of the top sheet, six lines from the second sheet, and so on.

**Staple**

Staple the eight half-sheets together to form a booklet.

**Label**

Label each of the tabs with a lesson number. The last tab is for the vocabulary.

Reading and Writing  
As you read and study the chapter, use each page to write notes and to graph examples for each lesson.
Vocabulary
- slope
- rate of change

Why is slope important in architecture?
The slope of a roof describes how steep it is. It is the number of units the roof rises for each unit of run. In the photo, the roof rises 8 feet for each 12 feet of run.

\[ \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{8}{12} = \frac{2}{3} \]

**FIND SLOPE** The slope of a line is a number determined by any two points on the line. This number describes how steep the line is. The greater the absolute value of the slope, the steeper the line. Slope is the ratio of the change in the \(y\)-coordinates (rise) to the change in the \(x\)-coordinates (run) as you move from one point to the other.

The graph shows a line that passes through (1, 3) and (4, 5).

\[ \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{5 - 3}{4 - 1} = \frac{2}{3} \]

So, the slope of the line is \(\frac{2}{3}\).

**Key Concept**
- **Words** The slope of a line is the ratio of the rise to the run.
- **Symbols** The slope \(m\) of a nonvertical line through any two points, \((x_1, y_1)\) and \((x_2, y_2)\), can be found as follows.
  \[ m = \frac{y_2 - y_1}{x_2 - x_1} \]
  - change in \(y\)
  - change in \(x\)
**Example 1**  Positive Slope

Find the slope of the line that passes through \((-1, 2) \) and \((3, 4)\).

Let \((-1, 2) = (x_1, y_1) \) and \((3, 4) = (x_2, y_2)\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 2}{3 - (-1)} = \frac{2}{4} = \frac{1}{2} \text{ Substitute.}
\]

The slope is \(\frac{1}{2}\).

**Example 2**  Negative Slope

Find the slope of the line that passes through \((-1, -2) \) and \((-4, 1)\).

Let \((-1, -2) = (x_1, y_1) \) and \((-4, 1) = (x_2, y_2)\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-2)}{-4 - (-1)} = \frac{3}{3} = 1 \text{ Simplify.}
\]

The slope is \(-1\).

**Example 3**  Zero Slope

Find the slope of the line that passes through \((1, 2)\) and \((-1, 2)\).

Let \((1, 2) = (x_1, y_1) \) and \((-1, 2) = (x_2, y_2)\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 2}{-1 - 1} = \frac{0}{-2} = 0 \text{ Simplify.}
\]

The slope is zero.

**Example 4**  Undefined Slope

Find the slope of the line that passes through \((1, -2)\) and \((1, 3)\).

Let \((1, -2) = (x_1, y_1) \) and \((1, 3) = (x_2, y_2)\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - (-2)}{1 - 1} = \frac{5}{0} \text{ or } \text{ undefined.}
\]

Since division by zero is undefined, the slope is undefined.
Find a Rate of Change

DINING OUT
The graph shows the amount spent on food and drink at U.S. restaurants in recent years.


Use the formula for slope.

$$\frac{\text{rise}}{\text{run}} = \frac{\text{change in quantity}}{\text{change in time}} \leftarrow \text{billion } \$ \quad \leftarrow \text{years}$$

If you know the slope of a line and the coordinates of one of the points on a line, you can find the coordinates of other points on the line.

**Example 5** Find Coordinates Given Slope

Find the value of \( r \) so that the line through \((r, 6)\) and \((10, -3)\) has a slope of \(-\frac{3}{2}\).

Let \((r, 6) = (x_1, y_1)\) and \((10, -3) = (x_2, y_2)\).

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Slope formula}$$

$$\frac{-3 - 6}{10 - r} = \frac{-3}{2} \quad \text{Substitute.}$$

$$\frac{-3}{2} = \frac{-9}{10 - r} \quad \text{Subtract.}$$

$$-3(10 - r) = 2(-9) \quad \text{Find the cross products.}$$

$$-30 + 3r = -18 \quad \text{Simplify.}$$

$$-30 + 3r + 30 = -18 + 30 \quad \text{Add 30 to each side.}$$

$$3r = 12 \quad \text{Simplify.}$$

$$\frac{3r}{3} = \frac{12}{3} \quad \text{Divide each side by 3.}$$

$$r = 4 \quad \text{Simplify.}$$

**RATE OF CHANGE** Slope can be used to describe a rate of change. The rate of change tells, on average, how a quantity is changing over time.

**Example 6** Find a Rate of Change

DINING OUT The graph shows the amount spent on food and drink at U.S. restaurants in recent years.


Use the formula for slope.

- **1980–1990**
  - \( \frac{\text{rise}}{\text{run}} = \frac{\$200 - \$100}{10 - 5} = \frac{\$100}{5} = \$20/\text{year} \)
  - Rate of change = \$20 per year

- **1990–2000**
  - \( \frac{\text{rise}}{\text{run}} = \frac{\$300 - \$200}{10 - 5} = \frac{\$100}{5} = \$20/\text{year} \)
  - Rate of change = \$20 per year
Check for Understanding

Concept Check

1. Explain how you would find the slope of the line at the right.

2. OPEN ENDED Draw the graph of a line having each slope.
   a. positive slope     b. negative slope
   c. slope of 0        d. undefined slope

3. Explain why the formula for determining slope using the coordinates of two points does not apply to vertical lines.

4. FIND THE ERROR Carlos and Allison are finding the slope of the line that passes through (2, 6) and (5, 3).

   **Carlos**
   \[
   \frac{5 - 6}{5 - 2} = \frac{-1}{3} \text{ or } -1
   \]

   **Allison**
   \[
   \frac{6 - 3}{5 - 2} = \frac{3}{3} \text{ or } 1
   \]

   Who is correct? Explain your reasoning.

Guided Practice

Find the slope of the line that passes through each pair of points.
5. (1, 1), (3, 4)  6. (0, 0), (5, 4)  7. (−2, 2), (−1, −2)
8. (7, −4), (9, −1)  9. (3, 5), (−2, 5)  10. (−1, 3), (−1, 0)

Find the value of \( r \) so the line that passes through each pair of points has the given slope.
11. (6, −2), (\( r \), −6), \( m = 4 \)  12. (9, \( r \)), (6, 3), \( m = \frac{1}{3} \)
Application  CABLE TV  For Exercises 13 and 14, use the graph at the right.


**Practice and Apply**

Find the slope of the line that passes through each pair of points.

15. \((-2, -4), (2, 1)\)
16. \((0, 3), (3, 2)\)

17. \((-4, -1), (-3, -3)\)
18. \((-3, 3), (1, 3)\)
19. \((-2, 1), (-2, 3)\)
20. \((2, 3), (9, 7)\)
21. \((5, 7), (-2, -3)\)
22. \((-3, 6), (2, 4)\)
23. \((-3, -4), (5, -1)\)
24. \((2, -1), (5, -3)\)
25. \((-5, 4), (-5, -1)\)
26. \((2, 6), (-1, -3)\)
27. \((-2, 3), (8, 3)\)
28. \((-3, 9), (-7, 6)\)
29. \((-8, 3), (-6, 2)\)
30. \((-2, 0), (1, -1)\)
31. \((4.5, -1), (5.3, 2)\)
32. \((0.75, 1), (0.75, -1)\)
33. \(\left(2\frac{1}{2}, -1\frac{1}{2}\right), \left(-\frac{1}{2}, \frac{1}{2}\right)\)
34. \(\left(\frac{3}{4}, \frac{1}{4}\right), \left(-\frac{1}{2}, -1\right)\)

ARCHITECTURE  Use a ruler to estimate the slope of each roof.

35.
36.

37. Find the slope of the line that passes through the origin and \((r, s)\).
38. What is the slope of the line that passes through \((a, b)\) and \((a, -b)\)?

39. PAINTING  A ladder reaches a height of 16 feet on a wall. If the bottom of the ladder is placed 4 feet away from the wall, what is the slope of the ladder as a positive number?
40. **PART-TIME JOBS** In 1991, the federal minimum wage rate was $4.25 per hour. In 1997, it was increased to $5.15. Find the annual rate of change in the federal minimum wage rate from 1991 to 1997.

Find the value of $r$ so the line that passes through each pair of points has the given slope.

41. $(6, 2), (9, \ r), \ m = -1$
42. $(4, -5), (3, \ r), \ m = 8$
43. $(5, \ r), (2, -3), \ m = \frac{4}{3}$
44. $(-2, 7), (r, 3), \ m = \frac{4}{3}$
45. $\left(\frac{1}{2}, -\frac{1}{4}\right), \left(r, -\frac{5}{4}\right), \ m = 4$
46. $\left(\frac{2}{3}, r\right), \left(1, \frac{1}{2}\right), \ m = \frac{1}{2}$
47. $(4, r), (r, 2), \ m = -\frac{5}{3}$
48. $(r, 5), (-2, r), \ m = -\frac{2}{9}$

49. **CRITICAL THINKING** Explain how you know that the slope of the line through $(-4, -5)$ and $(4, 5)$ is positive without calculating.

**HEALTH** For Exercises 50–52, use the table that shows Karen’s height from age 12 to age 20.

<table>
<thead>
<tr>
<th>Age (years)</th>
<th>12</th>
<th>14</th>
<th>16</th>
<th>18</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (inches)</td>
<td>60</td>
<td>64</td>
<td>66</td>
<td>67</td>
<td>67</td>
</tr>
</tbody>
</table>

50. Make a broken-line graph of the data.
51. Use the graph to determine the two-year period when Karen grew the fastest. Explain your reasoning.
52. Explain the meaning of the horizontal section of the graph.

**SCHOOL** For Exercises 53–55, use the graph that shows public school enrollment.

53. For which 5-year period was the rate of change the greatest? When was the rate of change the least?
54. Find the rate of change from 1985 to 1990.
55. Explain the meaning of the part of the graph with a negative slope.

56. **RESEARCH** Use the Internet or other reference to find the population of your city or town in 1930, 1940, …, 2000. For which decade was the rate of change the greatest?

57. **CONSTRUCTION** The slope of a stairway determines how easy it is to climb the stairs. Suppose the vertical distance between two floors is 8 feet 9 inches. Find the total run of the ideal stairway in feet and inches.
58. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

Why is slope important in architecture?
Include the following in your answer:
• an explanation of how to find the slope of a roof, and
• a comparison of the appearance of roofs with different slopes.

59. The slope of the line passing through (5, -4) and (5, -10) is
   A) positive.  B) negative.  C) zero.  D) undefined.

60. The slope of the line passing through (a, b) and (c, d) is
   A) \( \frac{d - c}{b - a} \)  B) \( \frac{b - d}{a - c} \)  C) \( \frac{d - b}{a - c} \)  D) \( \frac{a - c}{b - d} \).

61. Choose four different pairs of points from those labeled on the graph. Find the slope of the line using the coordinates of each pair of points. Describe your findings.

62. **MAKE A CONJECTURE** Determine whether Q(2, 3), R(-1, -1), and S(-4, -2) lie on the same line. Explain your reasoning.

---

**Maintain Your Skills**

**Mixed Review**

Write an equation for each function. **(Lesson 4-8)**

63. | x   | 1 | 2 | 3 | 4 | 5 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>25</td>
</tr>
</tbody>
</table>

64. | x   | -2| -1| 1 | 2 | 4 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>13</td>
<td>12</td>
<td>10</td>
<td>9</td>
<td>7</td>
</tr>
</tbody>
</table>

Determine whether each relation is a function. **(Lesson 4-6)**

65. \( y = -15 \)
66. \( x = 5 \)
67. \( \{(1, 0), (1, 4), (-1, 1)\} \)
68. \( \{(6, 3), (5, -2), (2, 3)\} \)
69. Graph \( x - y = 0 \). **(Lesson 4-4)**
70. What number is 40% of 37.5? **(Prerequisite Skill)**

Find each product. **(Lesson 2-4)**

71. \( 7(-3) \)
72. \( (-4)(-2) \)
73. \( (9)(-4) \)
74. \( (-8)(3.7) \)
75. \( \left(-\frac{7}{8}\right)\left\frac{1}{3}\right) \)
76. \( \left(\frac{1}{4}\right)\left(\frac{1}{2}\right)(-14) \)

**Getting Ready for the Next Lesson**

**PREREQUISITE SKILL** Find each quotient.
(To review dividing fractions, see pages 800 and 801.)

77. \( 6 \div \frac{2}{3} \)
78. \( 12 \div \frac{1}{4} \)
79. \( 10 \div \frac{3}{8} \)
80. \( \frac{1}{2} \div \frac{1}{3} \)
81. \( \frac{3}{4} \div \frac{1}{6} \)
82. \( \frac{3}{4} \div 6 \)
83. \( 18 \div \frac{7}{8} \)
84. \( \frac{3}{8} \div \frac{2}{5} \)
85. \( 2\frac{2}{3} \div \frac{1}{4} \)
Mathematical Words and Everyday Words

You may have noticed that many words used in mathematics are also used in everyday language. You can use the everyday meaning of these words to better understand their mathematical meaning. The table shows two mathematical words along with their everyday and mathematical meanings.

<table>
<thead>
<tr>
<th>Word</th>
<th>Everyday Meaning</th>
<th>Mathematical Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>expression</td>
<td>1. something that expresses or communicates in words, art, music, or movement</td>
<td>one or more numbers or variables along with one or more arithmetic operations</td>
</tr>
<tr>
<td></td>
<td>2. the manner in which one expresses oneself, especially in speaking, depicting, or performing</td>
<td></td>
</tr>
<tr>
<td>function</td>
<td>1. the action for which one is particularly fitted or employed</td>
<td>a relationship in which the output depends upon the input</td>
</tr>
<tr>
<td></td>
<td>2. an official ceremony or a formal social occasion</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3. something closely related to another thing and dependent on it for its existence, value, or significance</td>
<td></td>
</tr>
</tbody>
</table>

Source: The American Heritage Dictionary of the English Language

Notice that the mathematical meaning is more specific, but related to the everyday meaning. For example, the mathematical meaning of \textit{expression} is closely related to the first everyday definition. In mathematics, an expression communicates using symbols.

Reading to Learn

1. How does the mathematical meaning of \textit{function} compare to the everyday meaning?

2. \textbf{RESEARCH} Use the Internet or other reference to find the everyday meaning of each word below. How might these words apply to mathematics? Make a table like the one above and note the mathematical meanings that you learn as you study Chapter 5.
   a. slope
   b. intercept
   c. parallel
A standard showerhead uses about 6 gallons of water per minute. If you graph the ordered pairs from the table, the slope of the line is 6.

The equation is $y = 6x$. The number of gallons of water $y$ depends directly on the amount of time in the shower $x$.

**DIRECT VARIATION** A direct variation is described by an equation of the form $y = kx$, where $k \neq 0$. We say that $y$ varies directly with $x$ or $y$ varies directly as $x$. In the equation $y = kx$, $k$ is the constant of variation.

**Example 1**  **Slope and Constant of Variation**

Name the constant of variation for each equation. Then find the slope of the line that passes through each pair of points.

**a.**

The constant of variation is 3.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$  
Slope formula

$$m = \frac{3 - 0}{1 - 0} = \frac{3}{1} = 3$$

The slope is 3.

**b.**

The constant of variation is $-2$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$  
Slope formula

$$m = \frac{-2 - 0}{1 - 0} = \frac{-2}{1} = -2$$

The slope is $-2$.

