6 Solving Linear Inequalities

What You'll Learn

- **Lessons 6-1 through 6-3**   Solve linear inequalities.
- **Lesson 6-4**   Solve compound inequalities and graph their solution sets.
- **Lesson 6-5**   Solve absolute value equations and inequalities.
- **Lesson 6-6**   Graph inequalities in the coordinate plane.

Key Vocabulary

- **set-builder notation** (p. 319)
- **compound inequality** (p. 339)
- **intersection** (p. 339)
- **union** (p. 340)
- **half-plane** (p. 353)

Why It's Important

Inequalities are used to represent various real-world situations in which a quantity must fall within a range of possible values. For example, figure skaters and gymnasts frequently want to know what they need to score to win a competition. That score can be represented by an inequality. *You will learn how a competitor can determine what score is needed to win in Lesson 6-1.*

CONTENTS

Getting Started

▶ **Prerequisite Skills** To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 6.

For Lessons 6-1 and 6-3   Solve Equations

Solve each equation. *(For review, see Lessons 3-2, 3-4, and 3-5.)*

1. $t + 31 = 84$
2. $b - 17 = 23$
3. $18 = 27 + f$
4. $d - \frac{2}{3} = \frac{1}{2}$
5. $3r - 45 = 4r$
6. $5m + 7 = 4m - 12$
7. $3y + 4 = 16$
8. $2a + 5 - 3a = 4$
9. $\frac{1}{2}k - 4 = 7$
10. $4.3b + 1.8 = 8.25$
11. $6s - 12 = 2(s + 2)$
12. $n - 3 = \frac{n + 1}{2}$

For Lesson 6-5   Evaluate Absolute Values

Find each value. *(For review, see Lesson 2-1.)*

13. $|-8|$
14. $|20|$
15. $|-30|$
16. $|-1.5|$
17. $|14 - 7|$
18. $|1 - 16|$
19. $|2 - 3|$
20. $|7 - 10|$

For Lesson 6-6   Graph Equations with Two Variables
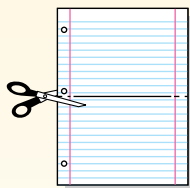
Graph each equation. *(For review, see Lesson 4-5.)*

21. $2x + 2y = 6$
22. $x - 3y = -3$
23. $y = 2x - 3$
24. $y = -4$
25. $x = -\frac{1}{2}y$
26. $3x - 6 = 2y$
27. $15 = 3(x + y)$
28. $2 - x = 2y$

FOLDABLES™
Study Organizer

Solving Linear Inequalities Make this Foldable to help you organize your notes. Begin with two sheets of notebook paper.
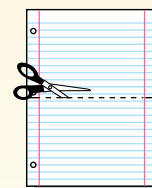
Step 1 **Fold and Cut**

Fold one sheet in half along the width. Cut along the fold from each edge to the margin.

Step 2 **Fold a New Paper and Cut**

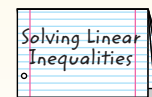Fold in half along the width. Cut along the fold between the margins.

Step 3 **Fold**

Insert the first sheet through the second sheet and align the folds.
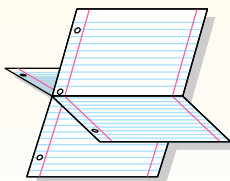
Step 4 **Label**

Label each page with a lesson number and title.

Solving Linear Inequalities

Reading and Writing As you read and study the chapter, fill the journal with notes, diagrams, and examples of linear inequalities.

CONTENTS

Solving Inequalities by Addition and Subtraction

What You'll Learn

- Solve linear inequalities by using addition.
- Solve linear inequalities by using subtraction.

Vocabulary
- set-builder notation

How are inequalities used to describe school sports?

In the 1999–2000 school year, more high schools offered girls' track and field than girls' volleyball.

$$14{,}587 > 13{,}426$$

If 20 schools added girls' track and field and 20 schools added girls' volleyball the next school year, there would still be more schools offering girls' track and field than schools offering girls' volleyball.

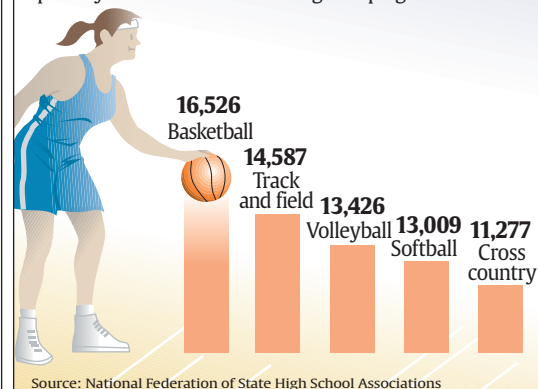$$14{,}587 + 20 \underline{\quad ? \quad} 13{,}426 + 20$$
$$14{,}607 > 13{,}446$$

USA TODAY Snapshots®

Girls gear up for high school sports

High school girls are playing sports in record numbers, almost 2.7 million in the 1999-2000 school year. Most popular girls sports by number of schools offering each program:

16,526 Basketball
14,587 Track and field
13,426 Volleyball
13,009 Softball
11,277 Cross country

Source: National Federation of State High School Associations

By Ellen J. Horrow and Alejandro Gonzalez, USA TODAY

SOLVE INEQUALITIES BY ADDITION
The sports application illustrates the **Addition Property of Inequalities**.

Key Concept — Addition Property of Inequalities

- **Words**   If any number is added to each side of a true inequality, the resulting inequality is also true.

- **Symbols**   For all numbers a, b, and c, the following are true.
  1. If $a > b$, then $a + c > b + c$.
  2. If $a < b$, then $a + c < b + c$.

- **Example**
  $$2 < 7$$
  $$2 + 6 < 7 + 6$$
  $$8 < 13$$

This property is also true when $>$ and $<$ are replaced with $\geq$ and $\leq$.

Example 1 Solve by Adding

Solve $t - 45 \leq 13$. Then check your solution.

$t - 45 \leq 13$	Original inequality
$t - 45 + 45 \leq 13 + 45$	Add 45 to each side.
$t \leq 58$	This means all numbers less than or equal to 58.

CHECK Substitute 58, a number less than 58, and a number greater than 58.

Let $t = 58$.
$$58 - 45 \stackrel{?}{\leq} 13$$
$$13 \leq 13 \quad \checkmark$$

Let $t = 50$.
$$50 - 45 \stackrel{?}{\leq} 13$$
$$5 \leq 13 \quad \checkmark$$

Let $t = 60$.
$$60 - 45 \stackrel{?}{\leq} 13$$
$$15 \not\leq 13$$

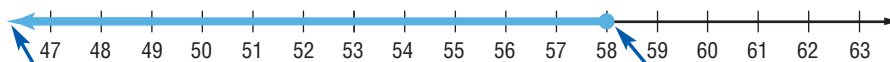The solution is the set {all numbers less than or equal to 58}.

The solution of the inequality in Example 1 was expressed as a set. A more concise way of writing a solution set is to use **set-builder notation**. The solution in set-builder notation is $\{t \mid t \le 58\}$.

The solution to Example 1 can also be represented on a number line.

The heavy arrow pointing to the left shows that the inequality includes all numbers less than 58.

The dot at 58 shows that 58 is included in the inequality.
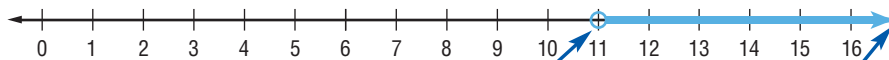
Example 2 *Graph the Solution*

Solve $7 < x - 4$. Then graph it on a number line.

$\begin{aligned}
7 &< x - 4 && \text{Original inequality} \\
7 + 4 &< x - 4 + 4 && \text{Add 4 to each side.} \\
11 &< x && \text{Simplify.}
\end{aligned}$

Since $11 < x$ is the same as $x > 11$, the solution set is $\{x \mid x > 11\}$.

The circle at 11 shows that 11 is *not* included in the inequality.

The heavy arrow pointing to the right shows that the inequality includes all numbers greater than 11.

SOLVE INEQUALITIES BY SUBTRACTION Subtraction can also be used to solve inequalities.

Key Concept — *Subtraction Property of Inequalities*

- **Words** If any number is subtracted from each side of a true inequality, the resulting inequality is also true.

- **Symbols** For all numbers a, b, and c, the following are true.
  1. If $a > b$, then $a - c > b - c$.
  2. If $a < b$, then $a - c < b - c$.

- **Example**
  $\begin{aligned}
  17 &> 8 \\
  17 - 5 &> 8 - 5 \\
  12 &> 3
  \end{aligned}$

This property is also true when $>$ and $<$ are replaced with $\ge$ and $\le$.

Example 3 *Solve by Subtracting*

Solve $19 + r \ge 16$. Then graph the solution.

$\begin{aligned}
19 + r &\ge 16 && \text{Original inequality} \\
19 + r - 19 &\ge 16 - 19 && \text{Subtract 19 from each side.} \\
r &\ge -3 && \text{Simplify.}
\end{aligned}$

The solution set is $\{r \mid r \ge -3\}$.

CONTENTS

Terms with variables can also be subtracted from each side to solve inequalities.

Example 4 Variables on Both Sides

Solve $5p + 7 > 6p$. Then graph the solution.

$$5p + 7 > 6p \qquad \text{Original inequality}$$
$$5p + 7 - 5p > 6p - 5p \qquad \text{Subtract } 5p \text{ from each side.}$$
$$7 > p \qquad \text{Simplify.}$$

Since $7 > p$ is the same as $p < 7$, the solution set is $\{p \mid p < 7\}$.

Verbal problems containing phrases like *greater than* or *less than* can often be solved by using inequalities. The following chart shows some other phrases that indicate inequalities.

Inequalities			
$<$	$>$	$\leq$	$\geq$
• less than • fewer than	• greater than • more than	• at most • no more than • less than or equal to	• at least • no less than • greater than or equal to

Example 5 Write and Solve an Inequality

Write an inequality for the sentence below. Then solve the inequality.

Four times a number is no more than three times that number plus eight.

Four times a number	is no more than	three times that number	plus	eight.
$4n$	$\leq$	$3n$	$+$	8

$$4n \leq 3n + 8 \qquad \text{Original inequality}$$
$$4n - 3n \leq 3n + 8 - 3n \qquad \text{Subtract } 3n \text{ from each side.}$$
$$n \leq 8 \qquad \text{Simplify.}$$

The solution set is $\{n \mid n \leq 8\}$.

Example 6 Write an Inequality to Solve a Problem

OLYMPICS **Yulia Raskina scored a total of 39.548 points in the four events of rhythmic gymnastics. Yulia Barsukova scored 9.883 in the rope competition, 9.900 in the hoop competition, and 9.916 in the ball competition. How many points did Barsukova need to score in the ribbon competition to surpass Raskina and win the gold medal?**

Words   Barsukova's total must be greater than Raskina's total.

Variable Let r = Barsukova's score in the ribbon competition.

Barsukova's total	is greater than	Raskina's total.
Inequality $9.883 + 9.900 + 9.916 + r$	$>$	39.548
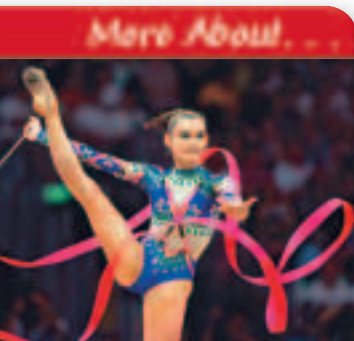
CONTENTS

Solve the inequality.

$9.883 + 9.900 + 9.916 + r > 39.548$	Original inequality
$29.699 + r > 39.548$	Simplify.
$29.699 + r - 29.699 > 39.548 - 29.699$	Subtract 29.699 from each side.
$r > 9.849$	Simplify.

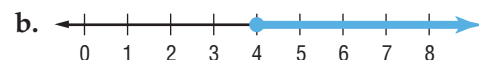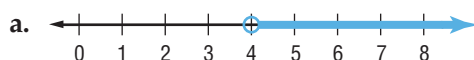Barsukova needed to score more than 9.849 points to win the gold medal.

Check for Understanding

Concept Check

1. **OPEN ENDED** List three inequalities that are equivalent to $y < -3$.

2. **Compare and contrast** the graphs of $a < 4$ and $a \leq 4$.

3. **Explain** what $\{b \mid b \geq -5\}$ means.

Guided Practice

4. Which graph represents the solution of $m + 3 > 7$?

   a.

   b.

   c.

   d.

Solve each inequality. Then check your solution, and graph it on a number line.

5. $a + 4 < 2$

6. $9 \leq b + 4$

7. $t - 7 \geq 5$

8. $y - 2.5 > 3.1$

9. $5.2r + 6.7 \geq 6.2r$

10. $7p \leq 6p - 2$

Define a variable, write an inequality, and solve each problem. Then check your solution.

11. A number decreased by 8 is at most 14.

12. A number plus 7 is greater than 2.

Application

13. **HEALTH** Chapa's doctor recommended that she limit her fat intake to no more than 60 grams per day. This morning, she ate two breakfast bars with 3 grams of fat each. For lunch she ate pizza with 21 grams of fat. If she follows her doctor's advice, how many grams of fat can she have during the rest of the day?

Practice and Apply
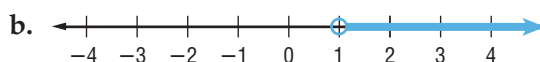
Match each inequality with its corresponding graph.

14. $x - 3 \geq -2$

   a.

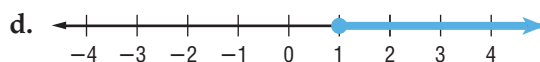15. $x + 7 \leq 6$

   b.

16. $4x > 3x - 1$

   c.
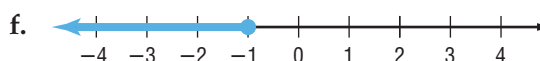
17. $8 + x < 9$

   d.

18. $5 \leq x + 6$

   e.

19. $x - 1 > 0$

   f.

Solve each inequality. Then check your solution, and graph it on a number line.

20. $t + 14 \geq 18$ **21.** $d + 5 \leq 7$ **22.** $n - 7 < -3$

23. $s - 5 > -1$ **24.** $5 < 3 + g$ **25.** $4 > 8 + r$

26. $-3 \geq q - 7$ **27.** $2 \leq m - 1$ **28.** $2y > -8 + y$

29. $3f < -3 + 2f$ **30.** $3b \leq 2b - 5$ **31.** $4w \geq 3w + 1$

32. $v - (-4) > 3$ **33.** $a - (-2) \leq -3$ **34.** $-0.23 < h - (-0.13)$

35. $x + 1.7 \geq 2.3$ **36.** $a + \frac{1}{4} > \frac{1}{8}$ **37.** $p - \frac{2}{3} \leq \frac{4}{9}$

38. If $d + 5 \geq 17$, then complete each inequality.

   a. $d \geq$ __?__   **b.** $d +$ __?__ ≥ 20 **c.** $d - 5 \geq$ __?__

39. If $z - 2 \leq 10$, then complete each inequality.

   a. $z \leq$ __?__   **b.** $z -$ __?__ ≤ 5 **c.** $z + 4 \leq$ __?__

Define a variable, write an inequality, and solve each problem. Then check your solution.

40. The sum of a number and 13 is at least 27.

41. A number decreased by 5 is less than 33.

42. Thirty is no greater than the sum of a number and -8.

43. Twice a number is more than the sum of that number and 14.

44. The sum of two numbers is at most 18, and one of the numbers is -7.

45. Four times a number is less than or equal to the sum of three times the number and -2.

46. BIOLOGY Adult Nile crocodiles weigh up to 2200 pounds. If a young Nile crocodile weighs 157 pounds, how many pounds might it be expected to gain in its lifetime?

47. ASTRONOMY There are at least 200 billion stars in the Milky Way. If 1100 of these stars can be seen in a rural area without the aid of a telescope, how many stars in the galaxy cannot be seen in this way?

48. BIOLOGY There are 3500 species of bees and more than 600,000 species of insects. How many species of insects are not bees?

49. BANKING City Bank requires a minimum balance of $1500 to maintain free checking services. If Mr. Hayashi knows he must write checks for $1300 and $947, how much money should he have in his account before writing the checks?

50. GEOMETRY The length of the base of the triangle at the right is less than the height of the triangle. What are the possible values of x?

4 + x in.

12 in.
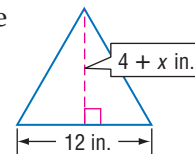
51. SHOPPING Terrell has $65 to spend at the mall. He bought a T-shirt for $18 and a belt for $14. If Terrell still wants to buy a pair of jeans, how much can he spend on the jeans?

52. SOCCER The Centerville High School soccer team plays 18 games in the season. The team has a goal of winning at least 60% of its games. After the first three weeks of the season, the team has won 4 games. How many more games must the team win to meet their goal?

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53. CRITICAL THINKING Determine whether each statement is *always*, *sometimes*, or *never* true.

    a. If $a < b$ and $c < d$, then $a + c < b + d$.

    b. If $a < b$ and $c < d$, then $a + c \geq b + d$.

    c. If $a < b$ and $c < d$, then $a - c = b - d$.

HEALTH **For Exercises 54 and 55, use the following information.**
Hector's doctor told him that his cholesterol level should be below 200. Hector's cholesterol is 225.

