Solving Systems of Linear Equations and Inequalities

What You'll Learn

- **Lesson 7-1** Solve systems of linear equations by graphing.
- **Lessons 7-2 through 7-4** Solve systems of linear equations algebraically.
- **Lesson 7-5** Solve systems of linear inequalities by graphing.

Key Vocabulary

- **system of equations** (p. 369)
- **substitution** (p. 376)
- **elimination** (p. 382)
- **system of inequalities** (p. 394)

Why It's Important

Business decision makers often use systems of linear equations to model a real-world situation in order to predict future events. Being able to make an accurate prediction helps them plan and manage their businesses.

Trends in the travel industry change with time. For example, in recent years, the number of tourists traveling to South America, the Caribbean, and the Middle East is on the rise. *You will use a system of linear equations to model the trends in tourism in Lesson 7-2.*

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Getting Started

▶ **Prerequisite Skills**   To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 7.

For Lesson 7-1   **Graph Linear Equations**

Graph each equation.   *(For review, see Lesson 4-5.)*

1. $y = 1$ **2.** $y = -2x$ **3.** $y = 4 - x$

4. $y = 2x + 3$ **5.** $y = 5 - 2x$ **6.** $y = \frac{1}{2}x + 2$

For Lesson 7-2   **Solve for a Given Variable**

Solve each equation or formula for the variable specified.   *(For review, see Lesson 3-8.)*

7. $4x + a = 6x$, for x **8.** $8a + y = 16$, for a

9. $\frac{7bc - d}{10} = 12$, for b **10.** $\frac{7m + n}{q} = 2m$, for q

For Lessons 7-3 and 7-4   **Simplify Expressions**

Simplify each expression. If not possible, write *simplified*.   *(For review, see Lesson 1-5.)*

11. $(3x + y) - (2x + y)$ **12.** $(7x - 2y) - (7x + 4y)$ **13.** $(16x - 3y) + (11x + 3y)$

14. $(8x - 4y) + (-8x + 5y)$ **15.** $4(2x + 3y) - (8x - y)$ **16.** $3(x - 4y) + (x + 12y)$

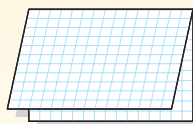**17.** $2(x - 2y) + (3x + 4y)$ **18.** $5(2x - y) - 2(5x + 3y)$ **19.** $3(x + 4y) + 2(2x - 6y)$

FOLDABLES™ Study Organizer

Solving Systems of Equations and Inequalities   Make this Foldable to help you organize your notes. Begin with five sheets of grid paper.
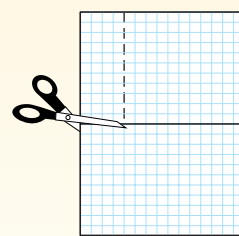
Step 1   **Fold**

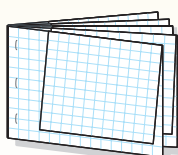Fold each sheet in half along the width.

Step 2   **Cut**

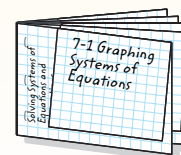Unfold and cut four rows from the left side of each sheet, from the top to the crease.

Step 3   **Stack and Staple**

Stack the sheets and staple to form a booklet.

Step 4   **Label**

Label each page with a lesson number and title.

Solving Systems of Equations and

7-1 Graphing Systems of Equations

Reading and Writing   As you read and study the chapter, unfold each page and fill the journal with notes, graphs, and examples for systems of equations and inequalities.

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Systems of Equations

You can use a spreadsheet to investigate when two quantities will be equal. Enter each formula into the spreadsheet and look for the row in which both formulas have the same result.

Example

Bill Winters is considering two job offers in telemarketing departments. The salary at the first job is $400 per week plus 10% commission on Mr. Winters' sales. At the second job, the salary is $375 per week plus 15% commission. For what amount of sales would the weekly salary be the same at either job?

Enter different amounts for Mr. Winters' weekly sales in column A. Then enter the formula for the salary at the first job in each cell in column B. In each cell of column C, enter the formula for the salary at the second job.