Compare the constant of variation with the slope of the graph for each example. Notice that the slope of the graph of $y = kx$ is $k$. 
The ordered pair \((0, 0)\) is a solution of \(y = kx\). Therefore, the graph of \(y = kx\) passes through the origin. You can use this information to graph direct variation equations.

**Example 2** Direct Variation with \(k > 0\)

Graph \(y = 4x\).

Step 1 Write the slope as a ratio.

\[
4 = \frac{4}{1} \quad \text{rise}
\]

\[
1 \quad \text{run}
\]

Step 2 Graph \((0, 0)\).

Step 3 From the point \((0, 0)\), move up 4 units and right 1 unit. Draw a dot.

Step 4 Draw a line containing the points.

**Example 3** Direct Variation with \(k < 0\)

Graph \(y = -\frac{1}{3}x\).

Step 1 Write the slope as a ratio.

\[
-\frac{1}{3} = \frac{-1}{3} \quad \text{rise}
\]

\[
\frac{1}{3} \quad \text{run}
\]

Step 2 Graph \((0, 0)\).

Step 3 From the point \((0, 0)\), move down 1 unit and right 3 units. Draw a dot.

Step 4 Draw a line containing the points.

A **family of graphs** includes graphs and equations of graphs that have at least one characteristic in common. The **parent graph** is the simplest graph in a family.

**Graphing Calculator Investigation**

**Family of Graphs**

The calculator screen shows the graphs of \(y = x\), \(y = 2x\), and \(y = 4x\).

**Think and Discuss**

1. Describe any similarities among the graphs.
2. Describe any differences among the graphs.
3. Write an equation whose graph has a steeper slope than \(y = 4x\). Check your answer by graphing \(y = 4x\) and your equation.
4. Write an equation whose graph lies between the graphs of \(y = x\) and \(y = 2x\). Check your answer by graphing the equations.
5. Write a description of this family of graphs. What characteristics do the graphs have in common? How are they different?
6. The equations whose graphs are in this family are all of the form \(y = mx\). How does the graph change as the absolute value of \(m\) increases?
Write and Solve a Direct Variation Equation

Suppose \( y \) varies directly as \( x \), and \( y = 28 \) when \( x = 7 \).

a. Write a direct variation equation that relates \( x \) and \( y \).

Find the value of \( k \).

\[
\begin{align*}
y &= kx & \text{Direct variation formula} \\
28 &= k(7) & \text{Replace } y \text{ with 28 and } x \text{ with 7.} \\
\frac{28}{7} &= \frac{k(7)}{7} & \text{Divide each side by 7.} \\
4 &= k & \text{Simplify.} \\
\text{Therefore, } y &= 4x.
\end{align*}
\]

b. Use the direct variation equation to find \( x \) when \( y = 52 \).

\[
\begin{align*}
y &= 4x & \text{Direct variation equation} \\
52 &= 4x & \text{Replace } y \text{ with 52.} \\
\frac{52}{4} &= \frac{4x}{4} & \text{Divide each side by 4.} \\
13 &= x & \text{Simplify.} \\
\text{Therefore, } x &= 13 \text{ when } y = 52.
\end{align*}
\]

**Example 5** Direct Variation Equation

**BIOLOGY** A flock of snow geese migrated 375 miles in 7.5 hours.

a. Write a direct variation equation for the distance flown in any time.

**Words** The distance traveled is 375 miles, and the time is 7.5 hours.

**Variables** Let \( r = \text{rate} \).

**Equation** \[ \text{Distance} = \text{rate} \times \text{time} \]

Solve for the rate.

\[
\begin{align*}
375 &= r(7.5) & \text{Original equation} \\
\frac{375}{7.5} &= \frac{r(7.5)}{7.5} & \text{Divide each side by 7.5.} \\
50 &= r & \text{Simplify.} \\
\text{Therefore, the direct variation equation is } d &= 50t.
\end{align*}
\]
b. Graph the equation.

The graph of \( d = 50t \) passes through the origin with slope 50.

\[
m = \frac{50}{1} \quad \text{rise over run}
\]

c. Estimate how many hours of flying time it would take the geese to migrate 3000 miles.

\[
\frac{d}{50} = t \quad \text{Original equation}
\]

\[
3000 = 50t \quad \text{Replace } d \text{ with 3000.}
\]

\[
\frac{3000}{50} = \frac{50t}{50} \quad \text{Divide each side by 50.}
\]

\[
t = 60 \quad \text{Simplify.}
\]

At this rate, it will take 60 hours of flying time to migrate 3000 miles.

Check for Understanding

Concept Check

1. OPEN ENDED Write a general equation for \( y \) varies directly as \( x \).

2. Choose the equations that represent direct variations. Then find the constant of variation for each direct variation.

   a. \( 15 = rs \)
   b. \( 4a = b \)
   c. \( z = \frac{1}{3}x \)
   d. \( s = \frac{9}{t} \)

3. Explain how the constant of variation and the slope are related in a direct variation equation.

Guided Practice

Name the constant of variation for each equation. Then determine the slope of the line that passes through each pair of points.

4. \[
y = -\frac{1}{3}x
\]

5. \[
y = \frac{1}{2}x
\]

6. \( y = 2x \)
7. \( y = -3x \)
8. \( y = \frac{1}{2}x \)

Write a direct variation equation that relates \( x \) and \( y \). Assume that \( y \) varies directly as \( x \). Then solve.

9. If \( y = 27 \) when \( x = 6 \), find \( x \) when \( y = 45 \).
10. If \( y = 10 \) when \( x = 9 \), find \( x \) when \( y = 9 \).
11. If \( y = -7 \) when \( x = -14 \), find \( y \) when \( x = 20 \).

Application

JOBS For Exercises 12–14, use the following information.

Suppose you work at a job where your pay varies directly as the number of hours you work. Your pay for 7.5 hours is $45.

12. Write a direct variation equation relating your pay to the hours worked.
13. Graph the equation.
14. Find your pay if you work 30 hours.
Name the constant of variation for each equation. Then determine the slope of the line that passes through each pair of points.

15. \( y = 2x \)  
16. \( y = 4x \)  
17. \( y = -\frac{1}{2}x \)

Graph each equation.

21. \( y = x \)  
22. \( y = 3x \)  
23. \( y = -x \)  
24. \( y = -4x \)  
25. \( y = \frac{1}{4}x \)  
26. \( y = \frac{3}{5}x \)  
27. \( y = \frac{5}{2}x \)  
28. \( y = \frac{7}{5}x \)  
29. \( y = \frac{1}{5}x \)  
30. \( y = -\frac{2}{3}x \)  
31. \( y = -\frac{4}{3}x \)  
32. \( y = -\frac{9}{2}x \)

Write a direct variation equation that relates \( x \) and \( y \). Assume that \( y \) varies directly as \( x \). Then solve.

33. If \( y = 8 \) when \( x = 4 \), find \( y \) when \( x = 5 \).
34. If \( y = 36 \) when \( x = 6 \), find \( x \) when \( y = 42 \).
35. If \( y = -16 \) when \( x = 4 \), find \( x \) when \( y = 20 \).
36. If \( y = -18 \) when \( x = 6 \), find \( x \) when \( y = 6 \).
37. If \( y = 4 \) when \( x = 12 \), find \( y \) when \( x = -24 \).
38. If \( y = 12 \) when \( x = 15 \), find \( x \) when \( y = 21 \).
39. If \( y = 2.5 \) when \( x = 0.5 \), find \( y \) when \( x = 20 \).
40. If \( y = -6.6 \) when \( x = 9.9 \), find \( y \) when \( x = 6.6 \).
41. If \( y = \frac{22}{3} \) when \( x = \frac{1}{4} \), find \( y \) when \( x = 1\frac{1}{8} \).
42. If \( y = 6 \) when \( x = \frac{2}{3} \), find \( x \) when \( y = 12 \).

Write a direct variation equation that relates the variables. Then graph the equation.

43. **GEOMETRY**  The circumference \( C \) of a circle is about 3.14 times the diameter \( d \).
44. **GEOMETRY**  The perimeter \( P \) of a square is 4 times the length of a side \( s \).
45. **SEWING**  The total cost is \( C \) for \( n \) yards of ribbon priced at $0.99 per yard.
46. **RETAIL**  Kona coffee beans are $14.49 per pound. The total cost of \( p \) pounds is \( C \).
47. **CRITICAL THINKING** Suppose $y$ varies directly as $x$. If the value of $x$ is doubled, what happens to the value of $y$? Explain.

**BIOLOGY** Which line in the graph represents the sprinting speeds of each animal?

48. elephant, 25 mph
49. reindeer, 32 mph
50. lion, 50 mph
51. grizzly bear, 30 mph

**SPACE** For Exercises 52 and 53, use the following information. The weight of an object on the moon varies directly with its weight on Earth. With all of his equipment, astronaut Neil Armstrong weighed 360 pounds on Earth, but weighed only 60 pounds on the moon.

52. Write an equation that relates weight on the moon $m$ with weight on Earth $e$.
53. Suppose you weigh 138 pounds on Earth. What would you weigh on the moon?

**ANIMALS** For Exercises 54 and 55, use the following information. Most animals age more rapidly than humans do. The chart shows equivalent ages for horses and humans.

54. Write an equation that relates human age to horse age.
55. Find the equivalent horse age for a human who is 16 years old.

56. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

**How is slope related to your shower?**

Include the following in your answer:
- an equation that relates the number of gallons $y$ to the time spent in the shower $x$ for a low-flow showerhead that uses only 2.5 gallons of water per minute, and
- a comparison of the steepness of the graph of this equation to the graph at the top of page 264.

57. Which equation best describes the graph at the right?

- **A** $y = 2x$
- **B** $y = -2x$
- **C** $y = \frac{1}{2}x$
- **D** $y = -\frac{1}{2}x$

58. Which equation does **not** model a direct variation?

- **A** $y = 4x$
- **B** $y = 22x$
- **C** $y = 3x + 1$
- **D** $y = \frac{1}{2}x$

**FAMILIES OF GRAPHS** For Exercises 59–62, use the graphs of $y = -1x$, $y = -2x$, and $y = -4x$, which form a family of graphs.

59. Graph $y = -1x$, $y = -2x$, and $y = -4x$ on the same screen.
60. How are these graphs similar to the graphs in the Graphing Calculator Investigation on page 265? How are they different?
61. Write an equation whose graph has a steeper slope than \( y = -4x \).

62. **MAKE A CONJECTURE** Explain how you can tell without graphing which of two direct variation equations has the graph with a steeper slope.

---

### Maintain Your Skills

**Mixed Review** Find the slope of the line that passes through each pair of points.  
(Lesson 5-1)

63.  
64.  
65.  
66. Find the value of \( r \) so that the line that passes through (1, 7) and (\( r \), 3) has a slope of 2.  
(Lesson 5-1)

---

Each table below represents points on a linear graph. Copy and complete each table.  
(Lesson 4-8)

67.  
68.  
69. \( 15 + (-12) \)  
70. \( 8 - (-5) \)  
71. \( -9 - 6 \)  
72. \( -18 - 12 \)

---

### Getting Ready for the Next Lesson

**PREREQUISITE SKILL** Solve each equation for \( y \).  
(To review rewriting equations, see Lesson 3-8.)

73. \( -3x + y = 8 \)  
74. \( 2x + y = 7 \)  
75. \( 4x = y + 3 \)  
76. \( 2y = 4x + 10 \)  
77. \( 9x + 3y = 12 \)  
78. \( x - 2y = 5 \)

---

### Practice Quiz 1

**Lessons 5-1 and 5-2**

Find the slope of the line that passes through each pair of points.  
(Lesson 5-1)

1. \( (-4, -6), (-3, -8) \)  
2. \( (8, 3), (-11, 3) \)  
3. \( (-4, 8), (5, 9) \)  
4. \( (0, 1), (7, 11) \)

Find the value of \( r \) so the line that passes through each pair of points has the given slope.  
(Lesson 5-1)

5. \( (5, -3), (r, -5), m = 2 \)  
6. \( (6, r), (-4, 9), m = \frac{3}{2} \)

Graph each equation.  
(Lesson 5-2)

7. \( y = -7x \)  
8. \( y = \frac{3}{4}x \)

Write a direct variation equation that relates \( x \) and \( y \). Assume that \( y \) varies directly as \( x \). Then solve.  
(Lesson 5-2)

9. If \( y = 24 \) when \( x = 8 \), find \( y \) when \( x = -3 \).  
10. If \( y = -10 \) when \( x = 15 \), find \( x \) when \( y = -6 \).
**Investigating Slope-Intercept Form**

**Collect the Data**

- Cut a small hole in a top corner of a plastic sandwich bag. Loop a long rubber band through the hole.
- Tape the free end of the rubber band to the desktop.
- Use a centimeter ruler to measure the distance from the desktop to the end of the bag. Record this distance for 0 washers in the bag using a table like the one below.

<table>
<thead>
<tr>
<th>Number of Washers</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

- Place one washer in the plastic bag. Then measure and record the new distance from the desktop to the end of the bag.
- Repeat the experiment, adding different numbers of washers to the bag. Each time, record the number of washers and the distance from the desktop to the end of the bag.

**Analyze the Data**

1. The domain contains values represented by the independent variable, washers. The range contains values represented by the dependent variable, distance. On grid paper, graph the ordered pairs (washers, distance).
2. Write a sentence that describes the points on the graph.
3. Describe the point that represents the trial with no washers in the bag.
4. The rate of change can be found by using the formula for slope.
   \[
   \text{rise} \quad \text{run} = \quad \text{change in distance} \quad \text{change in number of washers}
   \]
   Find the rate of change in the distance from the desktop to the end of the bag as more washers are added.
5. Explain how the rate of change is shown on the graph.

**Make a Conjecture**

- The graph shows sample data from a rubber band experiment. Draw a graph for each situation.
- A bag that hangs 10.5 centimeters from the desktop when empty and lengthens at the rate of the sample.
- A bag that has the same length when empty as the sample and lengthens at a faster rate.
- A bag that has the same length when empty as the sample and lengthens at a slower rate.
Slope-Intercept Form

Vocabulary
- slope-intercept form

What You’ll Learn
• Write and graph linear equations in slope-intercept form.
• Model real-world data with an equation in slope-intercept form.

How is a y-intercept related to a flat fee?
A cellular phone service provider charges $0.10 per minute plus a flat fee of $5.00 each month.

<table>
<thead>
<tr>
<th>x (minutes)</th>
<th>y (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5.00</td>
</tr>
<tr>
<td>1</td>
<td>5.10</td>
</tr>
<tr>
<td>2</td>
<td>5.20</td>
</tr>
<tr>
<td>3</td>
<td>5.30</td>
</tr>
<tr>
<td>4</td>
<td>5.40</td>
</tr>
<tr>
<td>5</td>
<td>5.50</td>
</tr>
<tr>
<td>6</td>
<td>5.60</td>
</tr>
<tr>
<td>7</td>
<td>5.70</td>
</tr>
</tbody>
</table>

The slope of the line is 0.1. It crosses the y-axis at (0, 5).
The equation of the line is \( y = 0.1x + 5 \).

Slope-Intercept Form
An equation of the form \( y = mx + b \) is in slope-intercept form. When an equation is written in this form, you can identify the slope and y-intercept of its graph.