54. Let p represent the number of points Hector should lower his cholesterol. Write an inequality with $225 - p$ on one side.

55. Solve the inequality.

56. WRITING IN MATH   Answer the question that was posed at the beginning of the lesson.

How are inequalities used to describe school sports?

Include the following in your answer:

- an inequality describing the number of schools needed to add girls' track and field so that the number is greater than the number of schools currently participating in girls' basketball.

Standardized Test Practice
Ⓐ Ⓑ Ⓒ Ⓓ

57. Which inequality is *not* equivalent to $x \leq 12$?

    Ⓐ $x - 7 \leq 5$ Ⓑ $x + 4 \leq 16$ Ⓒ $x - 1 \leq 13$ Ⓓ $12 \geq x$

58. Which statement is modeled by $n + 6 \geq 5$?

    Ⓐ The sum of a number and six is at least five.

    Ⓑ The sum of a number and six is at most five.

    Ⓒ The sum of a number and six is greater than five.

    Ⓓ The sum of a number and six is no greater than five.

Maintain Your Skills

Mixed Review
59. Would a scatter plot for the relationship of a person's height to the person's grade on the last math test show a *positive*, *negative*, or *no correlation*? *(Lesson 5-7)*

Write an equation in slope-intercept form of the line that passes through the given point and is parallel to the graph of each equation. *(Lesson 5-6)*

60. $(1, -3); y = 3x - 2$ **61.** $(0, 4); x + y = -3$ **62.** $(-1, 2); 2x - y = 1$

Find the next two terms in each sequence. *(Lesson 4-8)*

63. 7, 13, 19, 25, …   **64.** 243, 81, 27, 9, …   **65.** 3, 6, 12, 24, …

Solve each equation if the domain is {−1, 3, 5}. *(Lesson 4-4)*

66. $y = -2x$ **67.** $y = 7 - x$ **68.** $2x - y = 6$

Getting Ready for the Next Lesson
PREREQUISITE SKILL **Solve each equation.**
*(For review of **multiplication and division equations**, see Lesson 3-3.)*

69. $6g = 42$ **70.** $\dfrac{t}{9} = 14$ **71.** $\dfrac{2}{3}y = 14$ **72.** $3m = 435$

73. $\dfrac{4}{7}x = 28$ **74.** $5.3g = 11.13$ **75.** $\dfrac{a}{3.5} = 7$ **76.** $8p = 35$

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Solving Inequalities

You can use algebra tiles to solve inequalities.

Solve $-2x \geq 6$.

Step 1 *Model the inequality.*

Use a self-adhesive note to cover the equals sign on the equation mat. Then write a $\geq$ symbol on the note. Model the inequality.
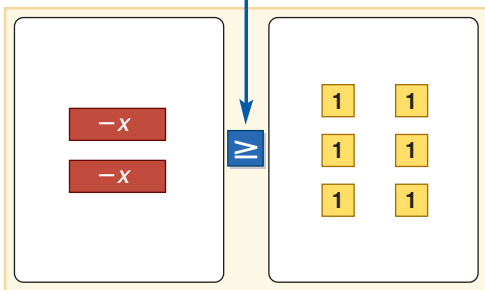
$$-2x \geq 6$$

Step 2 *Remove zero pairs.*

Since you do not want to solve for a negative x tile, eliminate the negative x tiles by adding 2 positive x tiles to each side. Remove the zero pairs.
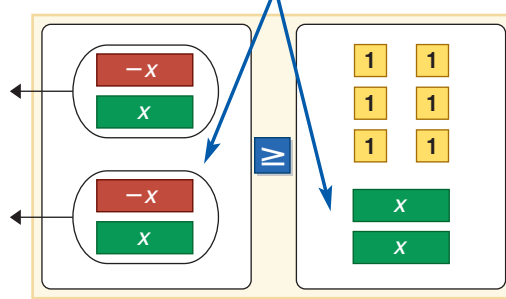
$$-2x + 2x \geq 6 + 2x$$

Step 3 *Remove zero pairs.*

Add 6 negative 1 tiles to each side to isolate the x tiles. Remove the zero pairs.
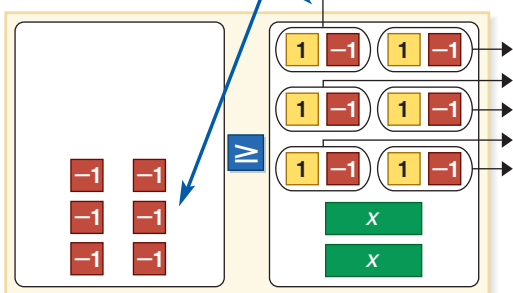
$$-6 \geq 2x$$

Step 4 *Group the tiles.*

Separate the tiles into 2 groups.
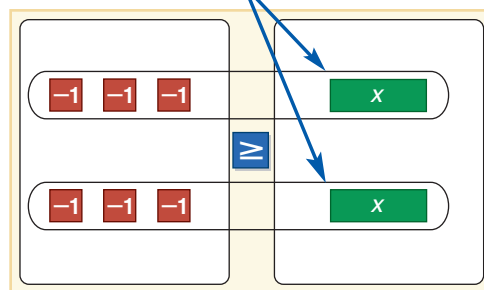
$$-3 \geq x \text{ or } x \leq -3$$

Model and Analyze

Use algebra tiles to solve each inequality.

1. $-4x < 12$
2. $-2x > 8$
3. $-3x \geq -6$
4. $-5x \leq -5$

5. In Exercises 1–4, is the coefficient of x in each inequality positive or negative?

6. Compare the inequality symbols and locations of the variable in Exercises 1–4 with those in their solutions. What do you find?

7. Model the solution for $2x \geq 6$. What do you find? How is this different from solving $-2x \geq 6$?

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Solving Inequalities by Multiplication and Division

- Solve linear inequalities by using multiplication.
- Solve linear inequalities by using division.

Why are inequalities important in landscaping?

Isabel Franco is a landscape architect. To beautify a garden, she plans to build a decorative wall of either bricks or blocks. Each brick is 3 inches high, and each block is 12 inches high. Notice that $3 < 12$.

48 in.

12 in.

12 in.

3 in.

A wall 4 bricks high would be lower than a wall 4 blocks high.

$$3 \times 4 \underline{\quad ? \quad} 12 \times 4$$
$$12 < 48$$

SOLVE INEQUALITIES BY MULTIPLICATION   If each side of an inequality is multiplied by a positive number, the inequality remains true.

$8 > 5$	$5 < 9$
$8(2) \underline{\ ?\ } 5(2)$ Multiply each side by 2.	$5(4) \underline{\ ?\ } 9(4)$ Multiply each side by 4.
$16 > 10$	$20 < 36$

This is *not* true when multiplying by negative numbers.
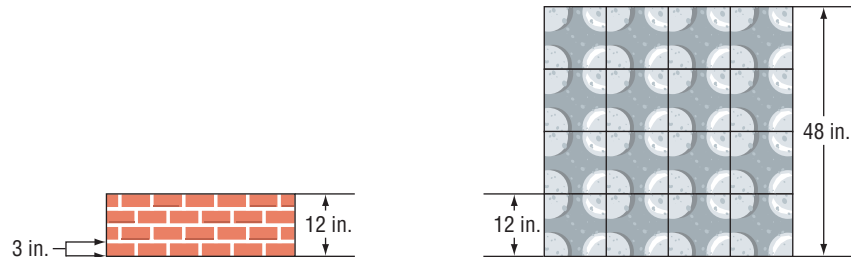
$5 > 3$	$-6 < 8$
$5(-2) \underline{\ ?\ } 3(-2)$ Multiply each side by -2.	$-6(-5) \underline{\ ?\ } 8(-5)$ Multiply each side by -5.
$-10 < -6$	$30 > -40$

If each side of an inequality is multiplied by a negative number, the direction of the inequality symbol changes. These examples illustrate the **Multiplication Property of Inequalities**.

Key Concept   *Multiplying by a Positive Number*

- **Words**   If each side of a true inequality is multiplied by the same positive number, the resulting inequality is also true.

- **Symbols**   If a and b are any numbers and c is a positive number, the following are true.
  If $a > b$, then $ac > bc$, and if $a < b$, then $ac < bc$.

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Key Concept — Multiplying by a Negative Number

- **Words** If each side of a true inequality is multiplied by the same negative number, the direction of the inequality symbol must be *reversed* so that the resulting inequality is also true.

- **Symbols** If a and b are any numbers and c is a negative number, the following are true.
  If $a > b$, then $ac < bc$, and if $a < b$, then $ac > bc$.

This property also holds for inequalities involving $\geq$ and $\leq$.

You can use this property to solve inequalities.

Example 1 Multiply by a Positive Number

Solve $\frac{b}{7} \geq 25$. Then check your solution.

$$\frac{b}{7} \geq 25 \qquad \text{Original inequality}$$

$$(7)\frac{b}{7} \geq (7)25 \qquad \text{Multiply each side by 7. Since we multiplied by a positive number, the inequality symbol stays the same.}$$

$$b \geq 175$$

CHECK To check this solution, substitute 175, a number less than 175, and a number greater than 175 into the inequality.

Let $b = 175$.   Let $b = 140$.   Let $b = 210$.

$\frac{175}{7} \overset{?}{\geq} 25$ $\frac{140}{7} \overset{?}{\geq} 25$ $\frac{210}{7} \overset{?}{\geq} 25$

$25 \geq 25$ ✓ $20 \ngeq 25$ $30 \geq 25$ ✓

The solution set is $\{b \mid b \geq 175\}$.

Example 2 Multiply by a Negative Number

Solve $-\frac{2}{5}p < -14$.

$$-\frac{2}{5}p < -14 \qquad \text{Original inequality}$$

$$\left(-\frac{5}{2}\right)\left(-\frac{2}{5}p\right) > \left(-\frac{5}{2}\right)(-14) \qquad \text{Multiply each side by } -\frac{5}{2} \text{ and change } < \text{ to } >.$$

$$p > 35 \qquad \text{The solution set is } \{p \mid p > 35\}.$$

Example 3 Write and Solve an Inequality

Write an inequality for the sentence below. Then solve the inequality.

One fourth of a number is less than -7.

One fourth,	of	a number,	is less than,	-7.
$\frac{1}{4}$	$\times$	n	$<$	-7

$$\frac{1}{4}n < -7 \qquad \text{Original inequality}$$

$$(4)\frac{1}{4}n < (4)(-7) \qquad \text{Multiply each side by 4 and do not change the inequality's direction.}$$

$$n < -28 \qquad \text{The solution set is } \{n \mid n < -28\}.$$

SOLVE INEQUALITIES BY DIVISION Dividing each side of an inequality by the same number is similar to multiplying each side of an equality by the same number. Consider the inequality $6 < 15$.

Divide each side by 3.

$$6 \quad < \quad 15$$
$$6 \div 3 \quad \underline{\quad ? \quad} \quad 15 \div 3$$
$$2 \quad < \quad 5$$

> Since each side is divided by a positive number, the direction of the inequality symbol remains the same.

Divide each side by -3.

$$6 \quad < \quad 15$$
$$6 \div (-3) \quad \underline{\quad ? \quad} \quad 15 \div (-3)$$
$$-2 \quad > \quad -5$$

> Since each side is divided by a negative number, the direction of the inequality symbol is reversed.

These examples illustrate the **Division Property of Inequalities**.

Key Concept   *Dividing by a Positive Number*

- **Words**   If each side of a true inequality is divided by the same positive number, the resulting inequality is also true.

- **Symbols**   If a and b are any numbers and c is a positive number, the following are true.

    If $a > b$, then $\frac{a}{c} > \frac{b}{c}$, and if $a < b$, then $\frac{a}{c} < \frac{b}{c}$.

Dividing by a Negative Number

- **Words**   If each side of a true inequality is divided by the same negative number, the direction of the inequality symbol must be *reversed* so that the resulting inequality is also true.

- **Symbols**   If a and b are any numbers and c is a negative number, the following are true.

    If $a > b$, then $\frac{a}{c} < \frac{b}{c}$, and if $a < b$, then $\frac{a}{c} > \frac{b}{c}$.

This property also holds for inequalities involving $\geq$ and $\leq$.

Example 4 *Divide by a Positive Number*

Solve $14h > 91$.

$14h > 91$ Original inequality

$\dfrac{14h}{14} > \dfrac{91}{14}$ Divide each side by 14 and do not change the direction of the inequality sign.

$h > 6.5$

CHECK   Let $h = 6.5$.   Let $h = 7$.   Let $h = 6$.

$14h > 91$ $14h > 91$ $14h > 91$

$14(6.5) \overset{?}{>} 91$ $14(7) \overset{?}{>} 91$ $14(6) \overset{?}{>} 91$

$91 \not> 91$ $98 > 91$ ✓ $84 \not> 91$

The solution set is $\{h \mid h > 6.5\}$.

Since dividing is the same as multiplying by the reciprocal, there are two methods to solve an inequality that involve multiplication.

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Example 5 *Divide by a Negative Number*

Solve $-5t \geq 275$ **using two methods.**

Method 1 Divide.

$-5t \geq 275$ Original inequality

$\dfrac{-5t}{-5} \leq \dfrac{275}{-5}$ Divide each side by -5 and change $\geq$ to $\leq$.

$t \leq -55$ Simplify.

Method 2 Multiply by the multiplicative inverse.

$-5t \geq 275$ Original inequality

$\left(-\dfrac{1}{5}\right)(-5t) \leq \left(-\dfrac{1}{5}\right)275$ Multiply each side by $-\dfrac{1}{5}$ and change $\geq$ to $\leq$.

$t \leq -55$ Simplify.

The solution set is $\{t \mid t \leq -55\}$.

You can use the Multiplication Property and the Division Property for Inequalities to solve standardized test questions.

Multiple-Choice Test Item

> Which inequality does *not* have the solution $\{y \mid y \leq -5\}$?
>
> Ⓐ $-7y \geq 35$ Ⓑ $2y \leq -10$ Ⓒ $\dfrac{7}{5}y \geq -7$ Ⓓ $-\dfrac{y}{4} \geq \dfrac{5}{4}$

Read the Test Item

You want to find the inequality that does *not* have the solution set $\{y \mid y \leq -5\}$.

Solve the Test Item

Consider each possible choice.

Ⓐ $-7y \geq 35$

$\dfrac{-7y}{-7} \leq \dfrac{35}{-7}$

$y \leq -5$ ✓

Ⓑ $2y \leq -10$

$\dfrac{2y}{2} \leq \dfrac{-10}{2}$

$y \leq -5$ ✓

Ⓒ $\dfrac{7}{5}y \geq -7$

$\left(\dfrac{5}{7}\right)\dfrac{7}{5}y \geq \left(\dfrac{5}{7}\right)(-7)$

$y \geq -5$

Ⓓ $-\dfrac{y}{4} \geq \dfrac{5}{4}$

$(-4)\left(-\dfrac{y}{4}\right) \leq (-4)\dfrac{5}{4}$

$y \leq -5$ ✓

The answer is C.

Test-Taking Tip

Always look for the word *not* in the questions. This indicates that you are looking for the one incorrect answer, rather than looking for the one correct answer. The word *not* is usually in italics or uppercase letters to draw your attention to it.

Check for Understanding

Concept Check

1. **Explain** why you can use either the Multiplication Property of Inequalities or the Division Property of Inequalities to solve $-7r \leq 28$.

2. **OPEN ENDED** Write a problem that can be represented by the inequality $\dfrac{3}{4}c > 9$.

◀ **CONTENTS** ▶

3. FIND THE ERROR Ilonia and Zachary are solving $-9b \leq 18$.

Ilonia	Zachary
$-9b \leq 18$	$-9b \leq 18$
$\dfrac{-9b}{-9} \geq \dfrac{18}{-9}$	$\dfrac{-9b}{-9} \leq \dfrac{18}{-9}$
$b \geq -2$	$b \leq -2$

Who is correct? Explain your reasoning.

Guided Practice

4. Which statement is represented by $7n \geq 14$?

   a. Seven times a number is at least 14.

   b. Seven times a number is greater than 14.

   c. Seven times a number is at most 14.

   d. Seven times a number is less than 14.

5. Which inequality represents *five times a number is less than 25*?

   a. $5n > 25$ **b.** $5n \geq 25$ **c.** $5n < 25$ **d.** $5n \leq 25$

Solve each inequality. Then check your solution.

   6. $-15g > 75$ **7.** $\dfrac{t}{9} < -12$ **8.** $-\dfrac{2}{3}b \leq -9$ **9.** $25f \geq 9$

Define a variable, write an inequality, and solve each problem. Then check your solution.

10. The opposite of four times a number is more than 12.

11. Half of a number is at least 26.

Standardized Test Practice
Ⓐ Ⓑ Ⓒ Ⓓ

12. Which inequality does *not* have the solution set $\{x \mid x > 4\}$?

   Ⓐ $-5x < -20$ Ⓑ $6x < 24$ Ⓒ $\dfrac{1}{5}x > \dfrac{4}{5}$ Ⓓ $-\dfrac{3}{4}x < -3$

Practice and Apply

Homework Help

For Exercises	See Examples
13–18, 39–44	3
19–38	1, 2, 4, 5
45–51	6

Extra Practice
See page 833.