The spreadsheet shows that for sales of $500 the total weekly salary for each job is $450.

	A	B	C
	Sales	Salary 1	Salary 2
1			
2	0	400	375
3	100	410	390
4	200	420	405
5	300	430	420
6	400	440	435
7	500	450	450
8	600	460	465
9	700	470	480
10	800	480	495
11	900	490	510
12	1000	500	525
13			

Job Salaries — Sheet1

Exercises

For Exercises 1–4, use the spreadsheet of weekly salaries above.

1. If x is the amount of Mr. Winters' weekly sales and y is his total weekly salary, write a linear equation for the salary at the first job.

2. Write a linear equation for the salary at the second job.

3. Which ordered pair is a solution for both of the equations you wrote for Exercises 1 and 2?
   a. (100, 410)   b. (300, 420)   c. (500, 450)   d. (900, 510)

4. Use the graphing capability of the spreadsheet program to graph the salary data using a line graph. At what point do the two lines intersect? What is the significance of that point in the real-world situation?

5. How could you find the sales for which Mr. Winters' salary will be equal without using a spreadsheet?

CONTENTS

7-1 Graphing Systems of Equations

- Determine whether a system of linear equations has 0, 1, or infinitely many solutions.
- Solve systems of equations by graphing.

Vocabulary

- **system of equations**
- **consistent**
- **inconsistent**
- **independent**
- **dependent**

How can you use graphs to compare the sales of two products?

During the 1990s, sales of cassette singles decreased, and sales of CD singles increased. Assume that the sales of these singles were linear functions. If x represents the years since 1991 and y represents the sales in millions of dollars, the following equations represent the sales of these singles.

Cassette singles: $y = 69 - 6.9x$
CD singles: $y = 5.7 + 6.3x$

These equations are graphed at the right.

The point at which the two graphs intersect represents the time when the sales of cassette singles equaled the sales of CD singles. The ordered pair of this point is a solution of both equations.

Cassette and CD Singles Sales

Sales (millions of dollars) vs Years Since 1991

$y = 69 - 6.9x$

$y = 5.7 + 6.3x$

Sales of cassette singles equals sales of CD singles.
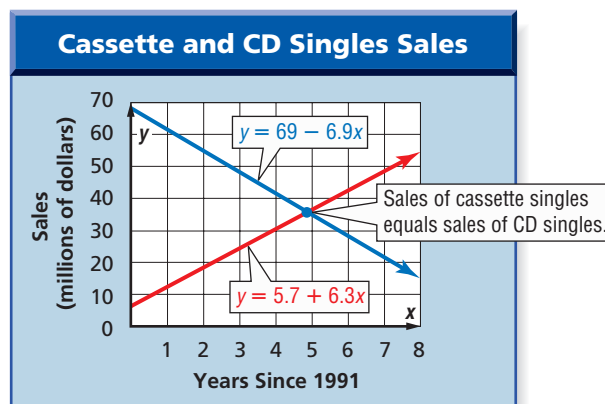
NUMBER OF SOLUTIONS Two equations, such as $y = 69 - 6.9x$ and $y = 5.7 + 6.3x$, together are called a **system of equations**. A solution of a system of equations is an ordered pair of numbers that satisfies both equations. A system of two linear equations can have 0, 1, or an infinite number of solutions.

- If the graphs intersect or coincide, the system of equations is said to be **consistent**. That is, it has at least one ordered pair that satisfies both equations.
- If the graphs are parallel, the system of equations is said to be **inconsistent**. There are *no* ordered pairs that satisfy both equations.
- Consistent equations can be **independent** or **dependent**. If a system has exactly one solution, it is independent. If the system has an infinite number of solutions, it is dependent.

Concept Summary — Systems of Equations

	Intersecting Lines	Same Line	Parallel Lines
Graph of a System			
Number of Solutions	exactly one solution	infinitely many	no solutions
Terminology	consistent and independent	consistent and dependent	inconsistent

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Example 1 Number of Solutions

Use the graph at the right to determine whether each system has *no* solution, *one* solution, or *infinitely many* solutions.