**Example 1** Write an Equation Given Slope and y-Intercept
Write an equation of the line whose slope is 3 and whose y-intercept is 5.

\[ y = mx + b \] Slope-intercept form
\[ y = 3x + 5 \] Replace \( m \) with 3 and \( b \) with 5.
Write an Equation Given Two Points

Step 1 You know the coordinates of two points on the line. Find the slope. Let \((x_1, y_1) = (0, 3)\) and \((x_2, y_2) = (2, -1)\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{rise}
\]

\[
m = \frac{-1 - 3}{2 - 0} \quad \text{run}
\]

\[
x_1 = 0, x_2 = 2 \quad y_1 = 3, y_2 = -1
\]

\[
m = -\frac{4}{2} \quad \text{or} \quad -2 \quad \text{Simplify.}
\]

The slope is \(-2\).

Step 2 The line crosses the \(y\)-axis at \((0, 3)\). So, the \(y\)-intercept is 3.

Step 3 Finally, write the equation.

\[
y = mx + b \quad \text{Slope-intercept form}
\]

\[
y = -2x + 3 \quad \text{Replace } m \text{ with } -2 \text{ and } b \text{ with 3.}
\]

The equation of the line is \(y = -2x + 3\).

One advantage of the slope-intercept form is that it allows you to graph an equation quickly.

Graph an Equation in Slope-Intercept Form

Graph \(y = -\frac{2}{3}x + 1\).

Step 1 The \(y\)-intercept is 1. So, graph \((0, 1)\).

Step 2 The slope is \(-\frac{2}{3}\) or \(-\frac{2}{3} \quad \text{rise}\). From \((0, 1)\), move down 2 units and right 3 units. Draw a dot.

Step 3 Draw a line connecting the points.

Graph an Equation in Standard Form

Graph \(5x - 3y = 6\).

Step 1 Solve for \(y\) to find the slope-intercept form.

\[
5x - 3y = 6 \quad \text{Original equation}
\]

\[
5x - 3y - 5x = 6 - 5x \quad \text{Subtract } 5x \text{ from each side.}
\]

\[
-3y = 6 - 5x \quad \text{Simplify.}
\]

\[
-3y = -5x + 6 \quad 6 - 5x = 6 + (-5x) \text{ or } -5x + 6
\]

\[
\frac{-3y}{-3} = \frac{-5x + 6}{-3} \quad \text{Divide each side by } -3.
\]

\[
\frac{-3y}{-3} = \frac{-5x}{-3} + \frac{6}{-3} \quad \text{Divide each term in the numerator by } -3.
\]

\[
y = \frac{5}{3}x - 2 \quad \text{Simplify.}
\]
Step 2  The \( y \)-intercept of \( y = \frac{5}{3}x - 2 \) is \(-2\).
So, graph \((0, -2)\).

Step 3  The slope is \(\frac{5}{3}\). From \((0, -2)\), move up 5 units
and right 3 units. Draw a dot.

Step 4  Draw a line containing the points.

MODEL REAL-WORLD DATA  If a quantity changes at a constant rate over
time, it can be modeled by a linear equation. The \( y \)-intercept represents a starting
point, and the slope represents the rate of change.

Example 5  Write an Equation in Slope-Intercept Form

AGRICULTURE  The natural sweeteners used in foods include sugar, corn
sweeteners, syrup, and honey. Use the information at the left about natural
sweeteners.

a. The amount of natural sweeteners consumed has increased by an average of
2.6 pounds per year. Write a linear equation to find the average consumption of
natural sweeteners in any year after 1989.

Words  The consumption increased 2.6 pounds per year, so the rate of change
is 2.6 pounds per year. In the first year, the average consumption was
133 pounds.

Variables  Let \( C \) = average consumption.
Let \( n \) = number of years after 1989.

Equation
\[
\begin{align*}
\text{Average consumption} & \quad = \quad \text{rate of change} \times \text{times} \quad \text{plus} \quad \text{amount at start.} \\
C & \quad = \quad 2.6 \times n \quad + \quad 133
\end{align*}
\]

b. Graph the equation.
The graph passes through \((0, 133)\) with slope 2.6.

c. Find the number of pounds of natural
sweeteners consumed by each person
in 1999.
The year 1999 is 10 years after 1989.
So, \( n = 10 \).
\[
\begin{align*}
C & = 2.6n + 133 \quad \text{Consumption equation} \\
C & = 2.6(10) + 133 \quad \text{Replace } n \text{ with 10.} \\
C & = 159 \quad \text{Simplify.}
\end{align*}
\]
So, the average person consumed 159 pounds of natural sweeteners in 1999.

CHECK  Notice that \((10, 159)\) lies on the graph.
Lesson 5-3  Slope-Intercept Form

**Check for Understanding**

**Concept Check**
1. **OPEN ENDED** Write an equation for a line with a slope of 7.
2. **Explain** why equations of vertical lines cannot be written in slope-intercept form, but equations of horizontal lines can.
3. **Tell** which part of the slope-intercept form represents the rate of change.

**Guided Practice**
Write an equation of the line with the given slope and $y$-intercept.
4. slope: $-3$, $y$-intercept: 1
5. slope: 4, $y$-intercept: $-2$

Write an equation of the line shown in each graph.
6. 
7.

Graph each equation.
8. $y = 2x - 3$
9. $y = -3x + 1$
10. $2x + y = 5$

**Application**
**MONEY** For Exercises 11–13, use the following information.
Suppose you have already saved $50 toward the cost of a new television set. You plan to save $5 more each week for the next several weeks.

11. Write an equation for the total amount $T$ you will have $w$ weeks from now.
12. Graph the equation.
13. Find the total amount saved after 7 weeks.

**Practice and Apply**

Write an equation of the line with the given slope and $y$-intercept.
14. slope: 2, $y$-intercept: $-6$
15. slope: 3, $y$-intercept: $-5$
16. slope: $\frac{1}{2}$, $y$-intercept: 3
17. slope: $-\frac{3}{5}$, $y$-intercept: 0
18. slope: $-1$, $y$-intercept: 10
19. slope: 0.5; $y$-intercept: 7.5

Write an equation of the line shown in each graph.
20. 
21. 
22.
Write an equation of the line shown in each graph.

23. 
24. 
25. 

26. Write an equation of a horizontal line that crosses the y-axis at (0, -5).

27. Write an equation of a line that passes through the origin with slope 3.

Graph each equation.

28. \(y = 3x + 1\)  
29. \(y = x - 2\)  
30. \(y = -4x + 1\)  
31. \(y = -x + 2\)  
32. \(y = \frac{1}{2}x + 4\)  
33. \(y = -\frac{1}{3}x - 3\)  
34. \(3x + y = -2\)  
35. \(2x - y = -3\)  
36. \(3y = 2x + 3\)  
37. \(-2y = 6x - 4\)  
38. \(2x + 3y = 6\)  
39. \(4x - 3y = 3\)

Write a linear equation in slope-intercept form to model each situation.

40. You rent a bicycle for $20 plus $2 per hour.
41. An auto repair shop charges $50 plus $25 per hour.
42. A candle is 6 inches tall and burns at a rate of \(\frac{1}{2}\) inch per hour.
43. The temperature is 15° and is expected to fall 2° each hour during the night.

44. CRITICAL THINKING The equations \(y = 2x + 3\), \(y = 4x + 3\), \(y = -x + 3\), and \(y = -10x + 3\) form a family of graphs. What characteristic do their graphs have in common?

SALES For Exercises 45 and 46, use the following information and the graph at the right.
In 1991, book sales in the United States totaled $16 billion. Sales increased by about $1 billion each year until 1999.

45. Write an equation to find the total sales \(S\) for any year \(t\) between 1991 and 1999.
46. If the trend continues, what will sales be in 2005?

TRAFFIC For Exercises 47–49, use the following information.
In 1966, the traffic fatality rate in the United States was 5.5 fatalities per 100 million vehicle miles traveled. Between 1966 and 1999, the rate decreased by about 0.12 each year.

47. Write an equation to find the fatality rate \(R\) for any year \(t\) between 1966 and 1999.
48. Graph the equation.
49. Find the fatality rate in 1999.
50. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

**How is a y-intercept related to a flat fee?**

Include the following in your answer:
- the point at which the graph would cross the y-axis if your cellular phone service provider charges a rate of $0.07 per minute plus a flat fee of $5.99,
- and a description of a situation in which the y-intercept of its graph is $25.

51. Which equation does not have a y-intercept of 5?

- **A** 2x = y - 5
- **B** 3x + y = 5
- **C** y = x + 5
- **D** 2x - y = 5

52. Which situation below is modeled by the graph?

- **A** You have $100 and plan to spend $5 each week.
- **B** You have $100 and plan to save $5 each week.
- **C** You need $100 for a new CD player and plan to save $5 each week.
- **D** You need $100 for a new CD player and plan to spend $5 each week.

53. The standard form of a linear equation is Ax + By = C, where A, B, and C are integers, A ≥ 0, and A and B are not both zero. Solve Ax + By = C for y. Your answer is written in slope-intercept form.

54. Use the slope-intercept equation in Exercise 53 to write expressions for the slope and y-intercept in terms of A, B, and C.

55. Use the expressions in Exercise 54 to find the slope and y-intercept of each equation.

- **a.** 2x + y = -4
- **b.** 3x + 4y = 12
- **c.** 2x - 3y = 9

**Maintain Your Skills**

**Mixed Review** Write a direct variation equation that relates x and y. Assume that y varies directly as x. Then solve. (Lesson 5-2)

- 56. If y = 45 when x = 60, find x when y = 8.
- 57. If y = 15 when x = 4, find y when x = 10.

Find the slope of the line that passes through each pair of points. (Lesson 5-1)

- 58. (-3, 0), (-4, 6)
- 59. (3, -1), (3, -4)
- 60. (5, -5), (9, 2)

- 61. Write the numbers 2.5, 3/4, -0.5, 7/8 in order from least to greatest. (Lesson 2-7)

Solve each equation. (Lesson 1-3)

- 62. \( x = \frac{15 - 9}{2} \)
- 63. 3(7) + 2 = b
- 64. \( q = 6^2 - 2^2 \)

**Getting Ready for the Next Lesson**

**PREREQUISITE SKILL** Find the slope of the line that passes through each pair of points. (To review slope, see Lesson 5-1)

- 65. (-1, 2), (1, -2)
- 66. (5, 8), (-2, 8)
- 67. (1, -1), (10, -13)
Families of Linear Graphs

A family of people is a group of people related by birth, marriage, or adoption. Recall that a family of graphs includes graphs and equations of graphs that have at least one characteristic in common.

Families of linear graphs fall into two categories—those with the same slope and those with the same y-intercept. A graphing calculator is a useful tool for studying a group of graphs to determine whether they form a family.

Example 1

Graph \( y = x \), \( y = x + 4 \), and \( y = x - 2 \) in the standard viewing window. Describe any similarities and differences among the graphs. Write a description of the family.

Enter the equations in the Y= list as \( Y_1 \), \( Y_2 \), and \( Y_3 \). Then graph the equations.

**KEYSTROKES:** Review graphing on pages 224 and 225.

- The graph of \( y = x \) has a slope of 1 and a y-intercept of 0.
- The graph of \( y = x + 4 \) has a slope of 1 and a y-intercept of 4.
- The graph of \( y = x - 2 \) has a slope of 1 and a y-intercept of \(-2\).

Notice that the graph of \( y = x + 4 \) is the same as the graph of \( y = x \), moved 4 units up. Also, the graph of \( y = x - 2 \) is the same as the graph of \( y = x \), moved 2 units down. All graphs have the same slope and different intercepts.

Because they all have the same slope, this family of graphs can be described as linear graphs with a slope of 1.

Example 2

Graph \( y = x + 1 \), \( y = 2x + 1 \), and \( y = -\frac{1}{3}x + 1 \) in the standard viewing window. Describe any similarities and differences among the graphs. Write a description of the family.

Enter the equations in the Y= list and graph.

- The graph of \( y = x + 1 \) has a slope of 1 and a y-intercept of 1.
- The graph of \( y = 2x + 1 \) has a slope of 2 and a y-intercept of 1.
- The graph of \( y = -\frac{1}{3}x + 1 \) has a slope of \(-\frac{1}{3}\) and a y-intercept of 1.

These graphs have the same intercept and different slopes. This family of graphs can be described as linear graphs with a y-intercept of 1.
Nonlinear functions can also be defined in terms of a family of graphs. Consider the absolute value function \( y = |x| \), where \( y \geq 0 \) for all values of \( x \).

**Example 3**

Graph \( y = |x|, y = |x| + 2 \), and \( y = |x + 3| \) in the standard viewing window. Describe any similarities or differences among the graphs.

Enter the equations in the Y= list and graph.

**KEYSTROKES:**

1. \( y = |x| \)
2. \( y = |x| + 2 \)
3. \( y = |x + 3| \)

- The graph of \( y = |x| \) is v-shaped with its vertex at the origin.
- The graph of \( y = |x| + 2 \) is v-shaped with its vertex at \((0, 2)\).
- The graph of \( y = |x + 3| \) is v-shaped with its vertex at \((-3, 0)\).

These graphs have the same shape, but they are positioned in different places on the coordinate plane.

**Exercises**

Graph each set of equations on the same screen. Describe any similarities or differences among the graphs. If the graphs are part of the same family, describe the family.

1. \( y = -4 \)
   - \( y = 0 \)
   - \( y = 7 \)
2. \( y = -x + 1 \)
   - \( y = 2x + 1 \)
   - \( y = \frac{1}{4}x + 1 \)
3. \( y = x + 4 \)
   - \( y = 2x + 4 \)
   - \( y = 2x - 4 \)
4. \( y = \frac{1}{2}x + 2 \)
   - \( y = \frac{1}{3}x + 3 \)
   - \( y = \frac{1}{4}x + 4 \)
5. \( y = -2x - 2 \)
   - \( y = 2x - 2 \)
   - \( y = \frac{1}{2}x - 2 \)
6. \( y = 3x \)
   - \( y = 3x + 6 \)
   - \( y = 3x - 7 \)
7. \( y = |x| \)
   - \( y = -3|x| \)
   - \( y = -3x \)
8. \( y = |x| \)
   - \( y = |x| + 3 \)
   - \( y = |x| - 2 \)
9. \( y = |x| \)
   - \( y = |2x| + 4 \)
   - \( y = 3|x| - 5 \)

10. **MAKE A CONJECTURE** Write a paragraph explaining how the values of \( m \) and \( b \) in the slope-intercept form affect the graph of a linear equation.

11. Families of graphs are also called **classes of functions**. Describe the similarities and differences in the class of functions \( f(x) = x + c \), where \( c \) is any real number.

12. Describe the similarities and differences in the classes of functions \( f(x) = |x| + c \), and \( f(x) = |x + c| \), where \( c \) is any real number.
You’ll Learn

• Write an equation of a line given the slope and one point on a line.
• Write an equation of a line given two points on the line.

**Vocabulary**

- linear extrapolation

**How** can slope-intercept form be used to make predictions?

In 1995, the population of Orlando, Florida, was about 175,000. At that time, the population was growing at a rate of about 2000 per year.

If you could write an equation based on the slope, 2000, and the point (1995, 175,000), you could predict the population for another year.

### Example 1

**Write an Equation Given Slope and One Point**

Write an equation of a line that passes through (1, 5) with slope 2.