Match each inequality with its corresponding statement.

13. $\dfrac{1}{5}n > 10$ **a.** Five times a number is less than or equal to ten.

14. $5n \leq 10$ **b.** One fifth of a number is no less than ten.

15. $5n > 10$ **c.** Five times a number is less than ten.

16. $-5n < 10$ **d.** One fifth of a number is greater than ten.

17. $\dfrac{1}{5}n \geq 10$ **e.** Five times a number is greater than ten.

18. $5n < 10$ **f.** Negative five times a number is less than ten.

Solve each inequality. Then check your solution.

19. $6g \leq 144$ **20.** $7t > 84$ **21.** $-14d \geq 84$ **22.** $-16z \leq -64$

23. $\dfrac{m}{5} \geq 7$ **24.** $\dfrac{b}{10} \leq 5$ **25.** $-\dfrac{r}{7} < -7$ **26.** $-\dfrac{a}{11} > 9$

27. $\dfrac{5}{8}y \geq -15$ **28.** $\dfrac{2}{3}v < 6$ **29.** $-\dfrac{3}{4}q \leq -33$ **30.** $-\dfrac{2}{5}p > 10$

31. $-2.5w < 6.8$ **32.** $-0.8s > 6.4$ **33.** $\dfrac{15c}{-7} > \dfrac{3}{14}$ **34.** $\dfrac{4m}{5} < \dfrac{-3}{15}$

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35. Solve $-\dfrac{y}{8} > \dfrac{1}{2}$. Then graph the solution.

36. Solve $-\dfrac{m}{9} \le -\dfrac{1}{3}$. Then graph the solution.

37. If $2a \ge 7$, then complete each inequality.

    a. $a \ge$ __?__   **b.** $-4a \le$ __?__   **c.** __?__ $a \le -21$

38. If $4t < -2$, then complete each inequality.

    a. $t <$ __?__   **b.** $-8t >$ __?__   **c.** __?__ $t > 14$

Define a variable, write an inequality, and solve each problem. Then check your solution.

39. Seven times a number is greater than 28.

40. Negative seven times a number is at least 14.

41. Twenty-four is at most a third of a number.

42. Two thirds of a number is less than -15.

43. Twenty-five percent of a number is greater than or equal to 90.

44. Forty percent of a number is less than or equal to 45.

45. GEOMETRY The area of a rectangle is less than 85 square feet. The length of the rectangle is 20 feet. What is the width of the rectangle?

46. FUND-RAISING The Middletown Marching Mustangs want to make at least $2000 on their annual mulch sale. The band makes $2.50 on each bag of mulch that is sold. How many bags of mulch should the band sell?

47. LONG-DISTANCE COSTS Juan's long-distance phone company charges him 9¢ for each minute or any part of a minute. He wants to call his friend, but he does not want to spend more than $2.50 on the call. How long can he talk to his friend?

48. EVENT PLANNING The Country Corner Reception Hall does not charge a rental fee as long as at least $4000 is spent on food. Shaniqua is planning a class reunion. If she has chosen a buffet that costs $28.95 per person, how many people must attend the reunion to avoid a rental fee for the hall?

49. LANDSCAPING Matthew is planning a circular flower garden with a low fence around the border. If he can use up to 38 feet of fence, what radius can he use for the garden? (*Hint*: $C = 2\pi r$)

50. DRIVING Average speed is calculated by dividing distance by time. If the speed limit on the interstate is 65 miles per hour, how far can a person travel legally in $1\dfrac{1}{2}$ hours?

51. ZOOS The yearly membership to the San Diego Zoo for a family with 2 adults and 2 children is $144. The regular admission to the zoo is $18 for each adult and $8 for each child. How many times should such a family plan to visit the zoo in a year to make a membership less expensive than paying regular admission?

52. CRITICAL THINKING Give a counterexample to show that each statement is not always true.

    a. If $a > b$, then $a^2 > b^2$.   **b.** If $a < b$ and $c < d$, then $ac < bd$.

53. CITY PLANNING The city of Santa Clarita requires that a parking lot can have no more than 20% of the parking spaces limited to compact cars. If a certain parking lot has 35 spaces for compact cars, how many spaces must the lot have to conform to the code?

Zoos •••••••••••

Dr. Harry Wegeforth founded the San Diego Zoo in 1916 with just 50 animals. Today, the zoo has over 3800 animals.

Source: www.sandiegozoo.org

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54. CIVICS For a candidate to run for a county office, he or she must submit a petition with at least 6000 signatures of registered voters. Usually only 85% of the signatures are valid. How many signatures should a candidate seek on a petition?

55. [WRITING IN MATH] Answer the question that was posed at the beginning of the lesson.

Why are inequalities important in landscaping?

Include the following in your answer:
- an inequality representing a brick wall that can be no higher than 4 feet, and
- an explanation of how to solve the inequality.

Standardized Test Practice
A B C D

56. The solution set for which inequality is *not* represented by the following graph?

$$-9\;-8\;-7\;-6\;-5\;-4\;-3\;-2\;-1\;\;0\;\;1\;\;2\;\;3\;\;4\;\;5\;\;6\;\;7\;\;8\;\;9$$

(A) $-\dfrac{x}{5} \le 1$ (B) $\dfrac{x}{5} \le -1$ (C) $-9x \le 45$ (D) $2.5x \ge -12.5$

57. Solve $-\dfrac{7}{8}t < \dfrac{14}{15}$.

(A) $\left\{t \,\middle|\, t > \dfrac{16}{15}\right\}$ (B) $\left\{t \,\middle|\, t < \dfrac{16}{15}\right\}$ (C) $\left\{t \,\middle|\, t > -\dfrac{16}{15}\right\}$ (D) $\left\{t \,\middle|\, t < -\dfrac{16}{15}\right\}$

Maintain Your Skills

Mixed Review Solve each inequality. Then check your solution, and graph it on a number line. *(Lesson 6-1)*

58. $s - 7 < 12$ **59.** $g + 3 \le -4$ **60.** $7 > n + 2$

61. Draw a scatter plot that shows a positive correlation. *(Lesson 5-7)*

Write an equation of the line that passes through each pair of points. *(Lesson 5-4)*

62. $(-1, 3), (2, 4)$ **63.** $(5, -2), (-1, -2)$ **64.** $(3, 3), (-1, 2)$

If $h(x) = 3x + 2$, find each value. *(Lesson 4-6)*

65. $h(-4)$ **66.** $h(2)$ **67.** $h(w)$ **68.** $h(r - 6)$

Solve each proportion. *(Lesson 3-6)*

69. $\dfrac{3}{4} = \dfrac{x}{8}$ **70.** $\dfrac{t}{1.5} = \dfrac{2.4}{1.6}$ **71.** $\dfrac{w + 2}{5} = \dfrac{7}{5}$ **72.** $\dfrac{x}{3} = \dfrac{x + 5}{15}$

Getting Ready for the Next Lesson **PREREQUISITE SKILL** Solve each equation.
*(To review **multi-step equations**, see Lessons 3-4 and 3-5.)*

73. $5x - 3 = 32$ **74.** $4t + 9 = 14$ **75.** $6y - 1 = 4y + 23$

76. $\dfrac{14g + 5}{6} = 9$ **77.** $5a + 6 = 9a - (7a + 18)$ **78.** $2(p - 4) = 7(p + 3)$

Practice Quiz 1   Lessons 6-1 and 6-2

Solve each inequality. Then check your solution, and graph it on a number line. *(Lesson 6-1)*

1. $h - 16 > -13$ **2.** $r + 3 \le -1$ **3.** $4 \ge p + 9$ **4.** $-3 < a - 5$ **5.** $7g \le 6g - 1$

Solve each inequality. Then check your solution. *(Lesson 6-2)*

6. $15z \ge 105$ **7.** $\dfrac{v}{5} < 7$ **8.** $-\dfrac{3}{7}q > 15$ **9.** $-156 < 12r$ **10.** $-\dfrac{2}{5}w \le -\dfrac{1}{2}$
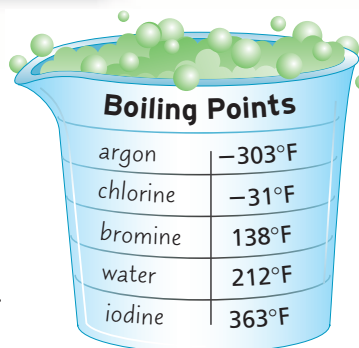
6-3 Solving Multi-Step Inequalities

What You'll Learn

- Solve linear inequalities involving more than one operation.
- Solve linear inequalities involving the Distributive Property.

How are linear inequalities used in science?

The boiling point of a substance is the temperature at which the element changes from a liquid to a gas. The boiling point of chlorine is $-31°F$. That means chlorine will be a gas for all temperatures greater than $-31°F$. If F represents temperature in degrees Fahrenheit, the inequality $F > -31$ represents the temperatures for which chlorine is a gas.

If C represents degrees Celsius, then $F = \frac{9}{5}C + 32$. You can solve $\frac{9}{5}C + 32 > -31$ to find the temperatures in degrees Celsius for which chlorine is a gas.

Boiling Points

argon	$-303°F$
chlorine	$-31°F$
bromine	$138°F$
water	$212°F$
iodine	$363°F$

Source: *World Book Encyclopedia*

SOLVE MULTI-STEP INEQUALITIES The inequality $\frac{9}{5}C + 32 > -31$ involves more than one operation. It can be solved by undoing the operations in the same way you would solve an equation with more than one operation.

Example 1 Solve a Real-World Problem

SCIENCE Find the temperatures in degrees Celsius for which chlorine is a gas.

$\frac{9}{5}C + 32 > -31$ — Original inequality

$\frac{9}{5}C + 32 - 32 > -31 - 32$ — Subtract 32 from each side.

$\frac{9}{5}C > -63$ — Simplify.

$\left(\frac{5}{9}\right)\frac{9}{5}C > \left(\frac{5}{9}\right)(-63)$ — Multiply each side by $\frac{5}{9}$.

$C > -35$ — Simplify.

Chlorine will be a gas for all temperatures greater than $-35°C$.

When working with inequalities, do not forget to reverse the inequality sign whenever you multiply or divide each side by a negative number.

Example 2 Inequality Involving a Negative Coefficient

Solve $-7b + 19 < -16$. Then check your solution.

$-7b + 19 < -16$ — Original inequality

$-7b + 19 - 19 < -16 - 19$ — Subtract 19 from each side.

$-7b < -35$ — Simplify.

$\frac{-7b}{-7} > \frac{-35}{-7}$ — Divide each side by -7 and change $<$ to $>$.

$b > 5$ — Simplify.

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CHECK To check this solution, substitute 5, a number less than 5, and a number greater than 5.

Let $b = 5$.

$-7b + 19 < -16$

$-7(5) + 19 \overset{?}{<} -16$

$-35 + 19 \overset{?}{<} -16$

$-16 \not< -16$

Let $b = 4$.

$-7b + 19 < -16$

$-7(4) + 19 \overset{?}{<} -16$

$-28 + 19 \overset{?}{<} -16$

$-9 \not< -16$

Let $b = 6$.

$-7b + 19 < -16$

$-7(6) + 19 \overset{?}{<} -16$

$-42 + 19 \overset{?}{<} -16$

$-23 < -16$ ✓

The solution set is $\{b \mid b > 5\}$.

Example 3 Write and Solve an Inequality

Write an inequality for the sentence below. Then solve the inequality.
Three times a number minus eighteen is at least five times the number plus twenty-one.

Three times a number	minus	eighteen	is at least	five times the number	plus	twenty one.
$3n$	$-$	18	$\geq$	$5n$	$+$	21

$3n - 18 \geq 5n + 21$	Original inequality
$3n - 18 - 5n \geq 5n + 21 - 5n$	Subtract $5n$ from each side.
$-2n - 18 \geq 21$	Simplify.
$-2n - 18 + 18 \geq 21 + 18$	Add 18 to each side.
$-2n \geq 39$	Simplify.
$\dfrac{-2n}{-2} \leq \dfrac{39}{-2}$	Divide each side by -2 and change $\geq$ to $\leq$.
$n \leq -19.5$	Simplify.

The solution set is $\{n \mid n \leq -19.5\}$.
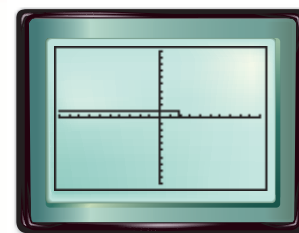
A graphing calculator can be used to solve inequalities.
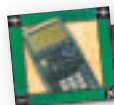
Graphing Calculator Investigation

Solving Inequalities

You can find the solution of an inequality in one variable by using a graphing calculator. On a TI-83 Plus, clear the Y= list. Enter $6x + 9 < -4x + 29$ as Y1. (The symbol $<$ is item 5 on the TEST menu.)

Press GRAPH.

[10, 10] scl: 1 by [10, 10] scl: 1

Think and Discuss

1. Describe what is shown on the screen.

2. Use the TRACE function to scan the values along the graph. What do you notice about the values of *y* on the graph?

3. Solve the inequality algebraically. How does your solution compare to the pattern you noticed in Exercise 2?

SOLVE INEQUALITIES INVOLVING THE DISTRIBUTIVE PROPERTY

When solving equations that contain grouping symbols, first use the Distributive Property to remove the grouping symbols.

Example 4 *Distributive Property*

Solve $3d - 2(8d - 9) > 3 - (2d + 7)$.

$3d - 2(8d - 9) > 3 - (2d + 7)$	Original inequality
$3d - 16d + 18 > 3 - 2d - 7$	Distributive Property
$-13d + 18 > -2d - 4$	Combine like terms.
$-13d + 18 + 13d > -2d - 4 + 13d$	Add 13d to each side.
$18 > 11d - 4$	Simplify.
$18 + 4 > 11d - 4 + 4$	Add 4 to each side.
$22 > 11d$	Simplify.
$\dfrac{22}{11} > \dfrac{11d}{11}$	Divide each side by 11.
$2 > d$	Simplify.

Since $2 > d$ is the same as $d < 2$, the solution set is $\{d \mid d < 2\}$.

If solving an inequality results in a statement that is always true, the solution is all real numbers. If solving an inequality results in a statement that is never true, the solution is the empty set $\varnothing$. The empty set has no members.

Example 5 *Empty Set*

Solve $8(t + 2) - 3(t - 4) < 5(t - 7) + 8$.

$8(t + 2) - 3(t - 4) < 5(t - 7) + 8$	Original inequality
$8t + 16 - 3t + 12 < 5t - 35 + 8$	Distributive Property
$5t + 28 < 5t - 27$	Combine like terms.
$5t + 28 - 5t < 5t - 27 - 5t$	Subtract 5t from each side.
$28 < -27$	This statement is false.

Since the inequality results in a false statement, the solution set is the empty set $\varnothing$.

Check for Understanding

Concept Check **1.** **Compare and contrast** the method used to solve $-5h + 6 = -7$ and the method used to solve $-5h + 6 \leq -7$.

2. **OPEN ENDED** Write a multi-step inequality with the solution graphed below.
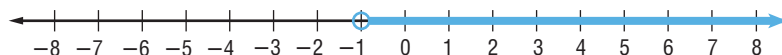
Guided Practice **3.** Justify each indicated step.

$$3(a - 7) + 9 \leq 21$$
$$3a - 21 + 9 \leq 21 \qquad \textbf{a.} \quad \underline{\quad ? \quad}$$
$$3a - 12 \leq 21$$
$$3a - 12 + 12 \leq 21 + 12 \qquad \textbf{b.} \quad \underline{\quad ? \quad}$$
$$3a \leq 33$$
$$\frac{3a}{3} \leq \frac{33}{3} \qquad \textbf{c.} \quad \underline{\quad ? \quad}$$
$$a \leq 11$$

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Solve each inequality. Then check your solution.

4. $-4y - 23 < 19$ 5. $\frac{2}{3}r + 9 \geq -3$ 6. $7b + 11 > 9b - 13$

7. $-5(g + 4) > 3(g - 4)$ 8. $3 + 5t \leq 3(t + 1) - 4(2 - t)$

9. Define a variable, write an inequality, and solve the problem below. Then check your solution.
   Seven minus two times a number is less than three times the number plus thirty-two.

Application 10. **SALES** A salesperson is paid $22,000 a year plus 5% of the amount of sales made. What is the amount of sales needed to have an annual income greater than $35,000?

Practice and Apply

Justify each indicated step.

11. $\frac{2}{5}w + 7 \leq -9$

$\frac{2}{5}w + 7 - 7 \leq -9 - 7$ **a.** __?__

$\frac{2}{5}w \leq -16$

$\left(\frac{5}{2}\right)\frac{2}{5}w \leq \left(\frac{5}{2}\right)(-16)$ **b.** __?__

$w \leq -40$

12. $m > \frac{15 - 2m}{-3}$

$(-3)m < (-3)\frac{15 - 2m}{-3}$ **a.** __?__

$-3m < 15 - 2m$

$-3m + 2m < 15 - 2m + 2m$ **b.** __?__

$-m < 15$

$(-1)(-m) > (-1)15$ **c.** __?__

$m > -15$

13. Solve $4(t - 7) \leq 2(t + 9)$. Show each step and justify your work.

14. Solve $-5(k + 4) > 3(k - 4)$. Show each step and justify your work.

Solve each inequality. Then check your solution.