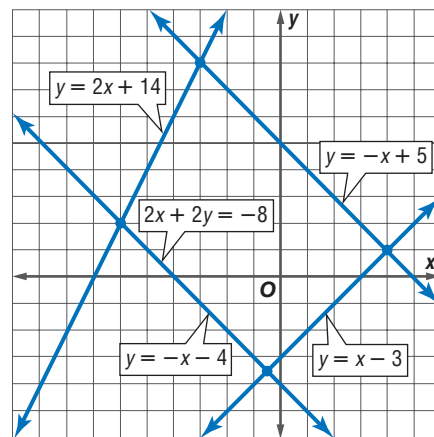**a.** $y = -x + 5$
$y = x - 3$

Since the graphs of $y = -x + 5$ and $y = x - 3$ are intersecting lines, there is one solution.

b. $y = -x + 5$
$2x + 2y = -8$

Since the graphs of $y = -x + 5$ and $2x + 2y = -8$ are parallel, there are no solutions.

c. $2x + 2y = -8$
$y = -x - 4$

Since the graphs of $2x + 2y = -8$ and $y = -x - 4$ coincide, there are infinitely many solutions.

SOLVE BY GRAPHING One method of solving systems of equations is to carefully graph the equations on the same coordinate plane.

Example 2 Solve a System of Equations

Graph each system of equations. Then determine whether the system has *no* solution, *one* solution, or *infinitely many* solutions. If the system has one solution, name it.

a. $y = -x + 8$
$y = 4x - 7$

The graphs appear to intersect at the point with coordinates (3, 5). Check this estimate by replacing x with 3 and y with 5 in each equation.

CHECK $\quad y = -x + 8 \qquad\qquad y = 4x - 7$
$\qquad\qquad 5 \stackrel{?}{=} -3 + 8 \qquad\qquad 5 \stackrel{?}{=} 4(3) - 7$
$\qquad\qquad 5 = 5 \;\; \checkmark \qquad\qquad\quad 5 \stackrel{?}{=} 12 - 7$
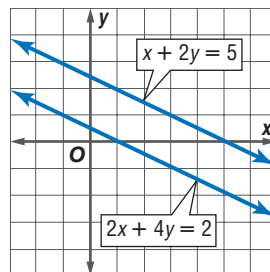$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad 5 = 5 \;\; \checkmark$

The solution is (3, 5).
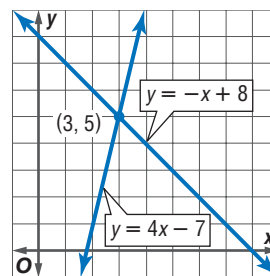
b. $x + 2y = 5$
$2x + 4y = 2$

The graphs of the equations are parallel lines. Since they do not intersect, there are no solutions to this system of equations. Notice that the lines have the same slope but different y-intercepts. *Recall that a system of equations that has no solution is said to be inconsistent.*
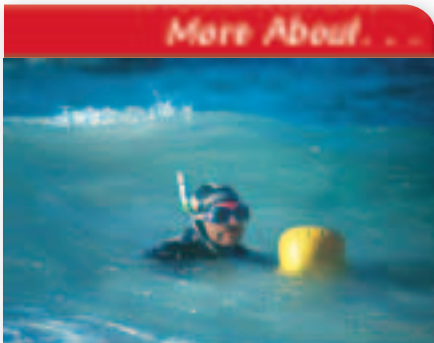
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Example 3 Write and Solve a System of Equations

WORLD RECORDS   Use the information on Guy Delage's swim at the left. If Guy can swim 3 miles per hour for an extended period and the raft drifts about 1 mile per hour, how many hours did he spend swimming each day?

Words   You have information about the amount of time spent swimming and floating. You also know the rates and the total distance traveled.

Variables   Let s = the number of hours Guy swam, and let f = the number of hours he floated each day. Write a system of equations to represent the situation.