**Step 1** The line has slope 2. To find the $y$-intercept, replace $m$ with 2 and $(x, y)$ with (1, 5) in the slope-intercept form. Then, solve for $b$.

\[
y = mx + b \quad \text{Slope-intercept form}
\]

\[
5 = 2(1) + b \quad \text{Replace } m \text{ with 2, } y \text{ with 5, and } x \text{ with 1.}
\]

\[
5 = 2 + b \quad \text{Multiply.}
\]

\[
5 - 2 = 2 + b - 2 \quad \text{Subtract 2 from each side.}
\]

\[
3 = b \quad \text{Simplify.}
\]

**Step 2** Write the slope-intercept form using $m = 2$ and $b = 3$.

\[
y = mx + b \quad \text{Slope-intercept form}
\]

\[
y = 2x + 3 \quad \text{Replace } m \text{ with 2 and } b \text{ with 3.}
\]

Therefore, the equation is $y = 2x + 3$. 

**WRITE AN EQUATION GIVEN THE SLOPE AND ONE POINT**

You have learned how to write an equation of a line when you know the slope and a specific point, the $y$-intercept. The following example shows how to write an equation when you know the slope and any point on the line.
The table of ordered pairs shows the coordinates of the two points on the graph of a function. Which equation describes the function?

\[
\begin{align*}
\text{(A)} & \quad y = -\frac{1}{3}x - 2 \\
\text{(B)} & \quad y = 3x - 2 \\
\text{(C)} & \quad y = -\frac{1}{3}x + 2 \\
\text{(D)} & \quad y = \frac{1}{3}x - 2
\end{align*}
\]

Read the Test Item
The table represents the ordered pairs \((-3, -1)\) and \((6, -4)\).

Solve the Test Item
Step 1 Find the slope of the line containing the points. Let \((x_1, y_1) = (-3, -1)\) and \((x_2, y_2) = (6, -4)\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Slope formula}
\]

\[
m = \frac{-4 - (-1)}{6 - (-3)} = \frac{-3}{9} = -\frac{1}{3}
\]

Step 2 You know the slope and two points. Choose one point and find the \(y\)-intercept. In this case, we chose \((6, -4)\).

\[
y = mx + b \quad \text{Slope-intercept form}
\]

\[
-4 = -\frac{1}{3}(6) + b \quad \text{Replace } m \text{ with } -\frac{1}{3}, \text{ } x \text{ with } 6, \text{ and } y \text{ with } -4.
\]

\[
-4 = -2 + b \quad \text{Multiply.}
\]

\[
-4 + 2 = -2 + b + 2 \quad \text{Add } 2 \text{ to each side.}
\]

\[
-2 = b \quad \text{Simplify.}
\]

Step 3 Write the slope-intercept form using \(m = -\frac{1}{3}\) and \(b = -2\).

\[
y = mx + b \quad \text{Slope-intercept form}
\]

\[
y = -\frac{1}{3}x - 2 \quad \text{Replace } m \text{ with } -\frac{1}{3} \text{ and } b \text{ with } -2.
\]

Therefore, the equation is \(y = -\frac{1}{3}x - 2\). The answer is A.
Example 3 Write an Equation to Solve a Problem

Baseball In the middle of the 1998 baseball season, Mark McGwire seemed to be on track to break the record for most runs batted in. After 40 games, McGwire had 45 runs batted in. After 86 games, he had 87 runs batted in. Write a linear equation to estimate the number of runs batted in for any number of games that season.

Explore You know the number of runs batted in after 40 and 86 games.

Plan Let \( x \) represent the number of games. Let \( y \) represent the number of runs batted in. Write an equation of the line that passes through \((40, 45)\) and \((86, 87)\).

Solve Find the slope.

\[
m = \frac{y_2 - y_1}{x_2 - x_1}
\]

Slope formula

\[
m = \frac{87 - 45}{86 - 40} \quad \text{Let } (x_1, y_1) = (40, 45) \text{ and } (x_2, y_2) = (86, 87).
\]

\[
m = \frac{42}{46} \text{ or about } 0.91
\]

Simplify.

Choose \((40, 45)\) and find the \( y \)-intercept of the line.

\[
y = mx + b
\]

Slope-intercept form

\[
45 = 0.91(40) + b \quad \text{Replace } m \text{ with } 0.91, x \text{ with } 40, \text{ and } y \text{ with } 45.
\]

\[
45 = 36.4 + b \quad \text{Multiply.}
\]

\[
45 - 36.4 = 36.4 + b - 36.4
\]

Subtract 36.4 from each side.

\[
8.6 = b
\]

Simplify.

Write the slope-intercept form using \( m = 0.91 \), and \( b = 8.6 \).

\[
y = mx + b
\]

Slope-intercept form

\[
y = 0.91x + 8.6
\]

Replace \( m \) with 0.91 and \( b \) with 8.6.

Therefore, the equation is \( y = 0.91x + 8.6 \).

Examine Check your result by substituting the coordinates of the point not chosen, \((86, 87)\), into the equation.

\[
y = 0.91x + 8.6
\]

Original equation

\[
87 \pm 0.91(86) + 8.6
\]

Replace \( y \) with 87 and \( x \) with 86.

\[
87 \pm 78.26 + 8.6
\]

Multiply.

\[
87 = 86.86 \checkmark
\]

The slope was rounded, so the answers vary slightly.
Lesson 5-4  Writing Equations in Slope-Intercept Form

When you use a linear equation to predict values that are beyond the range of the data, you are using **linear extrapolation**.

### Example 4  Linear Extrapolation

**SPORTS**  The record for most runs batted in during a single season is 190. Use the equation in Example 3 to decide whether a baseball fan following the 1998 season would have expected McGwire to break the record in the 162 games played that year.

\[
y = 0.91x + 8.6 \quad \text{Original equation}
\]

\[
y = 0.91(162) + 8.6 \quad \text{Replace } x \text{ with 162.}
\]

\[
y \approx 156 \quad \text{Simplify.}
\]

Since the record is 190 runs batted in, a fan would have predicted that Mark McGwire would not break the record.

Be cautious when making a prediction using just two given points. The model may be *approximately* correct, but still give inaccurate predictions. For example, in 1998, Mark McGwire had 147 runs batted in, which was nine less than the prediction.

### Check for Understanding

#### Concept Check

1. **Compare and contrast** the process used to write an equation given the slope and one point with the process used for two points.

2. **Open Ended**  Write an equation in slope-intercept form of a line that has a *y*-intercept of 3.

3. Tell whether the statement is *sometimes, always, or never* true. Explain.
   
   You can write the equation of a line given its *x*- and *y*-intercepts.

#### Guided Practice

Write an equation of the line that passes through each point with the given slope.

4. \((4, -2), m = 2\)  
5. \((3, 7), m = -3\)  
6. \((-3, 5), m = -1\)

Write an equation of the line that passes through each pair of points.

7. \((5, 1), (8, -2)\)  
8. \((6, 0), (0, 4)\)  
9. \((5, 2), (-7, -4)\)

10. The table of ordered pairs shows the coordinates of the two points on the graph of a line. Which equation describes the line?

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(-5)</td>
<td>2</td>
</tr>
<tr>
<td>0</td>
<td>7</td>
</tr>
</tbody>
</table>

- \(A\)  \(y = x + 7\)
- \(B\)  \(y = x - 7\)
- \(C\)  \(y = -5x + 2\)
- \(D\)  \(y = 5x + 2\)
Write an equation of the line that passes through each point with the given slope.

11. \((1, 2), m = 3\)  
12. \((5, -1), m = -1\)

13. \((5, -2), m = 3\)  
14. \((5, 4), m = -5\)  
15. \((3, 0), m = -2\)  
16. \((5, 3), m = \frac{1}{2}\)  
17. \((-3, -1), m = -\frac{2}{3}\)  
18. \((-3, -5), m = -\frac{5}{3}\)

Write an equation of the line that passes through each pair of points.

19. \((0, 2), (4, 1)\)  
20. \((0, 2), (2, 0)\)

21. \((4, 2), (-2, -4)\)  
22. \((3, -2), (6, 4)\)  
23. \((-1, 3), (2, -3)\)  
24. \((2, -2), (3, 2)\)  
25. \((7, -2), (-4, -2)\)  
26. \((0, 5), (-3, 5)\)  
27. \((1, 1), (7, 4)\)  
28. \((5, 7), (0, 6)\)  
29. \((-\frac{5}{4}, 1), (\frac{3}{4}, 4)\)

Write an equation of the line that has each pair of intercepts.

30. \(x\)-intercept: 3, \(y\)-intercept: 5  
31. \(x\)-intercept: 3, \(y\)-intercept: 4  
32. \(x\)-intercept: 6, \(y\)-intercept: 3  
33. \(x\)-intercept: 2, \(y\)-intercept: -2

MARRIAGE AGE  For Exercises 34–37, use the information in the graphic.

34. Write a linear equation to predict the median age that men marry \(M\) for any year \(t\).
35. Use the equation to predict the median age of men who marry for the first time in 2005.
36. Write a linear equation to predict the median age that women marry \(W\) for any year \(t\).
37. Use the equation to predict the median age of women who marry for the first time in 2005.

Waiting on weddings
Couples are marrying later. The median age of men and women who tied the knot for the first time in 1970 and 1998:

1970  
Men 23.2  
Women 20.8

1998  
Men 26.7  
Women 25

Source: Census Bureau, March 2000

By Hilary Wasson and Sam Ward, USA TODAY
For Exercises 38 and 39, use the data at the top of page 280.

38. Write a linear equation to find Orlando’s population for any year.
39. Predict what Orlando’s population will be in 2010.

40. **CANOE RENTAL** If you rent a canoe for 3 hours, you will pay $45. Write a linear equation to find the total cost \( C \) of renting the canoe for \( h \) hours.

For Exercises 41–43, consider line \( \ell \) that passes through (14, 2) and (28, 6).

41. Write an equation for line \( \ell \).
42. What is the slope of line \( \ell \)?
43. Where does line \( \ell \) intersect the \( x \)-axis? the \( y \)-axis?

44. **CRITICAL THINKING** The \( x \)-intercept of a line is \( p \), and the \( y \)-intercept is \( q \). Write an equation of the line.

45. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can slope-intercept form be used to make predictions?
Include the following in your answer:
• a definition of linear extrapolation, and
• an explanation of how slope-intercept form is used in linear extrapolation.

46. Which is an equation for the line with slope \( \frac{1}{3} \) through \((-2, 1)\)?
   A. \( y = \frac{1}{3}x + 1 \)  
   B. \( y = \frac{1}{3}x + \frac{5}{3} \)  
   C. \( y = \frac{1}{3}x - \frac{5}{3} \)  
   D. \( y = \frac{1}{3}x + \frac{1}{3} \)

47. About 20,000 fewer babies were born in California in 1996 than in 1995. In 1995, about 560,000 babies were born. Which equation can be used to predict the number of babies \( y \) (in thousands), born \( x \) years after 1995?
   A. \( y = 20x + 560 \)  
   B. \( y = -20x + 560 \)  
   C. \( y = -20x - 560 \)  
   D. \( y = 20x - 560 \)

**Maintain Your Skills**

**Graph each equation.**  
**Lesson 5-3**

48. \( y = 3x - 2 \)  
49. \( x + y = 6 \)  
50. \( x + 2y = 8 \)

51. **HEALTH** Each time your heart beats, it pumps 2.5 ounces of blood through your heart. Write a direct variation equation that relates the total volume of blood \( V \) with the number of times your heart beats \( b \).  
   **Lesson 5-2**

**State the domain of each relation.**  
**Lesson 4-3**

52. \{ (0, 8), (9, -2), (4, 2) \}  
53. \{ (-2, 1), (5, 1), (-2, 7), (0, -3) \}

54. \(-3 \bullet -5\)  
55. \(4 \bullet \frac{16}{3}\)  
56. \(\frac{3}{4} \bullet \frac{2}{3}\)

**Getting Ready for the Next Lesson**

**PREREQUISITE SKILL** Find each difference.  
**Lesson 2-7**

57. \(4 - 7\)  
58. \(5 - 12\)  
59. \(2 - (-3)\)

60. \(-1 - 4\)  
61. \(-7 - 8\)  
62. \(-5 - (-2)\)

**www.algebra1.com/self_check_quiz**
**What You’ll Learn**
- Write the equation of a line in point-slope form.
- Write linear equations in different forms.

**Vocabulary**
- point-slope form

**Point-Slope Form**
Remember, \((x_1, y_1)\) represents the given point, and \((x, y)\) represents any other point on the line.

**How can you use the slope formula to write an equation of a line?**

The graph shows a line with slope 2 that passes through \((3, 4)\). Another point on the line is \((x, y)\).

\[
\frac{y - 4}{x - 3} = 2
\]

**Key Concept**

**Point-Slope Form**

- **Words**
  The linear equation \(y - y_1 = m(x - x_1)\) is written in point-slope form, where \((x_1, y_1)\) is a given point on a nonvertical line and \(m\) is the slope of the line.

- **Symbols**
  \[y - y_1 = m(x - x_1)\]

**Example 1**

**Write an Equation Given Slope and a Point**

Write the point-slope form of an equation for a line that passes through \((-1, 5)\) with slope \(-3\).

\[
y - 5 = -3(x + 1)
\]

Therefore, the equation is \(y - 5 = -3(x + 1)\).
Vertical lines cannot be written in point-slope form because the slope is undefined. However, since the slope of a horizontal line is 0, horizontal lines can be written in point-slope form.

**Example 2** Write an Equation of a Horizontal Line

Write the point-slope form of an equation for a horizontal line that passes through \((6, -2)\).

\[
y - y_1 = m(x - x_1) \quad \text{Point-slope form}
\]

\[
y - (-2) = 0(x - 6) \quad (x_1, y_1) = (6, -2)
\]

\[
y + 2 = 0 \quad \text{Simplify.}
\]

Therefore, the equation is \(y + 2 = 0\).

**FORMS OF LINEAR EQUATIONS**

You have learned about three of the most common forms of linear equations.

<table>
<thead>
<tr>
<th>Form</th>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope-Intercept</td>
<td>(y = mx + b)</td>
<td>(m) is the slope, and (b) is the (y)-intercept.</td>
</tr>
<tr>
<td>Point-Slope</td>
<td>(y - y_1 = m(x - x_1))</td>
<td>(m) is the slope and ((x_1, y_1)) is a given point.</td>
</tr>
<tr>
<td>Standard</td>
<td>(Ax + By = C)</td>
<td>(A) and (B) are not both zero. Usually (A) is nonnegative and (A, B,) and (C) are integers whose greatest common factor is 1.</td>
</tr>
</tbody>
</table>

Linear equations in point-slope form can be written in slope-intercept or standard form.

**Example 3** Write an Equation in Standard Form

Write \(y + 5 = -\frac{5}{4}(x - 2)\) in standard form.

In standard form, the variables are on the left side of the equation. \(A, B,\) and \(C\) are all integers.

\[
y + 5 = -\frac{5}{4}(x - 2) \quad \text{Original equation}
\]

\[
4(y + 5) = 4\left(-\frac{5}{4}\right)(x - 2) \quad \text{Multiply each side by 4 to eliminate the fraction.}
\]

\[
4y + 20 = -5(x - 2) \quad \text{Distributive Property}
\]

\[
4y + 20 = -5x + 10 \quad \text{Distributive Property}
\]

\[
4y + 20 - 20 = -5x + 10 - 20 \quad \text{Subtract 20 from each side.}
\]

\[
4y = -5x - 10 \quad \text{Simplify.}
\]

\[
4y + 5x = -5x - 10 + 5x \quad \text{Add 5x to each side.}
\]

\[
5x + 4y = -10 \quad \text{Simplify.}
\]

The standard form of the equation is \(5x + 4y = -10\).
Example 4 Write an Equation in Slope-Intercept Form

Write \( y - 2 = \frac{1}{2}(x + 5) \) in slope-intercept form.