15. $-3t + 6 \leq -3$ 16. $-5 - 8f > 59$ 17. $-2 - \frac{d}{5} < 23$

18. $\frac{w}{8} - 13 > -6$ 19. $7q - 1 + 2q \leq 29$ 20. $8a + 2 - 10a \leq 20$

21. $9r + 15 \leq 24 + 10r$ 22. $13k - 11 > 7k + 37$ 23. $\frac{2v - 3}{5} \geq 7$

24. $\frac{3a + 8}{2} < 10$ 25. $\frac{3w + 5}{4} \geq 2w$ 26. $\frac{5b + 8}{3} < 3b$

27. $7 + 3t \leq 2(t + 3) - 2(-1 - t)$ 28. $5(2h - 6) - 7(h + 7) > 4h$

29. $3y + 4 > 2(y + 3) + y$ 30. $3 - 3(b - 2) < 13 - 3(b - 6)$

31. $3.1v - 1.4 \geq 1.3v + 6.7$ 32. $0.3(d - 2) - 0.8d > 4.4$

33. Solve $4(y + 1) - 3(y - 5) \geq 3(y - 1)$. Then graph the solution.

34. Solve $5(x + 4) - 2(x + 6) \geq 5(x + 1) - 1$. Then graph the solution.

Define a variable, write an inequality, and solve each problem. Then check your solution.

35. One eighth of a number decreased by five is at least thirty.

36. Two thirds of a number plus eight is greater than twelve.

37. Negative four times a number plus nine is no more than the number minus twenty-one.

38. Three times the sum of a number and seven is greater than five times the number less thirteen.

GEOMETRY For Exercises 39 and 40, use the following information.
By definition, the measure of any acute angle is less than 90 degrees. Suppose the measure of an acute angle is $3a - 15$.

39. Write an inequality to represent the situation.

40. Solve the inequality.

SCHOOL For Exercises 41 and 42, use the following information.
Carmen's scores on three math tests were 91, 95, and 88. The fourth and final test of the grading period is tomorrow. She needs an average (mean) of at least 92 to receive an A for the grading period.

41. If s is her score on the fourth test, write an inequality to represent the situation.

42. If Carmen wants an A in math, what must she score on the test?

PHYSICAL SCIENCE For Exercises 43 and 44, use the information at the left and the information below.
The melting point for an element is the temperature where the element changes from a solid to a liquid. If C represents degrees Celsius and F represents degrees Fahrenheit, then $C = \dfrac{5(F - 32)}{9}$.

43. Write an inequality that can be used to find the temperatures in degrees Fahrenheit for which mercury is a solid.

44. For what temperatures will mercury be a solid?

45. HEALTH Keith weighs 200 pounds. He wants to weigh less than 175 pounds. If he can lose an average of 2 pounds per week on a certain diet, how long should he stay on his diet to reach his goal weight?

46. CRITICAL THINKING Write a multi-step inequality that has no solution and one that has infinitely many solutions.

47. PERSONAL FINANCES Nicholas wants to order a pizza. He has a total of $13.00 to pay the delivery person. The pizza costs $7.50 plus $1.25 per topping. If he plans to tip 15% of the total cost of the pizza, how many toppings can he order?

LABOR For Exercises 48–50, use the following information.
A union worker made $500 per week. His union sought a one-year contract and went on strike. Once the new contract was approved, it provided for a 4% raise.

48. Assume that the worker was not paid during the strike. Given his raise in salary, how many weeks could he strike and still make at least as much for the next 52 weeks as he would have made without a strike?

49. How would your answer to Exercise 48 change if the worker had been making $600 per week?

50. How would your answer to Exercise 48 change if the worker's union provided him with $150 per week during the strike?

51. NUMBER THEORY Find all sets of two consecutive positive odd integers whose sum is no greater than 18.

52. NUMBER THEORY Find all sets of three consecutive positive even integers whose sum is less than 40.

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53. WRITING IN MATH   Answer the question that was posed at the beginning of the lesson.

How are linear inequalities used in science?

Include the following in your answer:

- an inequality for the temperatures in degrees Celsius for which bromine is a gas, and
- a description of a situation in which a scientist might use an inequality.

54. What is the first step in solving $\dfrac{y-5}{9} \geq 13$?

   Ⓐ Add 5 to each side.   Ⓑ Subtract 5 from each side.

   Ⓒ Divide each side by 9.   Ⓓ Multiply each side by 9.

55. Solve $4t + 2 < 8t - (6t - 10)$.

   Ⓐ $\{t \mid t < -6\}$ Ⓑ $\{t \mid t > -6\}$ Ⓒ $\{t \mid t < 4\}$ Ⓓ $\{t \mid t > 4\}$

Graphing Calculator   Use a graphing calculator to solve each inequality.

56. $3x + 7 > 4x + 9$ **57.** $13x - 11 \leq 7x + 37$ **58.** $2(x - 3) < 3(2x + 2)$

Maintain Your Skills

Mixed Review   **59. BUSINESS**   The charge per mile for a compact rental car at Great Deal Rentals is $0.12. Mrs. Ludlow must rent a car for a business trip. She has a budget of $50 for mileage charges. How many miles can she travel without going over her budget?   *(Lesson 6-2)*

Solve each inequality. Then check your solution, and graph it on a number line. *(Lesson 6-1)*

60. $d + 13 \geq 22$ **61.** $t - 5 < 3$ **62.** $4 > y + 7$

Write the point-slope form of an equation for a line that passes through each point with the given slope.   *(Lesson 5-5)*

63. $(1, -3), m = 2$ **64.** $(-2, -1), m = -\dfrac{2}{3}$ **65.** $(3, 6), m = 0$

Determine the slope of the line that passes through each pair of points.   *(Lesson 5-1)*

66. $(3, -1), (4, -6)$ **67.** $(-2, -4), (1, 3)$ **68.** $(0, 3), (-2, -5)$

Determine whether each equation is a linear equation. If an equation is linear, rewrite it in the form $Ax + By = C$.   *(Lesson 4-5)*

69. $4x = 7 + 2y$ **70.** $2x^2 - y = 7$ **71.** $x = 12$

Solve each equation. Then check your solution.   *(Lesson 3-5)*

72. $2(x - 2) = 3x - (4x - 5)$ **73.** $5t - 7 = t + 3$

Getting Ready for the Next Lesson   **PREREQUISITE SKILL**   Graph each set of numbers on a number line.
*(To review **graphing integers on a number line**, see Lesson 2-1.)*

74. $\{-2, 3, 5\}$ **75.** $\{-1, 0, 3, 4\}$ **76.** $\{-5, -4, -1, 1\}$

77. {integers less than 5}   **78.** {integers greater than -2}

79. {integers between 1 and 6}   **80.** {integers between -4 and 2}

81. {integers greater than or equal to -4}

82. {integers less than 6 but greater than -1}

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Reading Mathematics

Compound Statements

Two simple statements connected by the words *and* or *or* form a compound statement. Before you can determine whether a compound statement is true or false, you must understand what the words *and* and *or* mean. Consider the statement below.

A triangle has three sides, *and* a hexagon has five sides.
For a compound statement connected by the word *and* to be true, both simple statements must be true. In this case, it is true that a triangle has three sides. However, it is false that a hexagon has five sides; it has six. Thus, the compound statement is false.

A compound statement connected by the word *or* may be *exclusive* or *inclusive*. For example, the statement "With your dinner, you may have soup *or* salad," is exclusive. In everyday language, *or* means one or the other, but not both. However, in mathematics, *or* is inclusive. It means one or the other or both. Consider the statement below.

A triangle has three sides, *or* a hexagon has five sides.
For a compound statement connected by the word *or* to be true, at least one of the simple statements must be true. Since it is true that a triangle has three sides, the compound statement is true.

Triangle

Square

Pentagon

Hexagon

Octagon

Reading to Learn

Determine whether each compound statement is *true* or *false*. Explain your answer.

1. A hexagon has six sides, *or* an octagon has seven sides.

2. An octagon has eight sides, *and* a pentagon has six sides.

3. A pentagon has five sides, *and* a hexagon has six sides.

4. A triangle has four sides, *or* an octagon does *not* have seven sides.

5. A pentagon has three sides, *or* an octagon has ten sides.

6. A square has four sides, *or* a hexagon has six sides.

7. $5 < 4$ or $8 < 6$

8. $-1 > 0$ and $1 < 5$

9. $4 > 0$ and $-4 < 0$

10. $0 = 0$ or $-2 > -3$

11. $5 \neq 5$ or $-1 > -4$

12. $0 > 3$ and $2 > -2$

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Solving Compound Inequalities

What You'll Learn

- Solve compound inequalities containing the word *and* and graph their solution sets.
- Solve compound inequalities containing the word *or* and graph their solution sets.

How are compound inequalities used in tax tables?

Richard Kelley is completing his income tax return. He uses the table to determine the amount he owes in federal income tax.

2002 Tax Tables

If taxable income is—		Single	Married filing jointly	Married filing separately	Head of a household
At least	Less than				
41,000	41,050	7423	5554	7975	6083
41,050	41,100	7436	5561	7988	6096
41,100	41,150	7450	5569	8002	6110
41,150	41,200	7463	5576	8015	6123
41,200	41,250	7477	5584	8029	6137
41,250	41,300	7490	5591	8042	6150
41,300	41,350	7504	5599	8056	6164
41,350	41,400	7517	5606	8069	6177
41,400	41,450	7531	5614	8083	6191
41,450	41,500	7544	5621	8096	6204
41,500	41,550	7558	5629	8110	6218
41,550	41,600	7571	5636	8123	6231

Source: IRS

Let c represent the amount of Mr. Kelley's income. His income is at least $41,350 and it is less than $41,400. This can be written as $c \geq 41,350$ and $c < 41,400$. When considered together, these two inequalities form a **compound inequality**. This compound inequality can be written without using *and* in two ways.

$$41,350 \leq c < 41,400 \text{ or } 41,400 > c \geq 41,350$$

INEQUALITIES CONTAINING *AND*   A compound inequality containing *and* is true only if both inequalities are true. Thus, the graph of a compound inequality containing *and* is the **intersection** of the graphs of the two inequalities. In other words, the solution must be a solution of *both* inequalities.

The intersection can be found by graphing each inequality and then determining where the graphs overlap.

Example 1 Graph an Intersection

Graph the solution set of $x < 3$ and $x \geq -2$.

 Graph $x < 3$.
−5 −4 −3 −2 −1 0 1 2 3 4 5

Graph $x \geq -2$.
−5 −4 −3 −2 −1 0 1 2 3 4 5

Find the intersection.
−5 −4 −3 −2 −1 0 1 2 3 4 5

The solution set is $\{x \mid -2 \leq x < 3\}$. Note that the graph of $x \geq -2$ includes the point −2. The graph of $x < 3$ does *not* include 3.

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Solve $-5 < x - 4 < 2$. **Then graph the solution set.**

First express $-5 < x - 4 < 2$ using *and*. Then solve each inequality.

$$-5 < x - 4 \qquad \text{and} \qquad x - 4 < 2$$
$$-5 + 4 < x - 4 + 4 \qquad\qquad x - 4 + 4 < 2 + 4$$
$$-1 < x \qquad\qquad\qquad\qquad x < 6$$

The solution set is the intersection of the two graphs.

Graph $-1 < x$ or $x > -1$.

Graph $x < 6$.

Find the intersection.
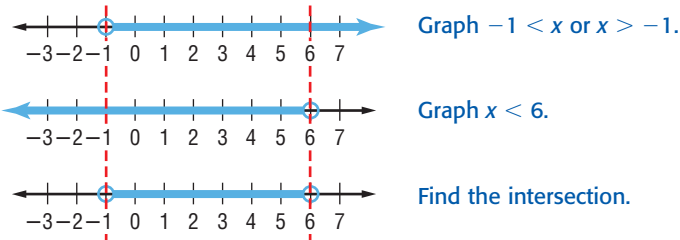
The solution set is $\{x \mid -1 < x < 6\}$.

INEQUALITIES CONTAINING *OR*   Another type of compound inequality contains the word *or*. A compound inequality containing *or* is true if one or more of the inequalities is true. The graph of a compound inequality containing *or* is the **union** of the graphs of the two inequalities. In other words, the solution of the compound inequality is a solution of *either* inequality, not necessarily both.

     The union can be found by graphing each inequality.

Example 3 Write and Graph a Compound Inequality

AVIATION   An airplane is experiencing heavy turbulence while flying at 30,000 feet. The control tower tells the pilot that he should increase his altitude to at least 33,000 feet or decrease his altitude to no more than 26,000 feet to avoid the turbulence. Write and graph a compound inequality that describes the altitude at which the airplane should fly.

Words   The pilot has been told to fly at an altitude of at least 33,000 feet or no more than 26,000 feet.

Variables   Let a be the plane's altitude.

The plane's altitude	is at least	33,000 feet	or	the altitude	is no more than	26,000 feet.
a	$\geq$	33,000	or	a	$\leq$	26,000

Inequality

Now, graph the solution set.
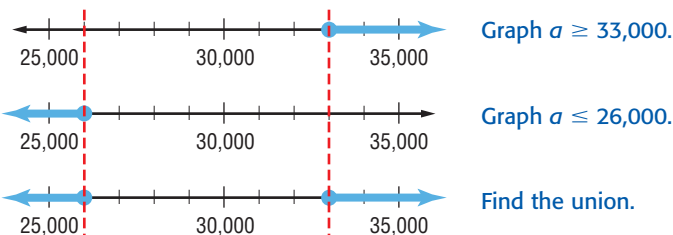
Graph $a \geq 33,000$.

Graph $a \leq 26,000$.

Find the union.

$a \geq 33,000$ or $a \leq 26,000$

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Example 4 **Solve and Graph a Union**

Solve $-3h + 4 < 19$ or $7h - 3 > 18$. Then graph the solution set.

$$-3h + 4 < 19 \qquad \text{or} \qquad 7h - 3 > 18$$
$$-3h + 4 - 4 < 19 - 4 \qquad 7h - 3 + 3 > 18 + 3$$
$$-3h < 15 \qquad\qquad 7h > 21$$
$$\frac{-3h}{-3} > \frac{15}{-3} \qquad\qquad \frac{7h}{7} > \frac{21}{7}$$
$$h > -5 \qquad\qquad h > 3$$

The solution set is the union of the two graphs.

Graph $h > -5$.

Graph $h > 3$.

Find the union.
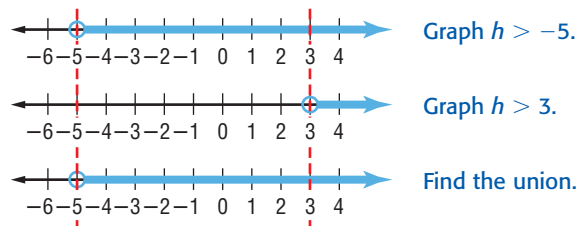
Notice that the graph of $h > -5$ contains every point in the graph of $h > 3$. So, the union is the graph of $h > -5$. The solution set is $\{h \mid h > -5\}$.

Check for Understanding

Concept Check

1. **Describe** the difference between a compound inequality containing *and* and a compound inequality containing *or*.

2. **Write** 7 *is less than* t, *which is less than* 12 as a compound inequality.

3. **OPEN ENDED** Give an example of a compound inequality containing *and* that has no solution.

Guided Practice

Graph the solution set of each compound inequality.

4. $a \leq 6$ and $a \geq -2$

5. $y > 12$ or $y < 9$

Write a compound inequality for each graph.

6.

7.
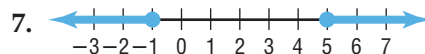
Solve each compound inequality. Then graph the solution set.

8. $6 < w + 3$ and $w + 3 < 11$

9. $n - 7 \leq -5$ or $n - 7 \geq 1$
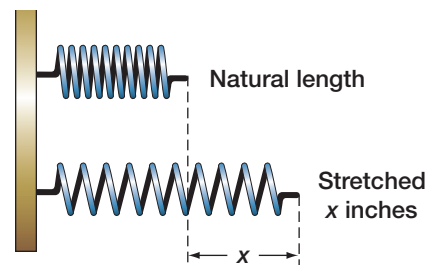
10. $3z + 1 < 13$ or $z \leq 1$

11. $-8 < x - 4 \leq -3$

12. Define a variable, write a compound inequality, and solve the following problem.
Three times a number minus 7 is less than 17 and greater than 5.

Application

13. **PHYSICAL SCIENCE** According to Hooke's Law, the force F in pounds required to stretch a certain spring x inches beyond its natural length is given by $F = 4.5x$. If forces between 20 and 30 pounds, inclusive, are applied to the spring, what will be the range of the increased lengths of the stretched spring?
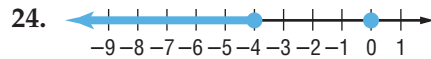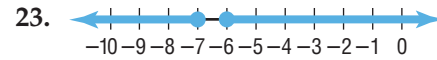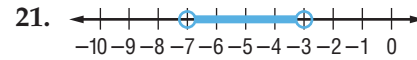

Natural length

Stretched
x inches

www.algebra1.com/extra_examples

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**Graph the solution set of each compound inequality.**

14. $x > 5$ and $x \le 9$ 15. $s < -7$ and $s \le 0$ 16. $r < 6$ or $r > 6$

17. $m \ge -4$ or $m > 6$ 18. $7 < d < 11$ 19. $-1 \le g < 3$

Write a compound inequality for each graph.