In slope-intercept form, \( y \) is on the left side of the equation. The constant and \( x \) are on the right side.

\[
y - 2 = \frac{1}{2}(x + 5) \quad \text{Original equation}
\]

\[
y - 2 = \frac{1}{2}x + \frac{5}{2} \quad \text{Distributive Property}
\]

\[
y - 2 + 2 = \frac{1}{2}x + \frac{5}{2} + 2 \quad \text{Add 2 to each side.}
\]

\[
y = \frac{1}{2}x + \frac{9}{2} \quad 2 = \frac{4}{2} \quad \text{and} \quad \frac{4}{2} + \frac{5}{2} = \frac{9}{2}
\]

The slope-intercept form of the equation is \( y = \frac{1}{2}x + \frac{9}{2} \).

You can draw geometric figures on a coordinate plane and use the point-slope form to write equations of the lines.

Example 5 Write an Equation in Point-Slope Form

GEOMETRY The figure shows right triangle \( ABC \).

a. Write the point-slope form of the line containing the hypotenuse \( AB \).

Step 1 First, find the slope of \( AB \).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Slope formula}
\]

\[
= \frac{4 - 1}{6 - 2} \quad \text{or} \quad \frac{3}{4} \quad (x_1, y_1) = (2, 1), (x_2, y_2) = (6, 4)
\]

Step 2 You can use either point for \((x_1, y_1)\) in the point-slope form.

Method 1 Use \((6, 4)\).

\[
y - y_1 = m(x - x_1)
\]

\[
y - 4 = \frac{3}{4}(x - 6)
\]

Method 2 Use \((2, 1)\).

\[
y - y_1 = m(x - x_1)
\]

\[
y - 1 = \frac{3}{4}(x - 2)
\]

b. Write each equation in standard form.

\[
y - 4 = \frac{3}{4}(x - 6) \quad \text{Original equation}
\]

\[
y - 1 = \frac{3}{4}(x - 2)
\]

\[
4(y - 4) = 4\left(\frac{3}{4}(x - 6)\right) \quad \text{Multiply each side by 4.}
\]

\[
4(y - 1) = 4\left(\frac{3}{4}(x - 2)\right)
\]

\[
4y - 16 = 3(x - 6) \quad \text{Multiply.}
\]

\[
4y - 4 = 3(x - 2)
\]

\[
4y - 16 = 3x - 18 \quad \text{Distributive Property}
\]

\[
4y - 4 = 3x - 6
\]

\[
4y - 3x = 2 \quad \text{Add to each side.}
\]

\[
4y = 3x + 2
\]

\[
-3x + 4y = -2 \quad \text{Subtract 3x from each side.}
\]

\[
-3x + 4y = -2
\]

\[
3x - 4y = 2 \quad \text{Multiply each side by -1.}
\]

\[
3x - 4y = 2
\]

Regardless of which point was used to find the point-slope form, the standard form results in the same equation.
**Concept Check**

1. Explain what \( x_1 \) and \( y_1 \) in the point-slope form of an equation represent.

2. **FIND THE ERROR** Tanya and Akira wrote the point-slope form of an equation for a line that passes through \((-2, -6)\) and \((1, 6)\). Tanya says that Akira’s equation is wrong. Akira says they are both correct. Who is correct? Explain.

   - **Tanya**
     \[ y - 6 = 4(x + 2) \]
   - **Akira**
     \[ y - 6 = 4(x - 1) \]

3. **OPEN ENDED** Write an equation in point-slope form. Then write an equation for the same line in slope-intercept form.

   **Guided Practice**

   Write the point-slope form of an equation for a line that passes through each point with the given slope.

   4. \((1, 3), \ m = -2\)
   5. \((-1, -2), \ m = 3\)
   6. \((2, -2), \ m = 0\)

   Write each equation in standard form.
   7. \(y - 5 = 4(x + 2)\)
   8. \(y + 3 = -\frac{3}{4}(x - 1)\)
   9. \(y - 3 = 2.5(x + 1)\)

   Write each equation in slope-intercept form.
   10. \(y + 6 = 2(x - 2)\)
    11. \(y + 3 = -\frac{2}{3}(x - 6)\)
    12. \(y - \frac{7}{2} = \frac{1}{2}(x - 4)\)

   **Application**

   **GEOMETRY** For Exercises 13 and 14, use parallelogram \(ABCD\).
   A parallelogram has opposite sides parallel.
   13. Write the point-slope form of the line containing \(AD\).
   14. Write the standard form of the line containing \(AD\).

   **Practice and Apply**

   Write the point-slope form of an equation for a line that passes through each point with the given slope.

   15. \((3, 8), \ m = 2\)
    16. \((-4, -3), \ m = 1\)
    17. \((-2, 4), \ m = -3\)

   18. \((-6, 1), \ m = -4\)
    19. \((-3, 6), \ m = 0\)
    20. \((9, 1), \ m = \frac{2}{3}\)

   21. \((8, -3), \ m = \frac{3}{4}\)
    22. \((-6, 3), \ m = -\frac{2}{3}\)
    23. \((1, -3), \ m = -\frac{5}{8}\)

   24. \((9, -5), \ m = 0\)
    25. \((-4, 8), \ m = \frac{7}{2}\)
    26. \((1, -4), \ m = -\frac{8}{3}\)
27. Write the point-slope form of an equation for a horizontal line that passes through $(5, -9)$.

28. A horizontal line passes through $(0, 7)$. Write the point-slope form of its equation.

29. Write each equation in standard form.
   \[ y - 13 = 4(x - 2) \quad 30. \quad y + 3 = 3(x + 5) \quad 31. \quad y - 5 = -2(x + 6) \]
   \[ 32. \quad y + 3 = -5(x + 1) \quad 33. \quad y + 7 = \frac{1}{2}(x + 2) \quad 34. \quad y - 1 = \frac{5}{6}(x - 4) \]
   \[ 35. \quad y - 2 = \frac{2}{5}(x - 8) \quad 36. \quad y + 4 = -\frac{1}{3}(x - 12) \quad 37. \quad y + 2 = \frac{5}{3}(x + 6) \]
   \[ 38. \quad y + 6 = \frac{3}{2}(x - 4) \quad 39. \quad y - 6 = 1.3(x + 7) \quad 40. \quad y - 2 = -2.5(x - 1) \]

30. Write each equation in slope-intercept form.
   \[ 41. \quad y - 2 = 3(x - 1) \quad 42. \quad y - 5 = 6(x + 1) \quad 43. \quad y + 2 = -2(x - 5) \]
   \[ 44. \quad y - 1 = -7(x - 3) \quad 45. \quad y + 3 = \frac{1}{2}(x + 4) \quad 46. \quad y - 1 = \frac{2}{3}(x + 9) \]
   \[ 47. \quad y + 3 = -\frac{1}{4}(x + 2) \quad 48. \quad y - 5 = -\frac{2}{5}(x + 15) \quad 49. \quad y + \frac{1}{2} = x - \frac{1}{2} \]
   \[ 50. \quad y - \frac{1}{3} = -2\left(x + \frac{1}{3}\right) \quad 51. \quad y + \frac{1}{4} = -3\left(x + \frac{1}{2}\right) \quad 52. \quad y + \frac{3}{5} = -4\left(x - \frac{1}{2}\right) \]

53. Write the point-slope form, slope-intercept form, and standard form of an equation for a line that passes through $(5, -3)$ with slope 10.

54. Line $\ell$ passes through $(1, -6)$ with slope $\frac{3}{2}$. Write the point-slope form, slope-intercept form, and standard form of an equation for line $\ell$.

**BUSINESS** For Exercises 55–57, use the following information.
A home security company provides security systems for $5 per week, plus an installation fee. The total fee for 12 weeks of service is $210.

55. Write the point-slope form of an equation to find the total fee $y$ for any number of weeks $x$.

56. Write the equation in slope-intercept form.

57. What is the flat fee for installation?

**MOVIES** For Exercises 58–60, use the following information.
Between 1990 and 1999, the number of movie screens in the United States increased by about 1500 each year. In 1996, there were 29,690 movie screens.

58. Write the point-slope form of an equation to find the total number of screens $y$ for any year $x$.

59. Write the equation in slope-intercept form.

60. Predict the number of movie screens in the United States in 2005.

---

**Online Research Data Update** What has happened to the number of movie screens since 1999? Visit [www.algebra1.com/data_update](http://www.algebra1.com/data_update) to learn more.
GEOMETRY  For Exercises 61–63, use square PQRS.

61. Write a point-slope equation of the line containing each side.

62. Write the slope-intercept form of each equation.

63. Write the standard form of each equation.

64. CRITICAL THINKING  A line contains the points (9, 1) and (5, 5). Write a convincing argument that the same line intersects the x-axis at (10, 0).

65. WRITING IN MATH  Answer the question that was posed at the beginning of the lesson.

How can you use the slope formula to write an equation of a line?
Include the following in your answer:
• an explanation of how you can use the slope formula to write the point-slope form.

66. Which equation represents a line that neither passes through (0, 1) nor has a slope of 3?
   A. \(-2x + y = 1\)  B. \(y + 1 = 3(x + 6)\)
   C. \(y - 3 = 3(x - 6)\)  D. \(x - 3y = -15\)

67. SHORT RESPONSE  Write the slope-intercept form of an equation of a line that passes through (2, -5).

EXTENDING THE LESSON  For Exercises 68–71, use the graph at the right.

68. Choose three different pairs of points from the graph. Write the slope-intercept form of the line using each pair.

69. Describe how the equations are related.

70. Choose a different pair of points from the graph and predict the equation of the line determined by these points. Check your conjecture by finding the equation.

71. MAKE A CONJECTURE  What conclusion can you draw from this activity?

MIXED REVIEW  Write the slope-intercept form of an equation of the line that satisfies each condition.  \(\text{Lessons 5-3 and 5-4}\)

72. slope -2 and y-intercept -5  
73. passes through (-2, 4) with slope 3  
74. passes through (2, -4) and (0, 6)  
75. a horizontal line through (1, -1)

SOLVE EACH EQUATION.  \(\text{Lesson 3-4}\)

76. \(4a - 5 = 15\)  
77. \(7 + 3c = -11\)  
78. \(\frac{2}{9}v - 6 = 14\)

79. Evaluate \((25 - 4) \div (2^2 - 1^3)\).  \(\text{Lesson 1-3}\)

GETTING READY FOR THE NEXT LESSON  PREREQUISITE SKILL  Write the multiplicative inverse of each number.  \(\text{For review of multiplicative inverses, see pages 800 and 801}\)

80. 2  
81. 10  
82. 1  
83. -1  
84. \(\frac{2}{3}\)  
85. \(-\frac{1}{9}\)  
86. \(\frac{5}{2}\)  
87. \(-\frac{2}{3}\)
What You’ll Learn

- Write an equation of the line that passes through a given point, parallel to a given line.
- Write an equation of the line that passes through a given point, perpendicular to a given line.

Vocabulary

- parallel lines
- perpendicular lines

How can you determine whether two lines are parallel?

The graphing calculator screen shows a family of linear graphs whose slope is 1. Notice that the lines do not appear to intersect.

PARALLEL LINES

Lines in the same plane that do not intersect are called parallel lines. Parallel lines have the same slope.

Key Concept

- **Words**
  Two nonvertical lines are parallel if they have the same slope. All vertical lines are parallel.

- **Model**

You can write the equation of a line parallel to a given line if you know a point on the line and an equation of the given line.

**Example 1 Parallel Line Through a Given Point**

Write the slope-intercept form of an equation for the line that passes through \((-1, -2)\) and is parallel to the graph of \(y = -3x - 2\).

The line parallel to \(y = -3x - 2\) has the same slope, \(-3\). Replace \(m\) with \(-3\), and \((x_1, y_1)\) with \((-1, -2)\) in the point-slope form.

\[
y - y_1 = m(x - x_1)\quad \text{Point-slope form}
\]
\[
y - (-2) = -3[x - (-1)]\quad \text{Replace } m \text{ with } -3, y \text{ with } -2, \text{ and } x \text{ with } -1.
\]
\[
y + 2 = -3(x + 1)\quad \text{Simplify.}
\]
\[
y + 2 = -3x - 3\quad \text{Distributive Property}
\]
\[
y + 2 - 2 = -3x - 3 - 2\quad \text{Subtract 2 from each side.}
\]
\[
y = -3x - 5\quad \text{Write the equation in slope-intercept form.}
\]

Therefore, the equation is \(y = -3x - 5\).
PERPENDICULAR LINES  Lines that intersect at right angles are called perpendicular lines. There is a relationship between the slopes of perpendicular lines.

**Algebra Activity**  Perpendicular Lines

**Model**
- A scalene triangle is one in which no two sides are equal. Cut out a scalene right triangle \(ABC\) so that \(\angle C\) is a right angle. Label the vertices and the sides as shown.
- Draw a coordinate plane on grid paper. Place \(\triangle ABC\) on the coordinate plane so that \(A\) is at the origin and side \(b\) lies along the positive \(x\)-axis.

**Analyze**
1. Name the coordinates of \(B\).
2. What is the slope of side \(c\)?
3. Rotate the triangle 90° counterclockwise so that \(A\) is still at the origin and side \(b\) is along the positive \(y\)-axis. Name the coordinates of \(B\).
4. What is the slope of side \(c\)?
5. Repeat the activity for two other different scalene triangles.
6. For each triangle and its rotation, what is the relationship between the first position of side \(c\) and the second?
7. For each triangle and its rotation, describe the relationship between the coordinates of \(B\) in the first and second positions.
8. Describe the relationship between the slopes of \(c\) in each position.

**Make a Conjecture**
9. Describe the relationship between the slopes of any two perpendicular lines.

**Key Concept**  Perpendicular Lines in a Coordinate Plane

**Words**  Two nonvertical lines are perpendicular if the product of their slopes is \(-1\). That is, the slopes are opposite reciprocals of each other. Vertical lines and horizontal lines are also perpendicular.

**Model**
**Example 2** Determine Whether Lines are Perpendicular

The outline of a kite is shown on a coordinate plane. Determine whether \( AC \) is perpendicular to \( BD \).

Find the slope of each segment.

**Slope of \( AC \):** \( m = \frac{5 - 1}{5 - 7} \) or \(-2\)

**Slope of \( BD \):** \( m = \frac{4 - 0}{8 - 0} \) or \(\frac{1}{2}\)

The line segments are perpendicular because \(\frac{1}{2}(-2) = -1\).

You can write the equation of a line perpendicular to a given line if you know a point on the line and the equation of the given line.

**Example 3** Perpendicular Line Through a Given Point

Write the slope-intercept form for an equation of the line that passes through \((-3, -2)\) and is perpendicular to the graph of \(x + 4y = 12\).

**Step 1** Find the slope of the given line.

\[
x + 4y = 12 \\
x + 4y - x = 12 - x \\
4y = -1x + 12 \\
4y = -1x + 12 \\
\frac{4y}{4} = \frac{-1x + 12}{4} \\
y = -\frac{1}{4}x + 3
\]

**Step 2** The slope of the given line is \(-\frac{1}{4}\). So, the slope of the line perpendicular to this line is the opposite reciprocal of \(-\frac{1}{4}\), or \(4\).

**Step 3** Use the point-slope form to find the equation.

\[
y - y_1 = m(x - x_1) \quad \text{Point-slope form} \\
y - (-2) = 4[x - (-3)] \quad (x_1, y_1) = (-3, -2) \text{ and } m = 4 \\
y + 2 = 4(x + 3) \quad \text{Simplify.} \\
y + 2 = 4x + 12 \quad \text{Distributive Property} \\
y + 2 - 2 = 4x + 12 - 2 \quad \text{Subtract 2 from each side.} \\
y = 4x + 10 \quad \text{Simplify.}
\]

Therefore, the equation of the line is \(y = 4x + 10\).

**CHECK** Graph both equations on a graphing calculator. Use the CALC menu to verify that \(y = 4x + 10\) passes through \((-3, -2)\).
Lesson 5-6
Geometry: Parallel and Perpendicular Lines

**Example 4 Perpendicular Line Through a Given Point**

Write the slope-intercept form for an equation of a line perpendicular to the graph of \( y = -\frac{1}{3}x + 2 \) and passes through the \( x \)-intercept of that line.

**Step 1** Find the slope of the perpendicular line. The slope of the given line is \(-\frac{1}{3}\), therefore a perpendicular line has slope 3 because \(-\frac{1}{3} \cdot 3 = -1\).

**Step 2** Find the \( x \)-intercept of the given line.

\[
\begin{align*}
y &= -\frac{1}{3}x + 2 & \text{Original equation} \\
0 &= -\frac{1}{3}x + 2 & \text{Replace } y \text{ with } 0. \\
-2 &= -\frac{1}{3}x & \text{Subtract } 2 \text{ from each side.} \\
6 &= x & \text{Multiply each side by } -3.
\end{align*}
\]

The \( x \)-intercept is at (6, 0).

**Step 3** Substitute the slope and the given point into the point-slope form of a linear equation. Then write the equation in slope-intercept form.

\[
\begin{align*}
y - y_1 &= m(x - x_1) & \text{Point-slope form} \\
y - 0 &= 3(x - 6) & \text{Replace } x \text{ with } 6, y \text{ with } 0, \text{ and } m \text{ with } 3. \\
y &= 3x - 18 & \text{Distributive Property}
\end{align*}
\]

**Check for Understanding**

**Concept Check**

1. **Explain** how to find the slope of a line that is perpendicular to the line shown in the graph.

2. **OPEN ENDED** Give an example of two numbers that are negative reciprocals.

3. Define parallel lines and perpendicular lines.

**Guided Practice**

Write the slope-intercept form of an equation of the line that passes through the given point and is parallel to the graph of each equation.

4. 

5. 

6. (1, -3), \( y = 2x - 1 \)

7. (2, 3), \( y = x + 5 \)

8. **GEOMETRY** Quadrilateral \( ABCD \) has vertices \( A(-2, 1), B(3, 3), C(5, 7), \) and \( D(0, 5) \). Determine whether \( AC \) is perpendicular to \( BD \).

Write the slope-intercept form of an equation that passes through the given point and is perpendicular to the graph of each equation.

9. (3, 1), \( y = \frac{1}{3}x + 2 \)

10. (6, -2), \( y = \frac{3}{5}x - 4 \)

11. (2, -2), \( 2x + y = 5 \)
**Application**

12. **GEOMETRY** The line with equation \( y = 3x - 4 \) contains side \( \overline{AC} \) of right triangle \( ABC \). If the vertex of the right angle \( C \) is at \((3, 5)\), what is an equation of the line that contains side \( \overline{BC} \)?

---

**Practice and Apply**

**Write the slope-intercept form of an equation of the line that passes through the given point and is parallel to the graph of each equation.**

13. \((2, -7), y = x - 2\)  
14. \((2, -1), y = 2x + 2\)  
15. \((-3, 2), y = x - 6\)  
16. \((4, -1), y = 2x + 1\)  
17. \((-5, -4), y = \frac{1}{2}x + 1\)  
18. \((3, 3), y = \frac{2}{3}x - 1\)  
19. \((-4, -3), y = -\frac{1}{3}x + 3\)  
20. \((-1, 2), y = -\frac{1}{2}x - 4\)  
21. \((-3, 0), 2y = x - 1\)  
22. \((2, 2), 3y = -2x + 6\)  
23. \((-2, 3), 6x + y = 4\)  
24. \((2, 2), 3x - 4y = -4\)

**Write the slope-intercept form of an equation that passes through the given point and is perpendicular to the graph of each equation.**

25. **GEOMETRY** A parallelogram is a quadrilateral in which opposite sides are parallel. Is \(ABCD\) a parallelogram? Explain.

26. Write an equation of the line parallel to the graph of \( y = 5x - 3 \) and through the origin.

27. Write an equation of the line with \( y\)-intercept \(-6\) that is parallel to the graph of \( x - 3y = 8\).

28. \((2, -6), y = x - 6\)  
29. \((1, 1), y = 4x + 6\)  
30. \((-3, 1), y = -3x + 7\)  
31. \((0, 5), y = -8x + 4\)  
32. \((1, -3), y = \frac{1}{2}x + 4\)  
33. \((4, 7), y = \frac{2}{3}x - 1\)  
34. \((0, 4), 3x + 8y = 4\)  
35. \((-2, 7), 2x - 5y = 3\)  
36. \((6, -1), 3y + x = 3\)  
37. \((0, -1), 5x - y = 3\)  
38. \((8, -2), 5x - 7 = 3y\)  
39. \((3, -3), 3x + 7 = 2x\)

40. Find an equation of the line that has a \( y\)-intercept of \(-2\) and is perpendicular to the graph of \(3x + 6y = 2\).

41. Write an equation of the line that is perpendicular to the line through \((9, 10)\) and \((3, -2)\) and passes through the \( x\)-intercept of that line.

**Determine whether the graphs of each pair of equations are parallel, perpendicular, or neither.**

42. \(y = -2x + 11\)  
43. \(3y = 2x + 14\)  
44. \(y = -5x\)  
\(y = 5x - 18\)

45. **GEOMETRY** The diagonals of a square are segments that connect the opposite vertices. Determine the relationship between the diagonals \( \overline{AC} \) and \( \overline{BD} \) of square \(ABCD\).

46. **CRITICAL THINKING** What is \( a \) if the lines with equations \( y = ax + 5 \) and \( 2y = (a + 4)x - 1 \) are parallel?
47. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can you determine whether two lines are parallel?

Include the following in your answer:
- an equation whose graph is parallel to the graph of \( y = -5x \), with an explanation of your reasoning, and
- an equation whose graph is perpendicular to the graph of \( y = -5x \), with an explanation of your reasoning.

48. What is the slope of a line perpendicular to the graph of \( 3x + 4y = 24 \)?

\( \text{A} \) \(-\frac{4}{3}\)  \( \text{B} \) \(-\frac{3}{4}\)  \( \text{C} \) \(\frac{3}{4}\)  \( \text{D} \) \(\frac{4}{3}\)

49. How can the graph of \( y = 3x + 4 \) be used to graph \( y = 3x + 2 \)?

- Move the graph of the line right 2 units.
- Change the slope of the graph from 4 to 2.
- Change the \( y \)-intercept from 4 to 2.
- Move the graph of the line left 2 units.

---

**Maintain Your Skills**

**Mixed Review** Write the point-slope form of an equation for a line that passes through each point with the given slope. *(Lesson 5-5)*

50. \((3, 5), m = -2\)  
51. \((-4, 7), m = 5\)  
52. \((-1, -3), m = -\frac{1}{2}\)

**TELEPHONE** For Exercises 53 and 54, use the following information.

An international calling plan charges a rate per minute plus a flat fee. A 10-minute call to the Czech Republic costs $3.19. A 15-minute call costs $4.29. *(Lesson 5-4)*

53. Write a linear equation in slope-intercept form to find the total cost \( C \) of an \( m \)-minute call.

54. Find the cost of a 12-minute call.

**Getting Ready for the Next Lesson** Write the slope-intercept form of an equation of the line that passes through each pair of points. *(To review slope-intercept form, see Lesson 5-4.)*

55. \((5, -1), (-3, 3)\)

56. \((0, 2), (8, 0)\)

57. \((2, 1), (3, -4)\)

58. \((5, 5), (8, -1)\)

59. \((6, 9), (4, 9)\)

60. \((-6, 4), (2, -2)\)

---

**Practice Quiz 2** *(Lessons 5-3 through 5-6)*

Write the slope-intercept form for an equation of the line that satisfies each condition.

1. slope 4 and \( y \)-intercept \(-3\) *(Lesson 5-3)*
2. passes through \((1, -3)\) with slope 2 *(Lesson 5-4)*
3. passes through \((-1, -2)\) and \((1, 3)\) *(Lesson 5-4)*
4. parallel to the graph of \( y = 2x - 2 \) and passes through \((-2, 3)\) *(Lesson 5-6)*
5. Write \( y - 4 = \frac{1}{2}(x + 3) \) in standard form and in slope-intercept form. *(Lesson 5-5)*
Statistics: Scatter Plots and Lines of Fit

What You’ll Learn

• Interpret points on a scatter plot.
• Write equations for lines of fit.

How do scatter plots help identify trends in data?

The points of a set of real-world data do not always lie on one line. But, you may be able to draw a line that seems to be close to all the points.

The line in the graph shows a linear relationship between the year \(x\) and the number of bushels of apples \(y\). As the years increase, the number of bushels of apples also increases.

INTERPRET POINTS ON A SCATTER PLOT  A scatter plot is a graph in which two sets of data are plotted as ordered pairs in a coordinate plane. Scatter plots are used to investigate a relationship between two quantities.

• In the first graph below, there is a positive correlation between \(x\) and \(y\). That is, as \(x\) increases, \(y\) increases.

• In the second graph below, there is a negative correlation between \(x\) and \(y\). That is, as \(x\) increases, \(y\) decreases.

• In the third graph below, there is no correlation between \(x\) and \(y\). That is, \(x\) and \(y\) are not related.

If the pattern in a scatter plot is linear, you can draw a line to summarize the data. This can help identify trends in the data.

Vocabulary

• scatter plot
• positive correlation
• negative correlation
• line of fit
• best-fit line
• linear interpolation

**INTERPRET POINTS ON A SCATTER PLOT**

**Scatter Plots**

**Positive Correlation**

**Negative Correlation**

**No Correlation**

Source: U.S. Apple Association

**Apples in Storage in U.S.**

<table>
<thead>
<tr>
<th>Year</th>
<th>Number (millions of bushels)</th>
</tr>
</thead>
<tbody>
<tr>
<td>'97</td>
<td>8</td>
</tr>
<tr>
<td>'98</td>
<td>10</td>
</tr>
<tr>
<td>'99</td>
<td>12.4</td>
</tr>
<tr>
<td>'00</td>
<td>13.6</td>
</tr>
</tbody>
</table>

**Source:** U.S. Apple Association
Example 1 Analyze Scatter Plots

Determine whether each graph shows a positive correlation, a negative correlation, or no correlation. If there is a positive or negative correlation, describe its meaning in the situation.

a. NUTRITION The graph shows fat grams and Calories for selected choices at a fast-food restaurant. The graph shows a positive correlation. As the number of fat grams increases, the number of Calories increases.

b. CARS The graph shows the weight and the highway gas mileage of selected cars. The graph shows a negative correlation. As the weight of the automobile increases, the gas mileage decreases.

Is there a relationship between the length of a person’s foot and his or her height? Make a scatter plot and then look for a pattern.

Algebra Activity Making Predictions

Collect the Data
• Measure your partner’s foot and height in centimeters. Then trade places.
• Add the points (foot length, height) to a class scatter plot.

Analyze the Data
1. Is there a correlation between foot length and height for the members of your class? If so, describe it.
2. Draw a line that summarizes the data and shows how the height changes as the foot length changes.

Make a Conjecture
3. Use the line to predict the height of a person whose foot length is 25 centimeters. Explain your method.
**Example 2** Find a Line of Fit

### BIRDS

The table shows an estimate for the number of bald eagle pairs in the United States for certain years since 1985.

<table>
<thead>
<tr>
<th>Years since 1985</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>9</th>
<th>11</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bald Eagle Pairs</td>
<td>2500</td>
<td>3000</td>
<td>3700</td>
<td>4500</td>
<td>5000</td>
<td>5800</td>
</tr>
</tbody>
</table>

Source: U.S. Fish and Wildlife Service

a. Draw a scatter plot and determine what relationship exists, if any, in the data.

Let the independent variable \( x \) be the number of years since 1985, and let the dependent variable \( y \) be the number of bald eagle pairs.

As the number of years increases, the number of bald eagle pairs increases. There is a positive correlation between the two variables.

b. Draw a line of fit for the scatter plot.

No one line will pass through all of the data points. Draw a line that passes close to the points. A line of fit is shown in the scatter plot at the right.

c. Write the slope-intercept form of an equation for the line of fit.

The line of fit shown above passes through the data points \( (3, 2500) \) and \( (11, 5000) \).

**Step 1** Find the slope.

\[
m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Slope formula}
\]

\[
m = \frac{5000 - 2500}{11 - 3} = \frac{2500}{8} = 312.5 \quad \text{Simplify.}
\]

**Step 2** Use \( m = 312.5 \) and either the point-slope form or the slope-intercept form to write the equation. You can use either data point. We chose \( (3, 2500) \).

**Point-slope form**

\[
y - y_1 = m(x - x_1)
\]

\[
y - 2500 = 312.5(x - 3)
\]

\[
y - 2500 = 312.5x - 937.5
\]

\[
y = 312.5x + 1562.5
\]

**Slope-intercept form**

\[
y = mx + b
\]

\[
2500 = 312.5(3) + b
\]

\[
2500 = 937.5 + b
\]

\[
1562.5 = b
\]

\[
y = 312.5x + 1562.5
\]

Using either method, \( y = 312.5x + 1562.5 \).
CHECK Check your result by substituting \((11, 5000)\) into \(y = 312.5x + 1562.5\).

\[
y = 312.5x + 1562.5 \quad \text{Line of fit equation}
\]

\[
5000 = 312.5(11) + 1562.5 \quad \text{Replace } x \text{ with } 11 \text{ and } y \text{ with } 5000.
\]

\[
5000 = 3437.5 + 1562.5 \quad \text{Multiply.}
\]

\[
5000 = 5000 \quad \text{Add.}
\]

The solution checks.

In Lesson 5-4, you learned about linear extrapolation, which is predicting values that are outside the range of the data. You can also use a linear equation to predict values that are inside the range of the data. This is called linear interpolation.

**Example 3 Linear Interpolation**

**BIRDS** Use the equation for the line of fit in Example 2 to estimate the number of bald eagle pairs in 1998.

Use the equation \(y = 312.5x + 1562.5\), where \(x\) is the number of years since 1985 and \(y\) is the number of bald eagle pairs.

\[
y = 312.5x + 1562.5 \quad \text{Original equation}
\]

\[
y = 312.5(13) + 1562.5 \quad \text{Replace } x \text{ with } 1998 - 1985 \text{ or } 13.
\]

\[
y = 5625 \quad \text{Simplify.}
\]

There were about 5625 bald eagle pairs in 1998.

---

**Check for Understanding**

**Concept Check**

1. Explain how to determine whether a scatter plot has a positive or negative correlation.

2. **OPEN ENDED** Sketch scatter plots that have each type of correlation.
   a. positive
   b. negative
   c. no correlation

3. Compare and contrast linear interpolation and linear extrapolation.

**Guided Practice**

Determine whether each graph shows a positive correlation, a negative correlation, or no correlation. If there is a positive or negative correlation, describe its meaning in the situation.

4. 

5. 

---
### Application

**BIOLOGY** For Exercises 6–9, use the table that shows the average body temperature in degrees Celsius of 9 insects at a given air temperature.