20.
```
+--+--+--+--•--+--+--+--+--•--+--+--+
-5 -4 -3 -2 -1  0  1  2  3  4  5
```

21.
```
+--+--+--○--+--+--+--○--+--+--+
-10 -9 -8 -7 -6 -5 -4 -3 -2 -1  0
```

22.
```
←--+--+--•--+--+--○--+--+--+--+→
   9 10 11 12 13 14 15 16 17 18 19
```

23.
```
←--+--+--+--+--•--+--+--+--+--+→
-10 -9 -8 -7 -6 -5 -4 -3 -2 -1  0
```

24.
```
←--+--+--+--•--+--+--•--+--+→
-9 -8 -7 -6 -5 -4 -3 -2 -1  0  1
```

25.
```
←--+--+--•--+--+--○--+--+--+→
-1  0  1  2  3  4  5  6  7  8  9
```
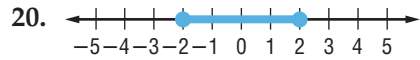
26. **WEATHER**   The Fujita Scale (F-scale) is the official classification system for tornado damage. One factor used to classify a tornado is wind speed. Use the information in the table to write an inequality for the range of wind speeds of an F3 tornado.
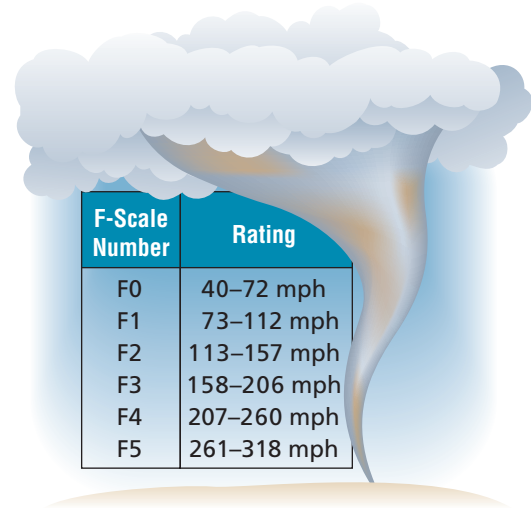
27. **BIOLOGY**   Each type of fish thrives in a specific range of temperatures. The optimum temperatures for sharks range from 18°C to 22°C, inclusive. Write an inequality to represent temperatures where sharks will *not* thrive.

F-Scale Number	Rating
F0	40–72 mph
F1	73–112 mph
F2	113–157 mph
F3	158–206 mph
F4	207–260 mph
F5	261–318 mph

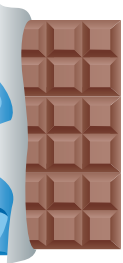**Solve each compound inequality. Then graph the solution set.**

28. $k + 2 > 12$ and $k + 2 \le 18$

29. $f + 8 \le 3$ and $f + 9 \ge -4$

30. $d - 4 > 3$ or $d - 4 \le 1$

31. $h - 10 < -21$ or $h + 3 < 2$

32. $3 < 2x - 3 < 15$

33. $4 < 2y - 2 < 10$

34. $3t - 7 \ge 5$ and $2t + 6 \le 12$

35. $8 > 5 - 3q$ and $5 - 3q > -13$

36. $-1 + x \le 3$ or $-x \le -4$

37. $3n + 11 \le 13$ or $-3n \ge -12$

38. $2p - 2 \le 4p - 8 \le 3p - 3$

39. $3g + 12 \le 6 + g \le 3g - 18$

40. $4c < 2c - 10$ or $-3c < -12$

41. $0.5b > -6$ or $3b + 16 < -8 + b$

Define a variable, write an inequality, and solve each problem.

42. Eight less than a number is no more than 14 and no less than 5.

43. The sum of 3 times a number and 4 is between -8 and 10.

44. The product of -5 and a number is greater than 35 or less than 10.

45. One half a number is greater than 0 and less than or equal to 1.

46. **HEALTH**   About 20% of the time you sleep is spent in rapid eye movement (REM) sleep, which is associated with dreaming. If an adult sleeps 7 to 8 hours, how much time is spent in REM sleep?

47. **SHOPPING**   A store is offering a $30 mail-in rebate on all color printers. Luisana is looking at different color printers that range in price from $175 to $260. How much can she expect to spend after the mail-in rebate?

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48. FUND-RAISING   Rashid is selling chocolates for his school's fund-raiser. He can earn prizes depending on how much he sells. So far, he has sold $70 worth of chocolates. How much more does he need to sell to earn a prize in category D?

Sales ($)	Prize
0–25	A
26–60	B
61–120	C
121–180	D
180+	E

49. CRITICAL THINKING   Write a compound inequality that represents the values of x which make the following expressions *false*.

 a. $x < 5$ or $x > 8$
 b. $x \leq 6$ and $x \geq 1$

HEARING   **For Exercises 50–52, use the following information.**
Humans hear sounds with sound waves within the 20 to 20,000 hertz range. Dogs hear sounds in the 15 to 50,000 hertz range.

50. Write a compound inequality for the hearing range of humans and one for the hearing range of dogs.

51. What is the union of the two solution sets? the intersection?

52. Write an inequality or inequalities for the range of sounds that dogs can hear, but humans cannot.

53. RESEARCH   Use the Internet or other resource to find the altitudes in miles of the layers of Earth's atmosphere, troposphere, stratosphere, mesosphere, thermosphere, and exosphere. Write inequalities for the range of altitudes for each layer.

54.   WRITING IN MATH   Answer the question that was posed at the beginning of the lesson.

How are compound inequalities used in tax tables?

Include the following in your answer:
- a description of the intervals used in the tax table shown at the beginning of the lesson, and
- a compound inequality describing the income of a head of a household paying $7024 in taxes.

Standardized Test Practice
Ⓐ Ⓑ Ⓒ Ⓓ

55. Ten pounds of fresh tomatoes make between 10 and 15 cups of cooked tomatoes. How many cups does one pound of tomatoes make?
 Ⓐ between 1 and $1\frac{1}{2}$ cups
 Ⓑ between 1 and 5 cups
 Ⓒ between 2 and 3 cups
 Ⓓ between 2 and 4 cups

56. Solve $-7 < x + 2 < 4$.
 Ⓐ $-5 < x < 6$
 Ⓑ $-9 < x < 2$
 Ⓒ $-5 < x < 2$
 Ⓓ $-9 < x < 6$

Graphing Calculator

57. SOLVE COMPOUND INEQUALITIES   In Lesson 6-3, you learned how to use a graphing calculator to find the values of x that make a given inequality true. You can also use this method to test compound inequalities. The words *and* and *or* can be found in the **LOGIC** submenu of the **TEST** menu of a TI-83 Plus. Use this method to solve each of the following compound inequalities using your graphing calculator.
 a. $x + 4 < -2$ or $x + 4 > 3$
 b. $x - 3 \leq 5$ and $x + 6 \geq 4$

Mixed Review

58. FUND-RAISING A university is running a drive to raise money. A corporation has promised to match 40% of whatever the university can raise from other sources. How much must the school raise from other sources to have a total of at least $800,000 after the corporation's donation? *(Lesson 6-3)*
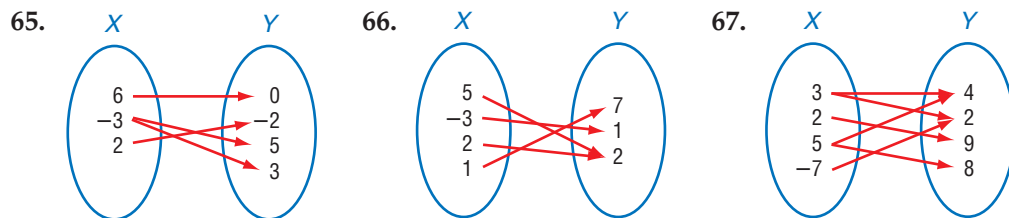
Solve each inequality. Then check your solution. *(Lesson 6-2)*

59. $18d \geq 90$ **60.** $-7v < 91$ **61.** $\frac{t}{13} < 13$ **62.** $-\frac{3}{8}b > 9$

Solve. Assume that y varies directly as x. *(Lesson 5-2)*

63. If $y = -8$ when $x = -3$, find x when $y = 6$.

64. If $y = 2.5$ when $x = 0.5$, find y when $x = 20$.

Express the relation shown in each mapping as a set of ordered pairs. Then state the domain, range, and inverse. *(Lesson 4-3)*

65.

X	Y
6	0
−3	−2
2	5
	3

66.

X	Y
5	7
−3	1
2	2
1	

67.

X	Y
3	4
2	2
5	9
−7	8

Find the odds of each outcome if a die is rolled. *(Lesson 2-6)*

68. a number greater than 2   **69.** not a 3

Find each product. *(Lesson 2-3)*

70. $-\frac{5}{6}\left(-\frac{2}{5}\right)$ **71.** $-100(4.7)$ **72.** $-\frac{7}{12}\left(\frac{6}{7}\right)\left(-\frac{3}{4}\right)$

Getting Ready for the Next Lesson

PREREQUISITE SKILL Find each value. *(To review **absolute value**, see Lesson 2-1.)*

73. $|-7|$ **74.** $|10|$ **75.** $|-1|$ **76.** $|-3.5|$

77. $|12 - 6|$ **78.** $|5 - 9|$ **79.** $|20 - 21|$ **80.** $|3 - 18|$

Practice Quiz 2   Lessons 6-3 and 6-4

Solve each inequality. Then check your solution. *(Lesson 6-3)*

1. $5 - 4b > -23$ **2.** $\frac{1}{2}n + 3 \geq -5$

3. $3(t + 6) < 9$ **4.** $9x + 2 > 20$

5. $2m + 5 \leq 4m - 1$ **6.** $a < \frac{2a - 15}{3}$

Solve each compound inequality. Then graph the solution set. *(Lesson 6-4)*

7. $x - 2 < 7$ and $x + 2 > 5$ **8.** $2b + 5 \leq -1$ or $b - 4 \geq -4$

9. $4m - 5 > 7$ or $4m - 5 < -9$ **10.** $a - 4 < 1$ and $a + 2 > 1$

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Solving Open Sentences Involving Absolute Value

What You'll Learn

- Solve absolute value equations.
- Solve absolute value inequalities.

How is absolute value used in election polls?

Voters in Hamilton will vote on a new tax levy in the next election. A poll conducted before the election found that 47% of the voters surveyed were for the tax levy, 45% were against the tax levy, and 8% were undecided. The poll has a 3-point margin of error.

Tax Levy Poll
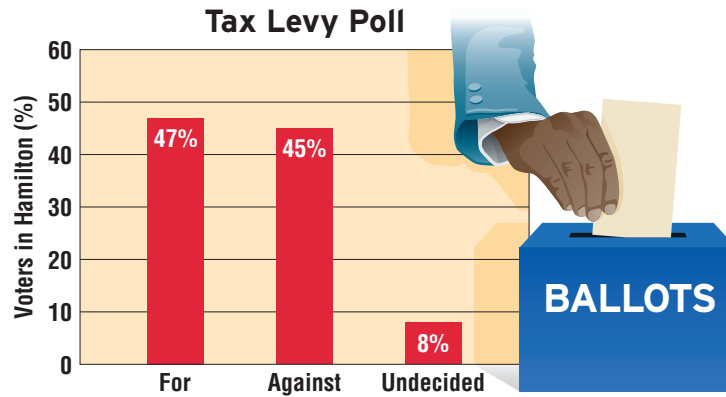
The margin of error means that the result may be 3 percentage points higher or lower. So, the number of people in favor of the tax levy may be as high as 50% or as low as 44%. This can be written as an inequality using absolute value.

$$|x - 47| \le 3 \quad \text{The difference between the actual number and 47 is within 3 points.}$$

ABSOLUTE VALUE EQUATIONS There are three types of open sentences that can involve absolute value.

$$|x| = n \qquad |x| < n \qquad |x| > n$$

Consider the case of $|x| = n$. $|x| = 5$ means the distance between 0 and x is 5 units.
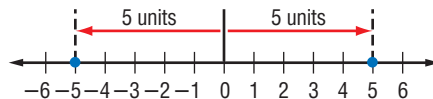
If $|x| = 5$, then $x = -5$ or $x = 5$. The solution set is $\{-5, 5\}$.

When solving equations that involve absolute value, there are two cases to consider.

Case 1 The value inside the absolute value symbols is positive.

Case 2 The value inside the absolute value symbols is negative.

Equations involving absolute value can be solved by graphing them on a number line or by writing them as a compound sentence and solving it.

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Example 1 Solve an Absolute Value Equation

Solve $|a - 4| = 3$.

Method 1 Graphing

$|a - 4| = 3$ means that the distance between a and 4 is 3 units. To find a on the number line, start at 4 and move 3 units in either direction.
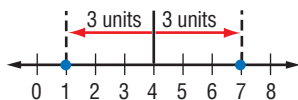
The distance from 4 to 1 is 3 units.
The distance from 4 to 7 is 3 units.

The solution set is {1, 7}.

Method 2 Compound Sentence

Write $|a - 4| = 3$ as $a - 4 = 3$ or $a - 4 = -3$.

Case 1

$$a - 4 = 3$$
$$a - 4 + 4 = 3 + 4 \quad \text{Add 4 to each side.}$$
$$a = 7 \quad \text{Simplify.}$$

Case 2

$$a - 4 = -3$$
$$a - 4 + 4 = -3 + 4 \quad \text{Add 4 to each side.}$$
$$a = 1 \quad \text{Simplify.}$$

The solution set is {1, 7}.

Example 2 Write an Absolute Value Equation

Write an equation involving absolute value for the graph.
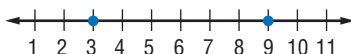
Find the point that is the same distance from 3 as the distance from 9.
The midpoint between 3 and 9 is 6.
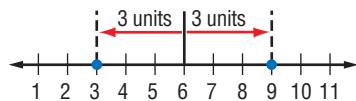
The distance from 6 to 3 is 3 units.
The distance from 6 to 9 is 3 units.

So, an equation is $|x - 6| = 3$.

CHECK Substitute 3 and 9 into $|x - 6| = 3$.
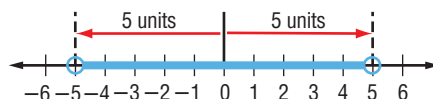
$$|x - 6| = 3 \qquad\qquad |x - 6| = 3$$
$$|3 - 6| \stackrel{?}{=} 3 \qquad\qquad |9 - 6| \stackrel{?}{=} 3$$
$$|-3| \stackrel{?}{=} 3 \qquad\qquad\quad |3| \stackrel{?}{=} 3$$
$$3 = 3 \checkmark \qquad\qquad\quad 3 = 3 \checkmark$$

ABSOLUTE VALUE INEQUALITIES Consider the inequality $|x| < n$.
$|x| < 5$ means that the distance from 0 to x is less than 5 units.

5 units   5 units

Therefore, $x > -5$ and $x < 5$. The solution set is $\{x \mid -5 < x < 5\}$.

CONTENTS

The Algebra Activity explores an inequality of the form $|x| < n$.

Algebra Activity

Absolute Value

Collect the Data
- Work in pairs. One person is the timekeeper.
- Start timing. The other person tells the timekeeper to stop timing after he or she thinks that one minute has elapsed.
- Write down the time in seconds.
- Switch places. Make a table that includes the results of the entire class.

Analyze the Data
1. Determine the error by subtracting 60 seconds from each student's time.
2. What does a negative error represent? a positive error?
3. The *absolute error* is the absolute value of the error. Since absolute value cannot be negative, the absolute error is positive. If the absolute error is 6 seconds, write two possibilities for a student's estimated time of one minute.
4. What estimates would have an absolute error less than 6 seconds?
5. Graph the responses and highlight all values such that $|60 - x| < 6$. How many guesses were within 6 seconds?

When solving inequalities of the form $|x| < n$, find the intersection of these two cases.

Case 1 The value inside the absolute value symbols is less than the positive value of n.

Case 2 The value inside the absolute value symbols is greater than the negative value of n.

Less Than
When an absolute value is on the left and the inequality symbol is $<$ or $\le$, the compound sentence uses *and*.

Example 3 Solve an Absolute Value Inequality ($<$)

Solve $|t + 5| < 9$. Then graph the solution set.

Write $|t + 5| < 9$ as $t + 5 < 9$ and $t + 5 > -9$.

Case 1
$$t + 5 < 9$$
$$t + 5 - 5 < 9 - 5 \qquad \text{Subtract 5 from each side.}$$
$$t < 4 \qquad \text{Simplify.}$$

Case 2
$$t + 5 > -9$$
$$t + 5 - 5 > -9 - 5 \qquad \text{Subtract 5 from each side.}$$
$$t > -14 \qquad \text{Simplify.}$$

The solution set is $\{t \mid -14 < t < 4\}$.

Consider the inequality $|x| > n$. $|x| > 5$ means that the distance from 0 to x is greater than 5 units.