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>Air</th>
<th>Body</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>25.7</td>
<td>27.0</td>
</tr>
<tr>
<td></td>
<td>30.4</td>
<td>31.5</td>
</tr>
<tr>
<td></td>
<td>28.7</td>
<td>28.9</td>
</tr>
<tr>
<td></td>
<td>31.2</td>
<td>31.0</td>
</tr>
<tr>
<td></td>
<td>31.5</td>
<td>31.5</td>
</tr>
<tr>
<td></td>
<td>26.2</td>
<td>25.6</td>
</tr>
<tr>
<td></td>
<td>30.1</td>
<td>28.4</td>
</tr>
<tr>
<td></td>
<td>31.5</td>
<td>31.7</td>
</tr>
<tr>
<td></td>
<td>18.2</td>
<td>18.7</td>
</tr>
</tbody>
</table>

6. Draw a scatter plot and determine what relationship exists, if any, in the data.
7. Draw a line of fit for the scatter plot.
8. Write the slope-intercept form of an equation for the line of fit.
9. Predict the body temperature of an insect if the air temperature is 40.2°F.

### Practice and Apply

Determine whether each graph shows a **positive correlation**, a **negative correlation**, or **no correlation**. If there is a positive or negative correlation, describe its meaning in the situation.

#### 10.

**Census Forms Returned**

![Graph of Census Forms Returned](Source: U.S. Census Bureau)

#### 11.

**Hurricanes**

![Graph of Hurricanes](Source: USA TODAY)

#### 12.

**Electronic Tax Returns**

![Graph of Electronic Tax Returns](Source: IRS)

#### 13.

**Cereal Bars**

![Graph of Cereal Bars](Source: Vitality)

### FARMING

For Exercises 14 and 15, refer to the graph at the top of page 298 about apple storage.

14. Use the points (1997, 8.1) and (1999, 12.4) to write the slope-intercept form of an equation for the line of fit.
15. Predict the number of bushels of apples in storage in 2002.
**USED CARS**  For Exercises 16 and 17, use the scatter plot that shows the ages and prices of used cars from classified ads.

16. Use the points (2, 9600) and (5, 6000) to write the slope-intercept form of an equation for the line of fit shown in the scatter plot.

17. Predict the price of a car that is 7 years old.

**PHYSICAL SCIENCE**  For Exercises 18–23, use the following information. Hydrocarbons are composed of only carbon and hydrogen atoms. The table gives the number of carbon atoms and the boiling points for several hydrocarbons.

18. Draw a scatter plot comparing the numbers of carbon atoms to the boiling points.

19. Draw a line of fit for the data.

20. Write the slope-intercept form of an equation for the line of fit.

21. Predict the boiling point for methane (CH₄), which has 1 carbon atom.

22. Predict the boiling point for pentane (C₅H₁₂), which has 5 carbon atoms.

23. The boiling point of heptane is 98.4°C. Use the equation of the line of fit to predict the number of carbon atoms in heptane.

**SPACE**  For Exercises 24–28, use the table that shows the amount the United States government has spent on space and other technologies in selected years.

<table>
<thead>
<tr>
<th></th>
<th></th>
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<tbody>
<tr>
<td>Spending (billions of dollars)</td>
<td>4.5</td>
<td>6.6</td>
<td>11.6</td>
<td>12.6</td>
<td>12.7</td>
<td>13.1</td>
<td>12.9</td>
<td>12.4</td>
</tr>
</tbody>
</table>

Source: U.S. Office of Management and Budget

24. Draw a scatter plot and determine what relationship, if any, exists in the data.

25. Draw a line of fit for the scatter plot.

26. Let $x$ represent the number of years since 1980. Let $y$ represent the spending in billions of dollars. Write the slope-intercept form of the equation for the line of fit.

27. Predict the amount that will be spent on space and other technologies in 2005.

28. The government projects spending of $14.3$ billion in space and other technologies in 2005. How does this compare to your prediction?
For Exercises 29–33, use the table that shows the number of acres burned by wildfires in Florida each year and the corresponding number of inches of spring rainfall.

<table>
<thead>
<tr>
<th>Year</th>
<th>Rainfall (inches)</th>
<th>Acres (thousands)</th>
<th>Year</th>
<th>Rainfall (inches)</th>
<th>Acres (thousands)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1988</td>
<td>17.5</td>
<td>194</td>
<td>1994</td>
<td>18.1</td>
<td>180</td>
</tr>
<tr>
<td>1989</td>
<td>12.0</td>
<td>645</td>
<td>1995</td>
<td>16.3</td>
<td>46</td>
</tr>
<tr>
<td>1990</td>
<td>14.0</td>
<td>250</td>
<td>1996</td>
<td>20.4</td>
<td>94</td>
</tr>
<tr>
<td>1991</td>
<td>30.1</td>
<td>87</td>
<td>1997</td>
<td>18.5</td>
<td>146</td>
</tr>
<tr>
<td>1992</td>
<td>16.0</td>
<td>83</td>
<td>1998</td>
<td>22.2</td>
<td>507</td>
</tr>
<tr>
<td>1993</td>
<td>19.6</td>
<td>80</td>
<td>1999</td>
<td>12.7</td>
<td>340</td>
</tr>
</tbody>
</table>

Source: Florida Division of Forestry

29. Draw a scatter plot with rainfall on the x-axis and acres on the y-axis.
30. Draw a line of fit for the data.
31. Write the slope-intercept form of an equation for the line of fit.
32. In 2000, there was only 8.25 inches of spring rainfall. Estimate the number of acres burned by wildfires in 2000.
33. In 1998, there was 22.2 inches of rainfall, yet 507,000 acres were burned. Where was this data graphed in the scatter plot? How did this affect the line of fit?

Online Research | Data Update | What has happened to the number of acres burned by wildfires in Florida since 1999? Visit www.algebra1.com/data_update to learn more.

34. **CRITICAL THINKING** A test contains 20 true-false questions. Draw a scatter plot that shows the relationship between the number of correct answers \( x \) and the number of incorrect answers \( y \).

RESEARCH For Exercises 35 and 36, choose a topic to research that you believe may be correlated, such as arm span and height. Find existing data or collect your own.
35. Draw a line of fit for the data.
36. Use the line to make a prediction about the data.

37. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How do scatter plots help identify trends in data?
Include the following in your answer:
• a scatter plot that shows a person’s height and his or her age, with a description of any trends, and
• an explanation of how you could use the scatter plot to predict a person’s age given his or her height.

38. Which graph is the best example of data that show a negative linear relationship between the variables \( x \) and \( y \)?

---

**WebQuest**
You can use a line of fit to describe the trend in winning Olympic times. Visit www.algebra1.com/webquest to continue work on your WebQuest project.
39. Choose the equation for the line that best fits the data in the table at the right.

(A) \( y = x + 4 \)
(B) \( y = 2x + 3 \)
(C) \( y = 7 \)
(D) \( y = 4x - 5 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
</tr>
</tbody>
</table>

Extending the Lesson

**GEOGRAPHY**
For Exercises 40–44, use the following information.
The latitude of a place on Earth is the measure of its distance from the equator.

40. **MAKE A CONJECTURE** What do you think is the relationship between a city’s latitude and its January temperature?

41. **RESEARCH** Use the Internet or other reference to find the latitude of 15 cities in the northern hemisphere and the corresponding January mean temperatures.

42. Make a scatter plot and draw a line of fit for the data.
43. Write an equation for the line of fit.
44. **MAKE A CONJECTURE** Find the latitude of your city and use the equation to predict its mean January temperature. Check your prediction by using another source such as the newspaper.

Maintain Your Skills

**Mixed Review**

Write the slope-intercept form of an equation for the line that satisfies each condition. *(Lesson 5-6)*

45. parallel to the graph of \( y = -4x + 5 \) and passes through \((-2, 5)\)
46. perpendicular to the graph of \( y = 2x + 3 \) and passes through \((0, 0)\)

Write the point-slope form of an equation for a line that passes through each point with the given slope. *(Lesson 5-5)*

47. \((-2, 3)\) \( m = -2 \)
48. \((1, -2)\) \( m = 3 \)
49. \((-3, -3)\) \( m = 1 \)

Find the \(x\)- and \(y\)-intercepts of the graph of each equation. *(Lesson 4-5)*

50. \(3x + 4y = 12\)
51. \(2x - 5y = 8\)
52. \(y = 3x + 6\)

Solve each equation. Then check your solution. *(Lesson 3-4)*

53. \(\frac{r + 7}{-4} = \frac{r + 2}{6}\)
54. \(\frac{n - (-4)}{-3} = 7\)
55. \(\frac{2x - 1}{5} = \frac{4x - 5}{7}\)
Regression and Median-Fit Lines

One type of equation of best-fit you can find is a linear regression equation. Linear regression is sometimes called the method of least squares.

**EARNINGS** The table shows the average hourly earnings of U.S. production workers for selected years.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Earnings</td>
<td>$2.09</td>
<td>$2.46</td>
<td>$3.23</td>
<td>$4.53</td>
<td>$6.66</td>
<td>$8.57</td>
<td>$10.01</td>
<td>$11.43</td>
<td>$13.24</td>
</tr>
</tbody>
</table>

Source: Bureau of Labor Statistics

Find and graph a linear regression equation. Then predict the average hourly earnings in 2010.

**Step 1** Find a regression equation.
- Enter the years in L1 and the earnings in L2.
  **KEYSTROKES:** Review entering a list on page 204.
- Find the regression equation by selecting LinReg(ax+b) on the STAT CALC menu.
  **KEYSTROKES:** `STAT` `4` `ENTER`

The equation is about \( y = 0.30x - 588.35 \).

Self-check: \( r \) is the **linear correlation coefficient**. The closer the absolute value of \( r \) is to 1, the better the equation models the data. Because the \( r \) value is close to 1, the model fits the data well.

**Step 2** Graph the regression equation.
- Use STAT PLOT to graph the scatter plot.
  **KEYSTROKES:** Review statistical plots on page 204.
- Copy the equation to the Y= list and graph.
  **KEYSTROKES:** `Y=` `VARS` `5` `1` `GRAPH`

The residual is the difference between actual and predicted data. The predicted earnings in 1970 using this model were $3.94. (To calculate, press `2nd` `[CALC]` 1 1970 `ENTER`.) So the residual for 1970 was $3.94 – $3.23 or $0.71.

**Step 3** Predict using the regression equation.
- Find \( y \) when \( x = 2010 \) using value on the CALC menu.
  **KEYSTROKES:** `2nd` `[CALC]` 1 `2010` `ENTER`

According to the regression equation, the average hourly earnings in 2010 will be about $15.97.
A second type of best-fit line that can be found using a graphing calculator is a median-fit line. The equation of a median-fit line is calculated using the medians of the coordinates of the data points.

Find and graph a median-fit equation for the data on hourly earnings. Then predict the average hourly earnings in 2010. Compare this prediction to the one made using the regression equation.

**Step 1** Find a median-fit equation.
- The data are already in Lists 1 and 2. Find the median-fit equation by using Med-Med on the STAT CALC menu.
  
  **KEYSTROKES:** [STAT] 3 ENTER

  ![](image1)

  The median-fit equation is \( y = 0.299x - 585.17 \).

**Step 2** Graph the median-fit equation.
- Copy the equation to the \( Y= \) list and graph.
  
  **KEYSTROKES:** \[ Y= \text{CLEAR} \text{ VARS} 5 \text{ ENTER} \]

  ![Graph](image2)

  According to the median-fit equation, the average hourly earnings in 2010 will be about $15.82. This is slightly less than the predicted value found using the regression equation.

**Step 3** Predict using the median-fit equation.
- \( \text{KEYSTROKES:} 2\text{nd} \text{ [CALC]} 1 \text{ ENTER} \)

  ![Graph](image3)

Exercises

Refer to the data on bald eagles in Example 2 on pages 300 and 301.

1. Find regression and median-fit equations for the data.

2. What is the correlation coefficient of the regression equation? What does it tell you about the data?

3. Use the regression and median-fit equations to predict the number of bald eagle pairs in 1998. Compare these to the number found in Example 3 on page 301.

For Exercises 4 and 5, use the table that shows the number of votes cast for the Democratic presidential candidate in selected North Carolina counties in the 1996 and 2000 elections.

4. Find regression and median-fit equations for the data.

5. In 1996, New Hanover County had 22,839 votes for the Democratic candidate. Use the regression and median-fit equations to estimate the number of votes for the Democratic candidate in that county in 2000. How do the predictions compare to the actual number of 29,292?

Source: NC State Board of Elections
Exercises

1. An equation of the form $y = kx$, where $k \neq 0$, describes a (direct variation, linear extrapolation).

2. The ratio of (rise, run), or vertical change, to the (rise, run), or horizontal change, as you move from one point on a line to another, is the slope of a nonvertical line.

3. The lines with equations $y = -2x + 7$ and $y = -2x - 6$ are (parallel, perpendicular).

4. The equation $y - 2 = -3(x - 1)$ is written in (point-slope, slope-intercept) form.

5. The equation $y = -\frac{1}{3}x + 6$ is written in (slope-intercept, standard) form.

6. The (x-intercept, y-intercept) of the equation $-x - 4y = 2$ is $(-\frac{1}{2}, -\frac{1}{2})$.

Lesson-by-Lesson Review

5-1 Slope

Concept Summary

- The slope of a nonvertical line is the ratio of the rise to the run.
- $m = \frac{y_2 - y_1}{x_2 - x_1}$

Determine the slope of the line that passes through $(0, -4)$ and $(3, 2)$.

Let $(0, -4) = (x_1, y_1)$ and $(3, 2) = (x_2, y_2)$.

$\frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$x_1 = 0, x_2 = 3, y_1 = -4, y_2 = 2$

$m = \frac{2 - (-4)}{3 - 0}$ Simplify.

$m = \frac{6}{3} = 2$

Exercises

Find the slope of the line that passes through each pair of points.

See Examples 1–4 on page 257.

7. $(1, 3), (-2, -6)$  8. $(0, 5), (6, 2)$  9. $(-6, 4), (-6, -2)$

10. $(8, -3), (-2, -3)$  11. $(2.9, 4.7), (0.5, 1.1)$  12. $(\frac{1}{2}, 1), (-1, \frac{2}{3})$
Slope and Direct Variation

**Concept Summary**
- A direct variation is described by an equation of the form \( y = kx \), where \( k \neq 0 \).
- In \( y = kx \), \( k \) is the constant of variation. It is also the slope of the related graph.

**Example**
Suppose \( y \) varies directly as \( x \), and \( y = -24 \) when \( x = 8 \). Write a direct variation equation that relates \( x \) and \( y \).

\[
\begin{align*}
\frac{y}{k} &= x \quad \text{Direct variation equation} \\
-24 &= k(8) \quad \text{Replace } y \text{ with } -24 \text{ and } x \text{ with } 8. \\
-\frac{24}{8} &= \frac{k(8)}{8} \quad \text{Divide each side by } 8. \\
-3 &= k \quad \text{Simplify.}
\end{align*}
\]
Therefore, \( y = -3x \).

**Exercises**
Graph each equation. See Examples 2 and 3 on page 265.

13. \( y = 2x \) 
14. \( y = -4x \) 
15. \( y = \frac{1}{3}x \) 
16. \( y = -\frac{1}{4}x \) 
17. \( y = \frac{3}{2}x \) 
18. \( y = -\frac{4}{3}x \)

Suppose \( y \) varies directly as \( x \). Write a direct variation equation that relates \( x \) and \( y \). See Example 4 on page 266.

19. \( y = -6 \) when \( x = 9 \) 
20. \( y = 15 \) when \( x = 2 \) 
21. \( y = 4 \) when \( x = -4 \) 
22. \( y = -6 \) when \( x = -18 \) 
23. \( y = -10 \) when \( x = 5 \) 
24. \( y = 7 \) when \( x = -14 \)

Slope-Intercept Form

**Concept Summary**
- The linear equation \( y = mx + b \) is written in slope-intercept form, where \( m \) is the slope, and \( b \) is the \( y \)-intercept.
- Slope-intercept form allows you to graph an equation quickly.

**Example**
Graph \( -3x + y = -1 \).

\[
\begin{align*}
-3x + y &= -1 \quad \text{Original equation} \\
-3x + y + 3x &= -1 + 3x \quad \text{Add } 3x \text{ to each side.} \\
y &= 3x - 1 \quad \text{Simplify.}
\end{align*}
\]

**Step 1** The \( y \)-intercept is \(-1\). So, graph \((0, -1)\).

**Step 2** The slope is \( 3 \) or \( \frac{3}{1} \). From \((0, -1)\), move up 3 units and right 1 unit. Then draw a line.
Exercises

Write an equation of the line with the given slope and \( y \)-intercept. See Examples 1 and 2 on pages 272 and 273.

25. slope: 3, \( y \)-intercept: 2

26. slope: 1, \( y \)-intercept: -3

27. slope: 0, \( y \)-intercept: 4

28. slope: \( \frac{1}{3} \), \( y \)-intercept: 2

29. slope: 0.5, \( y \)-intercept: -0.3

30. slope: -1.3, \( y \)-intercept: 0.4

Graph each equation. See Examples 3 and 4 on pages 273 and 274.

31. \( y = 2x + 1 \)

32. \( y = -x + 5 \)

33. \( y = \frac{1}{2}x + 3 \)

34. \( y = -\frac{4}{3}x - 1 \)

35. \( 5x - 3y = -3 \)

36. \( 6x + 2y = 9 \)

5-4

Writing Equations in Slope-Intercept Form

Concept Summary

- To write an equation given the slope and one point, substitute the values of \( m \), \( x \), and \( y \) into the slope-intercept form and solve for \( b \). Then, write the slope-intercept form using the values of \( m \) and \( b \).

- To write an equation given two points, find the slope. Then follow the steps above.

Example

Write an equation of a line that passes through \((-2, -3) \) with slope \( \frac{1}{2} \).

\[
\begin{align*}
\frac{1}{2}x + b & = \text{Slope-intercept form} \\
-3 & = \frac{1}{2}(-2) + b \quad \text{Replace } m \text{ with } \frac{1}{2}, y \text{ with } -3, \text{ and } x \text{ with } -2. \\
-3 & = -1 + b \quad \text{Multiply.} \\
-3 + 1 & = -1 + b + 1 \quad \text{Add 1 to each side.} \\
-2 & = b \quad \text{Simplify.}
\end{align*}
\]

Therefore, the equation is \( y = \frac{1}{2}x - 2 \).

Exercises

Write an equation of the line that satisfies each condition. See Examples 1 and 2 on pages 280 and 281.

37. passes through \((-3, 3) \) with slope 1

38. passes through \((0, 6) \) with slope -2

39. passes through \((1, 6) \) with slope \( \frac{1}{2} \)

40. passes through \((4, -3) \) with slope \( -\frac{3}{5} \)

41. passes through \((-4, 2) \) and \((1, 12) \)

42. passes through \((5, 0) \) and \((4, 5) \)

43. passes through \((8, -1) \) with slope 0

44. passes through \((4, 6) \) and has slope 0
Writing Equations in Point-Slope Form

**Concept Summary**
- The linear equation $y - y_1 = m(x - x_1)$ is written in point-slope form, where $(x_1, y_1)$ is a given point on a nonvertical line and $m$ is the slope.

**Example**
Write the point-slope form of an equation for a line that passes through $(-2, 5)$ with slope 3.

1. $y - y_1 = m(x - x_1)$ Use the point-slope form.
2. $y - 5 = 3[x - (-2)]$ $(x_1, y_1) = (-2, 5)$
3. $y - 5 = 3(x + 2)$ Subtract.

**Exercises**
Write the point-slope form of an equation for a line that passes through each point with the given slope. See Example 2 on page 287.

- 45. $(4, 6), m = 5$
- 46. $(-1, 4), m = -2$
- 47. $(5, -3), m = \frac{1}{2}$
- 48. $(1, -4), m = -\frac{5}{2}$
- 49. $(\frac{1}{4}, -2), m = 3$
- 50. $(4, -2), m = 0$

Write each equation in standard form. See Example 3 on page 287.

- 51. $y - 1 = 2(x + 1)$
- 52. $y + 6 = \frac{1}{3}(x - 9)$
- 53. $y + 4 = 1.5(x - 4)$

Geometry: Parallel and Perpendicular Lines

**Concept Summary**
- Two nonvertical lines are parallel if they have the same slope.
- Two nonvertical lines are perpendicular if the product of their slopes is $-1$.

**Example**
Write the slope-intercept form for an equation of the line that passes through $(5, -2)$ and is parallel to $y = 2x + 7$.

The line parallel to $y = 2x + 7$ has the same slope, 2.

1. $y - y_1 = m(x - x_1)$ Point-slope form
2. $y - (-2) = 2(x - 5)$ Replace $m$ with 2, $y$ with $-2$, and $x$ with 5.
3. $y + 2 = 2x - 10$ Simplify.
4. $y = 2x - 12$ Subtract 2 from each side.
Exercises  Write the slope-intercept form for an equation of the line parallel to the graph of the given equation and passing through the given point.  
See Example 1 on page 292.

54.  \( y = 3x - 2, (4, 6) \)  
55.  \( y = -2x + 4, (6, -6) \)  
56.  \( y = -6x - 1, (1, 2) \)

57.  \( y = \frac{5}{12}x + 2, (0, 4) \)  
58.  \( 4x - y = 7, (2, -1) \)  
59.  \( 3x + 9y = 1, (3, 0) \)

Write the slope-intercept form for an equation of the line perpendicular to the graph of the given equation and passing through the given point.  See Example 3 on page 294.

60.  \( y = 4x + 2, (1, 3) \)  
61.  \( y = -2x - 7, (0, -3) \)  
62.  \( y = 0.4x + 1, (2, -5) \)

63.  \( 2x - 7y = 1, (-4, 0) \)  
64.  \( 8x - 3y = 7, (4, 5) \)  
65.  \( 5y = -x + 1, (2, -5) \)

Statistics: Scatter Plots and Lines of Fit

Concept Summary

- If \( y \) increases as \( x \) increases, then there is a positive correlation between \( x \) and \( y \).
- If \( y \) decreases as \( x \) increases, then there is a negative correlation between \( x \) and \( y \).
- If there is no relationship between \( x \) and \( y \), then there is no correlation between \( x \) and \( y \).
- A line of fit describes the trend of the data.
- You can use the equation of a line of fit to make predictions about the data.

**Exercises**  For Exercises 66–70, use the table that shows the length and weight of several humpback whales.  See Examples 2 and 3 on pages 300 and 301.

<table>
<thead>
<tr>
<th>Length (ft)</th>
<th>40</th>
<th>42</th>
<th>45</th>
<th>46</th>
<th>50</th>
<th>52</th>
<th>55</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight (long tons)</td>
<td>25</td>
<td>29</td>
<td>34</td>
<td>35</td>
<td>35</td>
<td>43</td>
<td>45</td>
</tr>
</tbody>
</table>

66. Draw a scatter plot with length on the \( x \)-axis and weight on the \( y \)-axis.
67. Draw a line of fit for the data.
68. Write the slope-intercept form of an equation for the line of fit.
69. Predict the weight of a 48-foot humpback whale.
70. Most newborn humpback whales are about 12 feet in length. Use the equation of the line of fit to predict the weight of a newborn humpback whale. Do you think your prediction is accurate? Explain.
Chapter 5
Practice Test

Vocabulary and Concepts

1. Explain why the equation of a vertical line cannot be in slope-intercept form.
2. Draw a scatter plot that shows a positive correlation.
3. Name the part of the slope-intercept form that represents the rate of change.

Skills and Applications

Find the slope of the line that passes through each pair of points.

4. (5, 8), (3, 7) 5. (5, –2), (3, –2) 6. (6, –3), (6, 4)

7. BUSINESS A web design company advertises that it will design and maintain a website for your business for $9.95 per month. Write a direct variation equation to find the total cost $C$ for any number of months $m$.

Graph each equation.

8. $y = 3x - 1$ 9. $y = 2x + 3$ 10. $2x + 3y = 9$

11. WEATHER The temperature is 16°F at midnight and is expected to fall 2° each hour during the night. Write the slope-intercept form of an equation to find the temperature $T$ for any hour $h$ after midnight.

Suppose $y$ varies directly as $x$. Write a direct variation equation that relates $x$ and $y$.

12. $y = 6$ when $x = 9$ 13. $y = -12$ when $x = 4$ 14. $y = -8$ when $x = 8$

Write the slope-intercept form of an equation of the line that satisfies each condition.

15. has slope $-\frac{4}{3}$ and $y$-intercept 3 16. passes through $(-2, -5)$ and $(8, -3)$
17. parallel to $3x + 7y = 4$ and passes through $(5, -2)$ 18. a horizontal line passing through $(5, -8)$
19. perpendicular to the graph of $5x - 3y = 9$ and passes through the origin
20. Write the point-slope form of an equation for a line that passes through $(-4, 3)$ with slope $-2$.

ANIMALS For Exercises 21–24, use the table that shows the relationship between dog years and human years.

21. Draw a scatter plot and determine what relationship, if any, exists in the data.
22. Draw a line of fit for the scatter plot.
23. Write the slope-intercept form of an equation for the line of fit.
24. Determine how many human years are comparable to 13 dog years.

25. STANDARDIZED TEST PRACTICE A line passes through $(0, 4)$ and $(3, 0)$. Which equation does not represent the equation of this line?

- $y - 4 = -\frac{4}{3}(x - 0)$
- $y = -\frac{4}{3}x + 3$
- $\frac{x}{3} + \frac{y}{4} = 1$
- $y - 0 = -\frac{4}{3}(x - 3)$
- $4x + 3y = 12$

www.algebra1.com/chapter_test
Part 1  Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. If a person’s weekly salary is $x$ and she saves $y$, what fraction of her weekly salary does she spend? (Lesson 1-1)

A) \( \frac{x}{y} \)
B) \( \frac{x}{y} \)
C) \( \frac{x-y}{y} \)
D) \( \frac{y-x}{x} \)

2. Evaluate \(-2x + 7y\) if \(x = -5\) and \(y = 4\). (Lesson 2-6)

A) 38
B) 43
C) 227
D) 243

3. Find \(x\), if \(5x + 6 = 10\). (Lesson 3-3)

A) \( -\frac{5}{4} \)
B) \( \frac{1}{10} \)
C) \( \frac{5}{16} \)
D) \( \frac{4}{5} \)

4. According to the data in the table, which of the following statements is true? (Lesson 3-7)

<table>
<thead>
<tr>
<th>Age</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
</tr>
<tr>
<td>14</td>
<td>2</td>
</tr>
<tr>
<td>16</td>
<td>1</td>
</tr>
<tr>
<td>17</td>
<td>2</td>
</tr>
</tbody>
</table>

A) mean age = median age
B) mean age > median age
C) mean age < median age
D) median age < mode age

5. What relationship exists between the \(x\)- and \(y\)-coordinates of each of the data points shown in the table? (Lesson 4-8)

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>4</td>
</tr>
<tr>
<td>-2</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>-2</td>
</tr>
<tr>
<td>5</td>
<td>-4</td>
</tr>
</tbody>
</table>

A) \(x\) and \(y\) are opposites.
B) The sum of \(x\) and \(y\) is 2.
C) The \(y\)-coordinate is 1 more than the square of the \(x\)-coordinate.
D) The \(y\)-coordinate is 1 more than the opposite of the \(x\)-coordinate.

6. What is the \(y\)-intercept of the line with equation \(\frac{x}{3} - \frac{y}{2} = 1\)? (Lesson 4-5)

A) -3
B) -2
C) \( \frac{2}{3} \)
D) \( \frac{3}{2} \)

7. Find the slope of a line that passes through (2, 4) and \((-4, 7)\). (Lesson 5-1)

A) \( -\frac{1}{2} \)
B) \( \frac{1}{2} \)
C) -2
D) 2

8. Which equation represents the line that passes through (3, 7) and (10, 21)? (Lesson 5-4)

A) \( x + y = 10 \)
B) \( y = \frac{1}{2}x + \frac{11}{2} \)
C) \( y = 2x + 1 \)
D) \( y = 3x - 2 \)

9. Choose the equation of a line parallel to the graph of \(y = 3x + 4\). (Lesson 5-6)

A) \( y = -\frac{1}{3}x + 4 \)
B) \( y = -3x + 4 \)
C) \( y = -x + 1 \)
D) \( y = 3x + 5 \)
Part 2  Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

10. While playing a game with her friends, Ellen scored 12 points less than twice the lowest score. She scored 98. What was the lowest score in the game?  \(\text{(Lesson 3-4)}\)

11. Find the \(x\)- and \(y\)-intercepts of \(3x - 2y = 12\). \(\text{(Lesson 4-5)}\)

12. Find the slope of a line that passes through \((5, 2)\) and \((-7, -3)\). \(\text{(Lesson 5-1)}\)

13. Suppose \(y\) varies directly as \(x\), and \(y = -9\) when \(x = 3\). Find \(x\) when \(y = 18\). \(\text{(Lesson 5-2)}\)

14. The graph of \(3x + 2y = 3\) is shown at the right. What is the \(y\)-intercept? \(\text{(Lesson 5-3)}\)

15. The table of ordered pairs shows the coordinates of some of the points on the graph of a function.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>6</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>-2</td>
</tr>
</tbody>
</table>

What is the \(y\)-coordinate of a point \((5, y)\) that lies on the graph of the function? \(\text{(Lesson 5-4)}\)

16. The equation \(y - 3 = -2(x + 5)\) is written in point-slope form. What is the slope of the line? \(\text{(Lesson 5-5)}\)

17. Find the slope-intercept form of an equation that passes through \((5, -8)\) and is perpendicular to the graph of \(-5x + 2y = 14\). \(\text{(Lesson 5-6)}\)

Part 3  Extended Response

Record your answers on a sheet of paper. Show your work.

18. A friend wants to enroll for cellular phone service. Three different plans are available. \(\text{(Lesson 5-3)}\)

Plan 1 charges \$0.59 per minute.
Plan 2 charges a monthly fee of \$10, plus \$0.39 per minute.
Plan 3 charges a monthly fee of \$59.95.

a. For each plan, write an equation that represents the monthly cost \(C\) for \(m\) number of minutes per month.

b. Graph each of the three equations.

c. Your friend expects to use 100 minutes per month. In which plan do you think that your friend should enroll? Explain.

19. A record company is designing a new CD case in the shape of a parallelogram. \(\text{(Lesson 5-5)}\)

a. Find the point-slope form of the equation of the line containing side \(\overline{WZ}\).

b. Find the standard form of the equation of the line containing side \(\overline{WZ}\).

Test-Taking Tip

Questions 9 and 17
Recall that graphs of linear equations that are parallel have the same slope. Graphs of linear equations that are perpendicular have slopes that are opposite reciprocals of each other.