Therefore, $x < -5$ or $x > 5$. The solution set is $\{x \mid x < -5 \text{ or } x > 5\}$.

CONTENTS

When solving inequalities of the form $|x| > n$, find the union of these two cases.

Case 1 The value inside the absolute value symbols is greater than the positive value of n.

Case 2 The value inside the absolute value symbols is less than the negative value of n.

Example 4 Solve an Absolute Value Inequality ($>$)

Solve $|2x + 8| \geq 6$. Then graph the solution set.

Write $|2x + 8| \geq 6$ as $2x + 8 \geq 6$ or $2x + 8 \leq -6$.

Case 1

$$2x + 8 \geq 6$$

$2x + 8 - 8 \geq 6 - 8$ Subtract 8 from each side.

$\qquad\qquad 2x \geq -2$ Simplify.

$\qquad\qquad \dfrac{2x}{2} \geq \dfrac{-2}{2}$ Divide each side by 2.

$\qquad\qquad x \geq -1$ Simplify.

Case 2

$$2x + 8 \leq -6$$

$2x + 8 - 8 \leq -6 - 8$ Subtract 8 from each side.

$\qquad\qquad 2x \leq -14$ Simplify.

$\qquad\qquad \dfrac{2x}{2} \leq \dfrac{-14}{2}$ Divide each side by 2.

$\qquad\qquad x \leq -7$ Simplify.

The solution set is $\{x \mid x \leq -7 \text{ or } x \geq -1\}$.
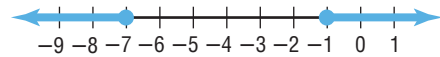
In general, there are three rules to remember when solving equations and inequalities involving absolute value.

Concept Summary   Absolute Value Equations and Inequalities

If $|x| = n$, then $x = -n$ or $x = n$.

If $|x| < n$, then $x < n$ and $x > -n$.

If $|x| > n$, then $x > n$ or $x < -n$.

These properties are also true when $>$ or $<$ is replaced with $\geq$ or $\leq$.

Check for Understanding

Concept Check
1. **Compare and contrast** the solution of $|x - 2| > 6$ and the solution of $|x - 2| < 6$.

2. **OPEN ENDED** Write an absolute value inequality and graph its solution set.

3. **FIND THE ERROR** Leslie and Holly are solving $|x + 3| = 2$.

Leslie		Holly	
$x + 3 = 2$ or $x + 3 = -2$		$x + 3 = 2$ or $x - 3 = 2$	
$x + 3 - 3 = 2 - 3$ $x + 3 - 3 = -2 - 3$		$x + 3 - 3 = 2 - 3$ $x - 3 + 3 = 2 + 3$	
$x = -1$ $x = -5$		$x = -1$ $x = 5$	

Who is correct? Explain your reasoning.

CONTENTS

4. Which graph represents the solution of $|k| \leq 3$?

a.
$$\begin{array}{c} \text{graph from } -5 \text{ to } 5 \end{array}$$
-5 -4 -3 -2 -1 0 1 2 3 4 5

b.
-5 -4 -3 -2 -1 0 1 2 3 4 5

c.
-5 -4 -3 -2 -1 0 1 2 3 4 5

d.
-5 -4 -3 -2 -1 0 1 2 3 4 5

5. Which graph represents the solution of $|x - 4| > 2$?

a.
-3 -2 -1 0 1 2 3 4 5 6 7

b.
-3 -2 -1 0 1 2 3 4 5 6 7

c.
-3 -2 -1 0 1 2 3 4 5 6 7

d.
-3 -2 -1 0 1 2 3 4 5 6 7
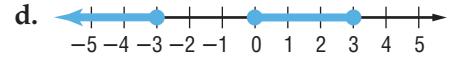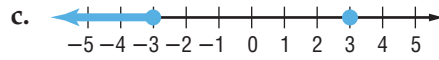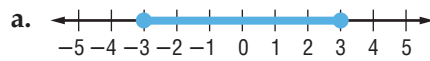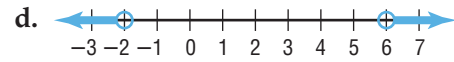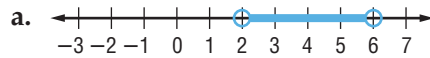
6. Express the statement in terms of an inequality involving absolute value. Do not solve.
A jar contains 832 gumballs. Amanda's guess was within 46 pieces.

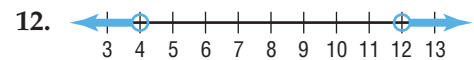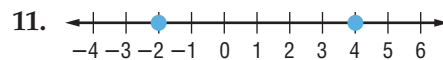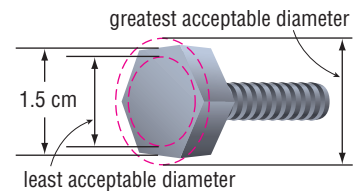**Solve each open sentence. Then graph the solution set.**

7. $|r + 3| = 10$

8. $|c - 2| < 6$

9. $|10 - w| > 15$

10. $|2g + 5| \geq 7$

For each graph, write an open sentence involving absolute value.

11.
-4 -3 -2 -1 0 1 2 3 4 5 6

12.
3 4 5 6 7 8 9 10 11 12 13

13. MANUFACTURING A manufacturer produces bolts which must have a diameter within 0.001 centimeter of 1.5 centimeters. What are the acceptable measurements for the diameter of the bolts?
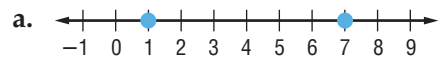
greatest acceptable diameter

1.5 cm

least acceptable diameter

Practice and Apply

Match each open sentence with the graph of its solution set.

14. $|x + 5| \leq 3$

a.
-1 0 1 2 3 4 5 6 7 8 9

15. $|x - 4| > 4$

b.
-5 -4 -3 -2 -1 0 1 2 3 4 5

16. $|2x - 8| = 6$

c.
-9 -8 -7 -6 -5 -4 -3 -2 -1 0 1

17. $|x + 3| \geq -1$

d.
2 3 4 5 6 7 8 9 10 11 12

18. $|x| < 2$

e.
-5 -4 -3 -2 -1 0 1 2 3 4 5

19. $|8 - x| = 2$

f.
-1 0 1 2 3 4 5 6 7 8 9
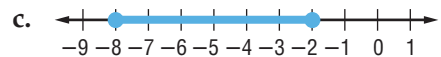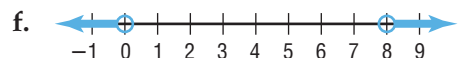
Express each statement using an inequality involving absolute value. Do *not* solve.

20. The pH of a buffered eye solution must be within 0.002 of a pH of 7.3.

21. The temperature inside a refrigerator should be within 1.5 degrees of 38°F.

22. Ramona's bowling score was within 6 points of her average score of 98.

23. The cruise control of a car set at 55 miles per hour should keep the speed within 3 miles per hour of 55.

Solve each open sentence. Then graph the solution set.

24. $|x - 5| = 8$ **25.** $|b + 9| = 2$

26. $|2p - 3| = 17$ **27.** $|5c - 8| = 12$

28. $|z - 2| \leq 5$ **29.** $|t + 8| < 2$

30. $|v + 3| > 1$ **31.** $|w - 6| \geq 3$

32. $|3s + 2| > -7$ **33.** $|3k + 4| \geq 8$

34. $|2n + 1| < 9$ **35.** $|6r + 8| < -4$

36. $|6 - (3d - 5)| \leq 14$ **37.** $|8 - (w - 1)| \leq 9$

38. $\left|\dfrac{5h + 2}{6}\right| = 7$ **39.** $\left|\dfrac{2 - 3x}{5}\right| \geq 2$

For each graph, write an open sentence involving absolute value.

40.

−5 −4 −3 −2 −1 0 1 2 3 4 5

41.

−2 −1 0 1 2 3 4 5 6 7 8

42.

−5 −4 −3 −2 −1 0 1 2 3 4 5

43.

−8 −7 −6 −5 −4 −3 −2 −1 0 1 2

44.

−5 −4 −3 −2 −1 0 1 2 3 4 5

45.

−15 −14 −13 −12 −11 −10 −9 −8 −7 −6 −5
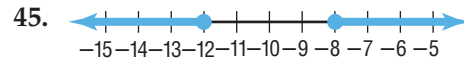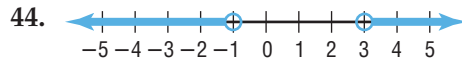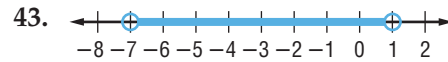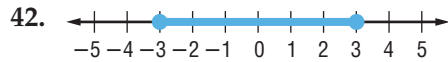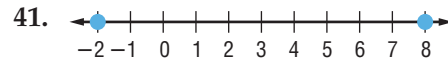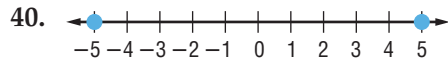
HEALTH For Exercises 46 and 47, use the following information.
The *average* length of a human pregnancy is 280 days. However, a healthy, full-term pregnancy can be 14 days longer or shorter.

46. Write an absolute value inequality for the length of a full-term pregnancy.

47. Solve the inequality for the length of a full-term pregnancy.

48. FIRE SAFETY The pressure of a typical fire extinguisher should be within 25 pounds per square inch (psi) of 195 psi. Write the range of pressures for safe fire extinguishers.

49. HEATING A thermostat with a 2-degree differential will keep the temperature within 2 degrees Fahrenheit of the temperature set point. Suppose your home has a thermostat with a 3-degree differential. If you set the thermostat at 68°F, what is the range of temperatures in the house?

50. ENERGY Use the margin of error indicated in the graph at the right to find the range of the percent of people who say protection of the environment should have priority over developing energy supplies.

51. TIRE PRESSURE Tire pressure is measured in pounds per square inch (psi). Tires should be kept within 2 psi of the manufacturer's recommended tire pressure. If the recommended inflation pressure for a tire is 30 psi, what is the range of acceptable pressures?

52. CRITICAL THINKING State whether each open sentence is *always*, *sometimes*, or *never* true.
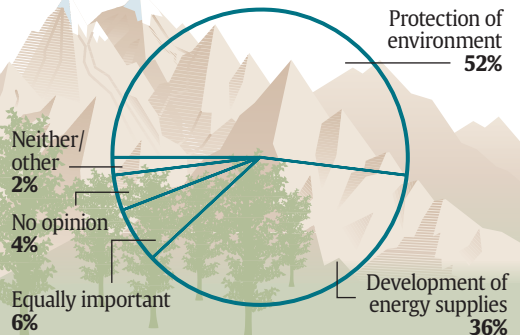
 a. $|x + 3| < -5$

 b. $|x - 6| > -1$

 c. $|x + 2| = 0$

USA TODAY Snapshots®

Environment first

Americans say protecting the environment should be given priority over developing U.S. energy supplies. Preferences:

Protection of environment **52%**

Neither/other **2%**

No opinion **4%**

Equally important **6%**

Development of energy supplies **36%**

Source: Gallup Poll of 1,060 adults; March 5-7, 2001.
Margin of error: plus or minus 3 percentage points.

By Marcy E. Mullins, USA TODAY

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53. PHYSICAL SCIENCE Li-Cheng must add 3.0 milliliters of sodium chloride to a solution. The sodium chloride must be within 0.5 milliliter of the required amount. How much sodium chloride can she add and obtain the correct results?

54. ENTERTAINMENT Luis Gomez is a contestant on a television game show. He must guess within $1500 of the actual price of a car without going over to win the car. The actual price of the car is $18,000. What is the range of guesses in which Luis can win the vehicle?

55. CRITICAL THINKING The symbol $\pm$ means *plus* or *minus*.
  a. If $x = 3 \pm 1.2$, what are the values of x?
  b. Write $x = 3 \pm 1.2$ as an expression involving absolute value.

56. `WRITING IN MATH` Answer the question that was posed at the beginning of the lesson.

How is absolute value used in election polls?

Include the following in your answer:
- an explanation of how to solve the inequality describing the percent of people who are against the tax levy, and
- a prediction of whether you think the tax levy will pass and why.

57. Choose the replacement set that makes $|x + 5| = 2$ true.
  Ⓐ $\{-3, 3\}$ Ⓑ $\{-3, -7\}$ Ⓒ $\{2, -2\}$ Ⓓ $\{3, -7\}$

58. What can you conclude about x if $-6 < |x| < 6$?
  Ⓐ $-x \geq 0$ Ⓑ $x \leq 0$ Ⓒ $-x < 6$ Ⓓ $-x > 6$

Maintain Your Skills

Mixed Review

59. FITNESS To achieve the maximum benefits from aerobic activity, your heart rate should be in your target zone. Your target zone is the range between 60% and 80% of your maximum heart rate. If Rafael's maximum heart rate is 190 beats per minute, what is his target zone? *(Lesson 6-4)*

Solve each inequality. Then check your solution. *(Lesson 6-3)*
60. $2m + 7 > 17$ **61.** $-2 - 3x \geq 2$ **62.** $\frac{2}{3}w - 3 \leq 7$

Find the slope and y-intercept of each equation. *(Lesson 5-4)*
63. $2x + y = 4$ **64.** $2y - 3x = 4$ **65.** $\frac{1}{2}x + \frac{3}{4}y = 0$

Solve each equation or formula for the variable specified. *(Lesson 3-8)*
66. $I = prt$, for r **67.** $ex - 2y = 3z$, for x **68.** $\frac{a + 5}{3} = 7x$, for x

Find each sum or difference. *(Lesson 2-2)*
69. $-13 + 8$ **70.** $-13.2 - 6.1$ **71.** $-4.7 - (-8.9)$

Name the property illustrated by each statement. *(Lesson 1-6)*
72. $10x + 10y = 10(x + y)$ **73.** $(2 + 3)a + 7 = 5a + 7$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Graph each equation.**
*(To review **graphing linear equations**, see Lesson 4-5.)*
74. $y = 3x + 4$ **75.** $y = -2$ **76.** $x + y = 3$
77. $y - 2x = -1$ **78.** $2y - x = -6$ **79.** $2(x + y) = 10$

CONTENTS

Graphing Inequalities in Two Variables

• Graph inequalities on the coordinate plane.

• Solve real-world problems involving linear inequalities.

Vocabulary
• half-plane
• boundary

How are inequalities used in budgets?

Hannah budgets $30 a month for lunch. On most days, she brings her lunch. She can also buy lunch at the cafeteria or at a fast-food restaurant. She spends an average of $3 for lunch at the cafeteria and an average of $4 for lunch at a restaurant. How many times a month can Hannah buy her lunch and remain within her budget?

My Monthly Budget

Lunch (school days)	$30
Entertainment	$55
Clothes	$50
Fuel	$60

Let x represent the number of days she buys lunch at the cafeteria, and let y represent the number of days she buys lunch at a restaurant. Then the following inequality can be used to represent the situation.

| The cost of eating in the cafeteria	plus	the cost of eating in a restaurant	is less than or equal to	$30.
$3x$	$+$	$4y$	$\leq$	30

There are many solutions of this inequality.

GRAPH LINEAR INEQUALITIES The solution set of an inequality in two variables is the set of all ordered pairs that satisfy the inequality. Like a linear equation in two variables, the solution set is graphed on a coordinate plane.

Example 1 Ordered Pairs that Satisfy an Inequality

From the set {(1, 6), (3, 0), (2, 2), (4, 3)}, which ordered pairs are part of the solution set for $3x + 2y < 12$?

Use a table to substitute the x and y values of each ordered pair into the inequality.

x	y	$3x + 2y < 12$	True or False
1	6	$3(1) + 2(6) < 12$ $15 < 12$	false
3	0	$3(3) + 2(0) < 12$ $9 < 12$	true
2	2	$3(2) + 2(2) < 12$ $10 < 12$	true
4	3	$3(4) + 2(3) < 12$ $18 < 12$	false
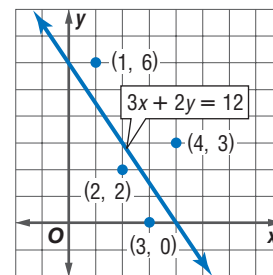
The ordered pairs {(3, 0), (2, 2)} are part of the solution set of $3x + 2y < 12$. In the graph, notice the location of the two ordered pairs that are solutions for $3x + 2y < 12$ in relation to the line.

The solution set for an inequality in two variables contains many ordered pairs when the domain and range are the set of real numbers. The graphs of all of these ordered pairs fill a region on the coordinate plane called a **half-plane**. An equation defines the **boundary** or edge for each half-plane.

Key Concept   *Half-Planes and Boundaries*

- **Words**   Any line in the plane divides the plane into two regions called half-planes. The line is called the boundary of each of the two half-planes.

- **Model**
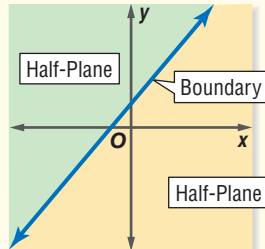
Study Tip

*Dashed Line*
- Like a circle on a number line, a dashed line on a coordinate plane indicates that the boundary is *not* part of the solution set.

Solid Line
- Like a dot on a number line, a solid line on a coordinate plane indicates that the boundary *is* included.

Consider the graph of $y > 4$. First determine the boundary by graphing $y = 4$, the equation you obtain by replacing the inequality sign with an equals sign. Since the inequality involves y-values greater than 4, but not equal to 4, the line should be dashed. The boundary divides the coordinate plane into two half-planes.
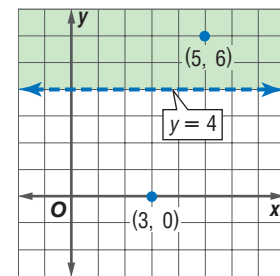
To determine which half-plane contains the solution, choose a point from each half-plane and test it in the inequality.

Try (3, 0).   Try (5, 6).

$y > 4$ $y = 0$ $y > 4$ $y = 6$

$0 > 4$ false $6 > 4$ true

The half-plane that contains (5, 6) contains the solution. Shade that half-plane.

Example 2 *Graph an Inequality*

Graph $y - 2x \leq -4$.

Step 1   Solve for y in terms of x.

$$y - 2x \leq -4 \qquad \text{Original inequality}$$

$$y - 2x + 2x \leq -4 + 2x \quad \text{Add 2x to each side.}$$

$$y \leq 2x - 4 \qquad \text{Simplify.}$$

Step 2   Graph $y = 2x - 4$. Since $y \leq 2x - 4$ means $y < 2x - 4$ or $y = 2x - 4$, the boundary is included in the solution set. The boundary should be drawn as a solid line.

(continued on the next page)

Origin as the Test Point
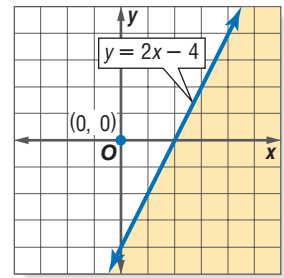Use the origin as a standard test point because the values are easy to substitute into the inequality.

Step 3 Select a point in one of the half-planes and test it. Let's use $(0, 0)$.

$y - 2x \leq -4$ Original inequality

$0 \leq 2(0) - 4$ $x = 0, y = 0$

$0 \leq -4$ false

Since the statement is false, the half-plane containing the origin is not part of the solution. Shade the other half-plane.

CHECK Test a point in the other half plane, for example, $(3, -3)$.

$y - 2x \leq -4$ Original inequality

$-3 \leq 2(3) - 4$ $x = 3, y = -3$

$-3 \leq 2$ ✓

Since the statement is true, the half-plane containing $(3, -3)$ should be shaded. The graph of the solution is correct.

SOLVE REAL-WORLD PROBLEMS When solving real-world inequalities, the domain and range of the inequality are often restricted to nonnegative numbers or whole numbers.

Example 3 **Write and Solve an Inequality**

ADVERTISING Rosa Padilla sells radio advertising in 30-second and 60-second time slots. During every hour, there are up to 15 minutes available for commercials. How many commercial slots can she sell for one hour of broadcasting?

Step 1 Let x equal the number of 30-second commercials. Let y equal the number of 60-second or 1-minute commercials. Write an open sentence representing this situation.

| $\frac{1}{2}$ min	times	the number of 30-s commercials	plus	the number of 1-min commercials	is up to	15 min.
$\frac{1}{2}$	$\cdot$	x	$+$	y	$\leq$	15

Step 2 Solve for y in terms of x.

$\frac{1}{2}x + y \leq 15$ Original inequality

$\frac{1}{2}x + y - \frac{1}{2}x \leq 15 - \frac{1}{2}x$ Subtract $\frac{1}{2}x$ from each side.

$y \leq 15 - \frac{1}{2}x$ Simplify.

Step 3 Since the open sentence includes the equation, graph $y = 15 - \frac{1}{2}x$ as a solid line. Test a point in one of the half-planes, for example $(0, 0)$. Shade the half-plane containing $(0, 0)$ since $0 \leq 15 - \frac{1}{2}(0)$ is true.
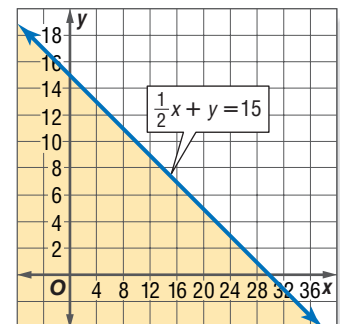

Advertising

A typical one-hour program on television contains 40 minutes of the program and 20 minutes of commercials. During peak periods, a 30-second commercial can cost an average of $2.3 million.

Source: www.superbowl-ads.com

CONTENTS

Step 4 Examine the solution.

- Rosa cannot sell a negative number of commercials. Therefore, the domain and range contain only nonnegative numbers.

- She also cannot sell half of a commercial. Thus, only points in the shaded half-plane whose x- and y-coordinates are whole numbers are possible solutions.
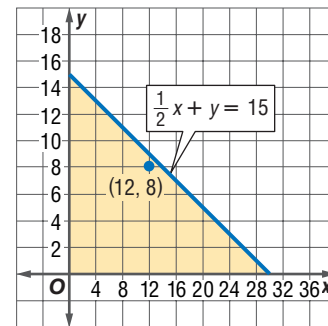
One solution is (12, 8). This represents twelve 30-second commercials and eight 60-second commercials in a one hour period.

Check for Understanding

Concept Check
1. **Compare and contrast** the graph of $y = x + 2$ and the graph of $y < x + 2$.

2. **OPEN ENDED** Write an inequality in two variables and graph it.

3. **Explain** why it is usually only necessary to test one point when graphing an inequality.
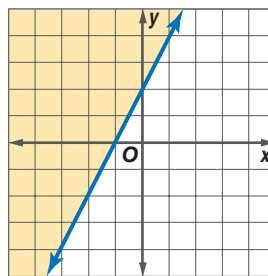
Guided Practice
Determine which ordered pairs are part of the solution set for each inequality.

4. $y \le x + 1$, $\{(-1, 0), (3, 2), (2, 5), (-2, 1)\}$

5. $y > 2x$, $\{(2, 6), (0, -1), (3, 5), (-1, -2)\}$
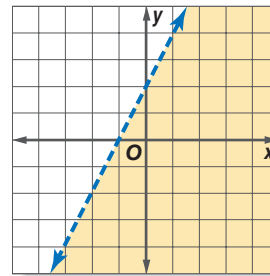
6. Which graph represents $y - 2x \ge 2$?
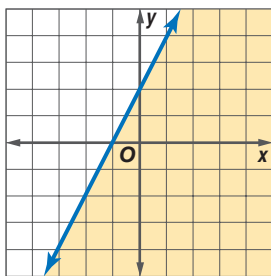
a.   b.   c.

Graph each inequality.

7. $y \ge 4$ 8. $y \le 2x - 3$

9. $4 - 2x < -2$ 10. $1 - y > x$

Application
11. **ENTERTAINMENT** Coach Riley wants to take her softball team out for pizza and soft drinks after the last game of the season. She doesn't want to spend more than $60. Write an inequality that represents this situation and graph the solution set.

Welcome to Angelo's Pizza!

Large Pizza $12

Pitcher of soft drink $3

CONTENTS
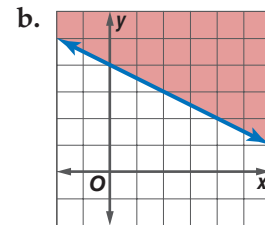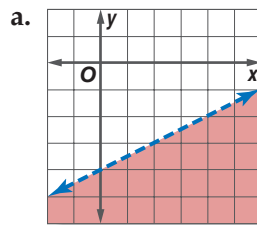
Determine which ordered pairs are part of the solution set for each inequality.

12. $y \le 3 - 2x$, $\{(0, 4), (-1, 3), (6, -8), (-4, 5)\}$

13. $y < 3x$, $\{(-3, 1), (-3, 2), (1, 1), (1, 2)\}$

14. $x + y < 11$, $\{(5, 7), (-13, 10), (4, 4), (-6, -2)\}$

15. $2x - 3y > 6$, $\{(3, 2), (-2, -4), (6, 2), (5, 1)\}$

16. $4y - 8 \ge 0$, $\{(5, -1), (0, 2), (2, 5), (-2, 0)\}$

17. $3x + 4y < 7$, $\{(1, 1), (2, -1), (-1, 1), (-2, 4)\}$

18. $|x - 3| \ge y$, $\{(6, 4), (-1, 8), (-3, 2), (5, 7)\}$

19. $|y + 2| < x$, $\{(2, -4), (-1, -5), (6, -7), (0, 0)\}$
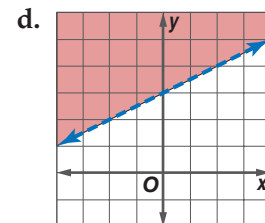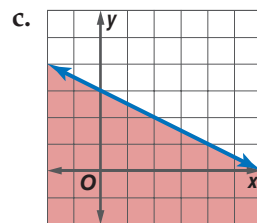
Match each inequality with its graph.

20. $2y + x \le 6$

21. $\frac{1}{2}x - y > 4$

22. $y > 3 + \frac{1}{2}x$

23. $4y + 2x \ge 16$

a.

b.

c.

d.

24. Is the point $A(2, 3)$ on, above, or below the graph of $-2x + 3y = 5$?

25. Is the point $B(0, 1)$ on, above, or below the graph of $4x - 3y = 4$?

Graph each inequality.

26. $y < -3$

27. $x \ge 2$

28. $5x + 10y > 0$

29. $y < x$

30. $2y - x \le 6$

31. $6x + 3y > 9$

32. $3y - 4x \ge 12$

33. $y \le -2x - 4$

34. $8x - 6y < 10$

35. $3x - 1 \ge y$

36. $3(x + 2y) > -18$

37. $\frac{1}{2}(2x + y) < 2$

POSTAGE For Exercises 38 and 39, use the following information.
The U.S. Postal Service limits the size of packages to those in which the length of the longest side plus the distance around the thickest part is less than or equal to 108 inches.

38. Write an inequality that represents this situation.

39. Are there any restrictions on the domain or range?

Online Research **Data Update** What are the current postage rates and regulations? Visit www.algebra1.com/data_update to learn more.

SHIPPING For Exercises 40 and 41, use the following information.
A delivery truck is transporting televisions and microwaves to an appliance store. The weight limit for the truck is 4000 pounds. The televisions weigh 77 pounds, and the microwaves weigh 55 pounds.

40. Write an inequality for this situation.

41. Will the truck be able to deliver 35 televisions and 25 microwaves at once?

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FALL DANCE   For Exercises 42–44, use the following information.
Tickets for the fall dance are $5 per person or $8 for couples. In order to cover expenses, at least $1200 worth of tickets must be sold.

42. Write an inequality that represents this situation.

43. Graph the inequality.

44. If 100 single tickets and 125 couple tickets are sold, will the committee cover its expenses?

45. CRITICAL THINKING   Graph the intersection of the graphs of $y \le x - 1$ and $y \ge -x$.

46. WRITING IN MATH   Answer the question that was posed at the beginning of the lesson.

How are inequalities used in budgets?

Include the following in your answer:

- an explanation of the restrictions placed on the domain and range of the inequality used to describe the number of times Hannah can buy her lunch, and
- three possible solutions of the inequality.

Standardized Test Practice
Ⓐ Ⓑ Ⓒ Ⓓ

47. Which ordered pair is *not* a solution of $y - 2x < -5$?
  Ⓐ $(2, -2)$ Ⓑ $(-1, -8)$ Ⓒ $(4, 1)$ Ⓓ $(5, 6)$

48. Which inequality is represented by the graph at the right?
  Ⓐ $2x + y < 1$ Ⓑ $2x + y > 1$
  Ⓒ $2x + y \le 1$ Ⓓ $2x + y \ge 1$
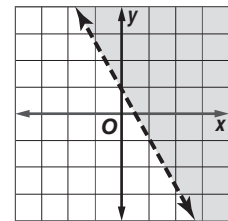
Maintain Your Skills

Mixed Review   **Solve each open sentence. Then graph the solution set.**   *(Lesson 6-5)*

49. $|3 + 2t| = 11$ **50.** $|x + 8| < 6$ **51.** $|2y + 5| \ge 3$

Solve each compound inequality. Then graph the solution.   *(Lesson 6-4)*

52. $y + 6 > -1$ and $y - 2 < 4$ **53.** $m + 4 < 2$ or $m - 2 > 1$

State whether each percent of change is a percent of *increase* or *decrease*. Then find the percent of change. Round to the nearest whole percent.   *(Lesson 3-7)*

54. original: 200   **55.** original: 100   **56.** original: 53
     new: 172   new: 142   new: 75

Solve each equation.   *(Lesson 3-4)*

57. $\dfrac{d - 2}{3} = 7$ **58.** $3n + 6 = -15$ **59.** $35 + 20h = 100$

Simplify.   *(Lesson 2-4)*

60. $\dfrac{-64}{4}$ **61.** $\dfrac{27c}{-9}$ **62.** $\dfrac{12a - 14b}{-2}$ **63.** $\dfrac{18y - 9}{3}$

Graphing Inequalities

You can use a TI-83 Plus graphing calculator to investigate the graphs of inequalities. Since graphing calculators only shade between two functions, enter a lower boundary as well as an upper boundary for each inequality.

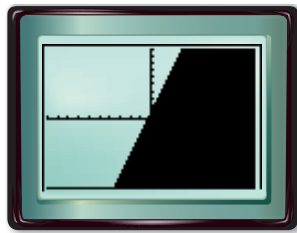**Graph two different inequalities on your graphing calculator.**

Step 1 *Graph $y \leq 3x + 1$.*

- Clear all functions from the Y= list.

  KEYSTROKES: Y= CLEAR

- Graph $y \leq 3x + 1$ in the standard window.

  KEYSTROKES: 2nd [DRAW] 7 (–) 10 , 3 X,T,θ,n + 1) ENTER

The lower boundary is Ymin or -10. The upper boundary is $y = 3x + 1$. All ordered pairs for which y is *less than or equal to* $3x + 1$ lie *below or on* the line and are solutions.
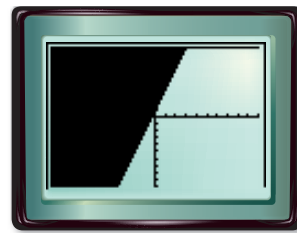
Step 2 *Graph $y - 3x \geq 1$.*

- Clear the drawing that is currently displayed.

  KEYSTROKES: 2nd [DRAW] 1

- Rewrite $y - 3x \geq 1$ as $y \geq 3x + 1$ and graph it.

  KEYSTROKES: 2nd [DRAW] 7 3 X,T,θ,n + 1 , 10) ENTER

This time, the lower boundary is $y = 3x + 1$. The upper boundary is Ymax or 10. All ordered pairs for which y is *greater than or equal to* $3x + 1$ lie *above or on* the line and are solutions.

Exercises

1. Compare and contrast the two graphs shown above.

2. Graph the inequality $y \geq -2x + 4$ in the standard viewing window.
   a. What functions do you enter as the lower and upper boundaries?
   b. Using your graph, name four solutions of the inequality.

3. Suppose student movie tickets cost $4 and adult movie tickets cost $8. You would like to buy at least 10 tickets, but spend no more than $80.
   a. Let $x =$ number of student tickets and $y =$ number of adult tickets. Write two inequalities, one representing the total number of tickets and the other representing the total cost of the tickets.
   b. Which inequalities would you use as the lower and upper boundaries?
   c. Graph the inequalities. Use the viewing window [0, 20] scl: 1 by [0, 20] scl: 1.
   d. Name four possible combinations of student and adult tickets.

www.algebra1.com/other_calculator_keystrokes

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Study Guide and Review

Vocabulary and Concept Check

Addition Property of Inequalities (p. 318)
boundary (p. 353)
compound inequality (p. 339)
Division Property of Inequalities (p. 327)

half-plane (p. 353)
intersection (p. 339)
Multiplication Property of
   Inequalities (p. 325)

set-builder notation (p. 319)
Subtraction Property of
   Inequalities (p. 319)
union (p. 340)

Choose the letter of the term that best matches each statement, algebraic expression, or algebraic sentence.

1. $\{w \mid w \geq -14\}$
2. If $x \leq y$, then $-5x \geq -5y$.
3. $p > -5$ and $p \leq 0$
4. If $a < b$, then $a + 2 < b + 2$.
5. the graph on one side of a boundary
6. If $s \geq t$, then $s - 7 \geq t - 7$.
7. $g \geq 7$ or $g < 2$
8. If $m > n$, then $\frac{m}{7} > \frac{n}{7}$.

a. Addition Property of Inequalities
b. Division Property of Inequalities
c. half-plane
d. intersection
e. Multiplication Property of Inequalities
f. set-builder notation
g. Subtraction Property of Inequalities
h. union

Lesson-by-Lesson Review

6-1 Solving Inequalities by Addition and Subtraction

See pages
318–323.

Concept Summary

- If any number is added to each side of a true inequality, the resulting inequality is also true.
- If any number is subtracted from each side of a true inequality, the resulting inequality is also true.

Examples Solve each inequality.

1 $f + 9 \leq -23$

$f + 9 \leq -23$	Original inequality
$f + 9 - 9 \leq -23 - 9$	Subtract.
$f \leq -32$	Simplify.

The solution set is $\{f \mid f \leq -32\}$.

2 $v - 19 > -16$

$v - 19 > -16$	Original inequality
$v - 19 + 19 > -16 + 19$	Add.
$v > 3$	Simplify.

The solution set is $\{v \mid v > 3\}$.

Exercises Solve each inequality. Then check your solution, and graph it on a number line. *See Examples 1–5 on pages 318–320.*

9. $c + 51 > 32$
10. $r + 7 > -5$
11. $w - 14 \leq 23$
12. $a - 6 > -10$
13. $-0.11 \geq n - (-0.04)$
14. $2.3 < g - (-2.1)$
15. $7h \leq 6h - 1$
16. $5b > 4b + 5$

17. Define a variable, write an inequality, and solve the problem. Then check your solution. *Twenty-one is no less than the sum of a number and negative two.*

6-2 Solving Inequalities by Multiplication and Division

Concept Summary

- If each side of a true inequality is multiplied or divided by the same positive number, the resulting inequality is also true.
- If each side of a true inequality is multiplied or divided by the same negative number, the direction of the inequality must be *reversed*.

Examples Solve each inequality.

1 $-14g \geq 126$

$-14g \geq 126$ Original inequality

$\dfrac{-14g}{-14} \leq \dfrac{126}{-14}$ Divide and change $\geq$ to $\leq$.

$g \leq -9$ Simplify.

The solution set is $\{g \mid g \leq -9\}$.

2 $\dfrac{3}{4}d < 15$

$\dfrac{3}{4}d < 15$ Original inequality

$\left(\dfrac{4}{3}\right)\dfrac{3}{4}d < \left(\dfrac{4}{3}\right)15$ Multiply each side by $\dfrac{4}{3}$.

$d < 20$ Simplify.

The solution set is $\{d \mid d < 20\}$.

Exercises Solve each inequality. Then check your solution.
See Examples 1–5 on pages 326–328.

18. $15v > 60$ **19.** $12r \leq 72$ **20.** $-15z \geq -75$ **21.** $-9m < 99$

22. $\dfrac{b}{-12} \leq 3$ **23.** $\dfrac{d}{-13} > -5$ **24.** $\dfrac{2}{3}w > -22$ **25.** $\dfrac{3}{5}p \leq -15$

26. Define a variable, write an inequality, and solve the problem. Then check your solution. *Eighty percent of a number is greater than or equal to 24.*

6-3 Solving Multi-Step Inequalities

Concept Summary

- Multi-step inequalities can be solved by undoing the operations.
- Remember to reverse the inequality sign when multiplying or dividing each side by a negative number.
- When solving equations that contain grouping symbols, first use the Distributive Property to remove the grouping symbols.

Example Solve $4(n-1) < 7n + 8$.

$4(n-1) < 7n + 8$ Original inequality

$4n - 4 < 7n + 8$ Distributive Property

$4n - 4 - 7n < 7n + 8 - 7n$ Subtract $7n$ from each side.

$-3n - 4 < 8$ Simplify.

$-3n - 4 + 4 < 8 + 4$ Add 4 to each side.

$-3n < 12$ Simplify.

$\dfrac{-3n}{-3} > \dfrac{12}{-3}$ Divide each side by -3 and change $<$ to $>$.

$n > -4$ Simplify.

The solution set is $\{n \mid n > -4\}$.

CONTENTS

Exercises   **Solve each inequality. Then check your solution.**
See Examples 1–5 on pages 332–334.

27. $-4h + 7 > 15$
28. $5 - 6n > -19$
29. $-5x + 3 < 3x + 19$
30. $15b - 12 > 7b + 60$
31. $-5(q + 12) < 3q - 4$
32. $7(g + 8) < 3(g + 2) + 4g$
33. $\dfrac{2(x + 2)}{3} \geq 4$
34. $\dfrac{1 - 7n}{5} > 10$

35. Define a variable, write an inequality, and solve the problem. Then check your solution. *Two thirds of a number decreased by 27 is at least 9.*

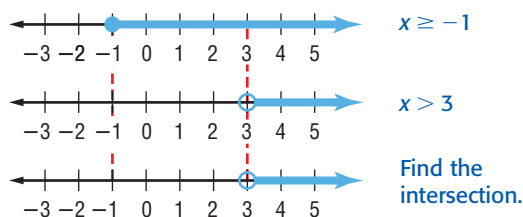## 6-4 Solving Compound Inequalities

See pages 339–344.

Concept Summary

- The solution of a compound inequality containing *and* is the intersection of the graphs of the two inequalities.
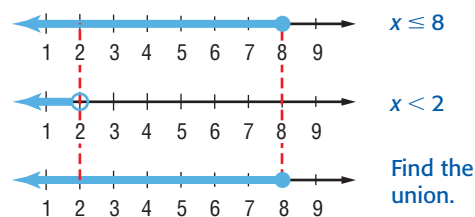- The solution of a compound inequality containing *or* is the union of the graphs of the two inequalities.

Examples   **Graph the solution set of each compound inequality.**

1 $x \geq -1$ and $x > 3$

$x \geq -1$

$x > 3$

Find the intersection.

The solution set is $\{x \,|\, x > 3\}$.

2 $x \leq 8$ or $x < 2$

$x \leq 8$

$x < 2$

Find the union.

The solution set is $\{x \,|\, x \leq 8\}$.

Exercises   **Solve each compound inequality. Then graph the solution set.**
See Examples 1–4 on pages 339–341.

36. $-1 < p + 3 < 5$
37. $-3 < 2k - 1 < 5$
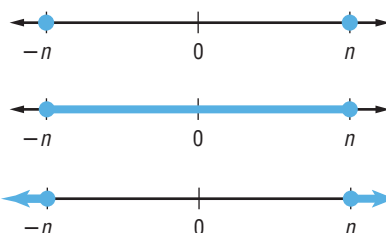38. $3w + 8 < 2$ or $w + 12 > 2 - w$
39. $a - 3 \leq 8$ or $a + 5 \geq 21$
40. $m + 8 < 4$ and $3 - m < 5$
41. $10 - 2y > 12$ and $7y < 4y + 9$

6-5 Solving Open Sentences Involving Absolute Value

See pages 345–351.

Concept Summary

- If $|x| = n$, then $x = -n$ or $x = n$.

- If $|x| < n$, then $x > -n$ and $x < n$.

- If $|x| > n$, then $x < -n$ or $x > n$.

CONTENTS

Chapter

6 For More ...

• Extra Practice, see pages 833–835.
• Mixed Problem Solving, see page 858.

Example **Solve** $|x + 6| = 15$.

$|x + 6| = 15$

$x + 6 = 15$ or $x + 6 = -15$

$x + 6 - 6 = 15 - 6$ $x + 6 - 6 = -15 - 6$

$x = 9$ $x = -21$

The solution set is $\{-21, 9\}$.

Exercises   **Solve each open sentence. Then graph the solution set.**
See Examples 1, 3, and 4 on pages 346–348.

42. $|w - 8| = 12$ **43.** $|q + 5| = 2$ **44.** $|h + 5| > 7$ **45.** $|w + 8| \geq 1$

46. $|r + 10| < 3$ **47.** $|t + 4| \leq 3$ **48.** $|2x + 5| < 4$ **49.** $|3d + 4| < 8$

6-6 *Graphing Inequalities in Two Variables*

Concept Summary

• To graph an inequality in two variables:

    Step 1   Determine the boundary and draw a dashed or solid line.

    Step 2   Select a test point. Test that point.

    Step 3   Shade the half-plane that contains the solution.

Example **Graph** $y \geq x - 2$.

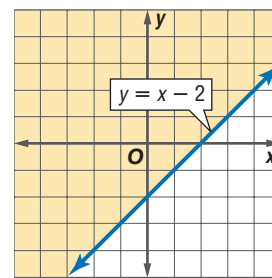Since the boundary is included in the solution, draw a solid line.

Test the point $(0, 0)$.

$y \geq x - 2$ Original inequality

$0 \geq 0 - 2$ $x = 0, y = 0$

$0 \geq -2$ true

The half plane that contains $(0, 0)$ should be shaded.

Exercises   **Determine which ordered pairs are part of the solution set for each inequality.**   *See Example 1 on page 352.*

50. $3x + 2y < 9, \{(1, 3), (3, 2), (-2, 7), (-4, 11)\}$

51. $5 - y \geq 4x, \left\{(2, -5), \left(\frac{1}{2}, 7\right), (-1, 6), (-3, 20)\right\}$

52. $\frac{1}{2}y \leq 6 - x, \{(-4, 15), (5, 1), (3, 8), (-2, 25)\}$

53. $-2x < 8 - y, \{(5, 10), (3, 6), (-4, 0), (-3, 6)\}$

Graph each inequality.   *See Example 2 on pages 353 and 354.*

54. $y - 2x < -3$ **55.** $x + 2y \geq 4$ **56.** $y \leq 5x + 1$ **57.** $2x - 3y > 6$

Vocabulary and Concepts

1. **Write** *the set of all numbers t such that t is greater than or equal to 17* in set-builder notation.
2. **Show** how to solve $6(a + 5) < 2a + 8$. Justify your work.
3. **OPEN ENDED** Give an example of a compound inequality that is an intersection and an example of a compound inequality that is a union.
4. **Compare and contrast** the graphs of $|x| \leq 3$ and $|x| \geq 3$.

Skills and Applications

Solve each inequality. Then check your solution.

5. $-23 \geq g - 6$
6. $9p < 8p - 18$
7. $d - 5 < 2d - 14$
8. $\frac{7}{8}w \geq -21$
9. $-22b \leq 99$
10. $4m - 11 \geq 8m + 7$
11. $-3(k - 2) > 12$
12. $\frac{f-5}{3} > -3$
13. $0.3(y - 4) \leq 0.8(0.2y + 2)$

14. **REAL ESTATE** A homeowner is selling her house. She must pay 7% of the selling price to her real estate agent after the house is sold. To the nearest dollar, what must be the selling price of her house to have at least $110,000 after the agent is paid?

15. Solve $6 + |r| = 3$.
16. Solve $|d| > -2$.

Solve each compound inequality. Then graph the solution set.

17. $r + 3 > 2$ and $4r < 12$
18. $3n + 2 \geq 17$ or $3n + 2 \leq -1$
19. $9 + 2p > 3$ and $-13 > 8p + 3$
20. $|2a - 5| < 7$
21. $|7 - 3s| \geq 2$
22. $|7 - 5z| > 3$

Define a variable, write an inequality, and solve each problem. Then check your solution.

23. One fourth of a number is no less than -3.
24. Three times a number subtracted from 14 is less than two.
25. Five less than twice a number is between 13 and 21.

26. **TRAVEL** Megan's car gets between 18 and 21 miles per gallon of gasoline. If her car's tank holds 15 gallons, what is the range of distance that Megan can drive her car on one tank of gasoline?

Graph each inequality.

27. $y \geq 3x - 2$
28. $2x + 3y < 6$
29. $x - 2y > 4$

30. **STANDARDIZED TEST PRACTICE** Which inequality is represented by the graph?
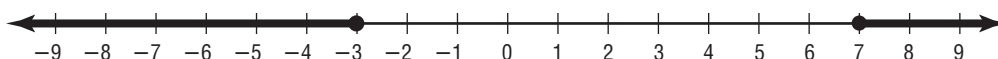
(A) $|x - 2| \leq 5$
(B) $|x - 2| \geq 5$
(C) $|x + 2| \leq 5$
(D) $|x + 2| \geq 5$

CONTENTS

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. Which of the following is a correct statement? (Lesson 2-4)

(A) $-\dfrac{9}{3} > \dfrac{3}{9}$

(B) $-\dfrac{3}{9} > -\dfrac{9}{3}$

(C) $-\dfrac{3}{9} < -\dfrac{9}{3}$

(D) $\dfrac{9}{3} < \dfrac{3}{9}$

2. $(-6)(-7) = $ (Lesson 2-3)

(A) -42

(B) -13

(C) 13

(D) 42

3. A cylindrical can has a volume of 5625π cubic centimeters. Its height is 25 centimeters. What is the radius of the can? Use the formula $V = \pi r^2 h$. (Lessons 2-8 and 3-8)

(A) 4.8 cm

(B) 7.5 cm

(C) 15 cm

(D) 47.1 cm

4. A furnace repair service charged a customer $80 for parts and $65 per hour worked. The bill totaled $177.50. About how long did the repair technician work on the furnace? (Lessons 3-1 and 3-4)

(A) 0.5 hour

(B) 1.5 hours

(C) 2 hours

(D) 4 hours

5. The formula $P = \dfrac{4(220 - A)}{5}$ determines the recommended maximum pulse rate P during exercise for a person who is A years old. Cameron is 15 years old. What is his recommended maximum pulse rate during exercise?   (Lesson 3-8)

(A) 162

(B) 164

(C) 173

(D) 263

6. The graph of the function $y = 2x - 1$ is shown. If the graph is translated 3 units up, which equation will best represent the new line? (Lesson 4-2)
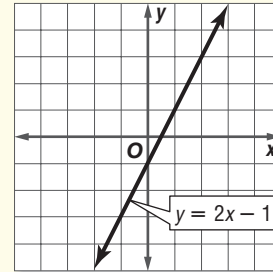
(A) $y = 2x + 2$

(B) $y = 2x - 3$

(C) $y = 2x + 3$

(D) $y = 2x - 4$

7. The table shows a set of values for x and y. Which equation best represents this set of data?   (Lesson 4-8)
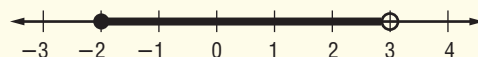
x	−4	−1	2	5	8
y	−16	−4	8	20	32

(A) $y = 3x - 4$

(B) $y = 3x + 2$

(C) $y = 2x - 10$

(D) $y = 4x$

8. Ali's grade depends on 4 test scores. On the first 3 tests, she earned scores of 78, 82, and 75. She wants to average at least 80. Which inequality can she use to find the score x that she needs on the fourth test in order to earn a final grade of at least 80?   (Lesson 6-3)

(A) $\dfrac{78 + 82 + 75 + x}{3} \ge 80$

(B) $\dfrac{78 + 82 + 75 + x}{4} \ge 80$

(C) $\dfrac{78 + 82 + 75 - x}{4} \ge 80$

(D) $\dfrac{78 + 82 + 75 + x}{4} \le 80$

9. Which inequality is represented by the graph? (Lesson 6-4)

(A) $-2 < x < 3$

(B) $-2 < x \le 3$

(C) $-2 \le x < 3$

(D) $-2 \le x \le 3$

CONTENTS

Preparing for Standardized Tests
For test-taking strategies and more
practice, see pages 867–884.

Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

10. A die is rolled. What are the odds of rolling a number less than 5? (Lesson 2-6)

11. A car is traveling at an average speed of 54 miles per hour. How many minutes will it take the car to travel 117 miles? (Lesson 3-6)

12. The price of a tape player was cut from $48 to $36. What was the percent of decrease? (Lesson 3-7)

13. Quadrilateral *MNOP* has vertices $M(0, -4)$, $N(-2, 8)$, $O(5, 3)$, and $P(2, -9)$. Find the coordinates of the vertices of the image if it is reflected over the *y*-axis. (Lesson 4-2)

14. Write an equation in slope-intercept form that describes the graph. (Lesson 5-4)
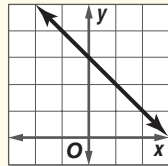
15. A line is parallel to the graph of the equation $\frac{1}{3}y = \frac{2}{3}x - 1$. What is the slope of the parallel line? (Lessons 5-4 and 5-6)

16. Solve $\frac{1}{2}(10x - 8) - 3(x - 1) \geq 15$ for *x*. (Lesson 6-3)

Test-Taking Tip Ⓐ Ⓑ Ⓒ Ⓓ
Questions 14 and 15
• Know the slope-intercept form of linear equations: $y = mx + b$.
• Understand the definition of slope.
• Recognize the relationships between the slopes of parallel lines and between the slopes of perpendicular lines.

17. Find all values of *x* that make the inequality $|x - 3| > 5$ true. (Lesson 6-5)

18. Graph the equation $y = -2x + 4$ and indicate which region represents $y < -2x + 4$. (Lesson 6-6)

Part 3 Extended Response

Record your answers on a sheet of paper. Show your work.

19. The Carlson family is building a house on a lot that is 91 feet long and 158 feet wide. (Lessons 6-1, 6-2, and 6-4)

  a. Town law states that the sides of a house cannot be closer than 10 feet to the edges of a lot. Write an inequality for the possible lengths of the Carlson family's house, and solve the inequality.

  b. The Carlson family wants their house to be at least 2800 square feet and no more than 3200 square feet. They also want their house to have the maximum possible length. Write an inequality for the possible widths of their house, and solve the inequality. Round your answer to the nearest whole number of feet.

20. For the graph below, write an open sentence involving absolute value. (Lesson 6-5)

21. A street vendor sells hot dogs for $3 each and bratwurst for $5 each. In order to cover his daily expenses, he must sell at least $400 worth of food. (Lesson 6-6)

  a. Write an inequality that represents this situation.

  b. If 68 hot dogs and 38 bratwursts are sold, will the street vendor cover his costs?

  c. Find a number of hotdogs and bratwursts that could be sold and cover the daily costs.

  d. Are there any restrictions on the domain and range? Explain.