Equations

The number of hours swimming	plus	the number of hours floating	equals	the total number of hours in a day.
s	$+$	f	$=$	24

The daily miles traveled swimming	plus	the daily miles traveled floating	equals	the total miles traveled in a day.
$3s$	$+$	$1f$	$=$	44

Graph the equations $s + f = 24$ and $3s + f = 44$. The graphs appear to intersect at the point with coordinates $(10, 14)$. Check this estimate by replacing s with 10 and f with 14 in each equation.
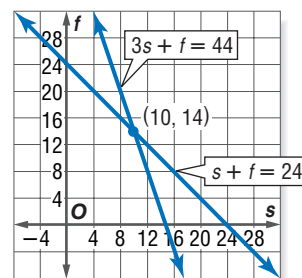
CHECK

$$s + f = 24 \qquad\qquad 3s + f = 44$$
$$10 + 14 \overset{?}{=} 24 \qquad 3(10) + 14 \overset{?}{=} 44$$
$$24 = 24 \quad \checkmark \qquad 30 + 14 \overset{?}{=} 44$$
$$44 = 44 \quad \checkmark$$

Guy Delage spent about 10 hours swimming each day.

Check for Understanding

Concept Check

1. **OPEN ENDED**   Draw the graph of a system of equations that has one solution at $(-2, 3)$.

2. **Determine** whether a system of equations with $(0, 0)$ and $(2, 2)$ as solutions *sometimes*, *always*, or *never* has other solutions. Explain.

3. **Find a counterexample** for the following statement.

   If the graphs of two linear equations have the same slope, then the system of equations has no solution.

Guided Practice   Use the graph at the right to determine whether each system has *no solution, one solution,* or *infinitely many* solutions.

4. $y = x - 4$
   $y = \frac{1}{3}x - 2$
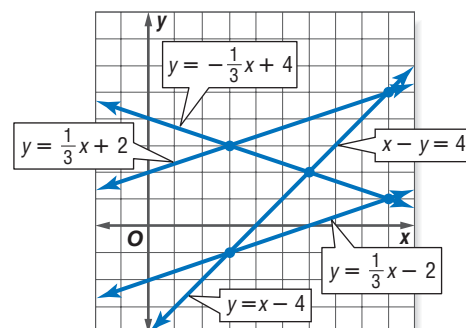
5. $y = \frac{1}{3}x + 2$
   $y = \frac{1}{3}x - 2$

6. $x - y = 4$
   $y = x - 4$

7. $x - y = 4$
   $y = -\frac{1}{3}x + 4$

Graph each system of equations. Then determine whether the system has *no solution*, *one* solution, or *infinitely many* solutions. If the system has one solution, name it.

8. $y = -x$
$y = 2x$

9. $x + y = 8$
$x - y = 2$

10. $2x + 4y = 2$
$3x + 6y = 3$

11. $x + y = 4$
$x + y = 1$

12. $x - y = 2$
$3y + 2x = 9$

13. $x + y = 2$
$y = 4x + 7$

Application **14. RESTAURANTS** A restaurant charges one price for adults and another price for children. The Rodriguez family has two adults and three children, and their bill was $40.50. The Wong family has three adults and one child. Their bill was $38. Determine the price of the buffet for an adult and the price for a child.
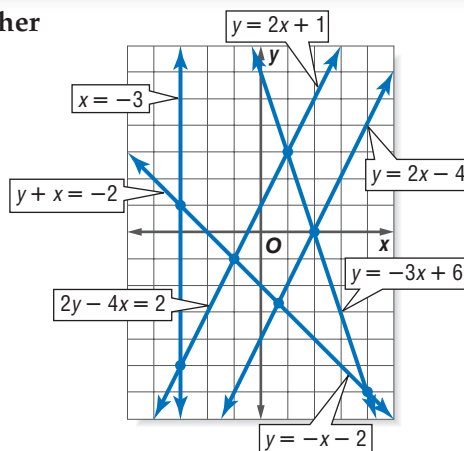
Practice and Apply

Use the graph at the right to determine whether each system has *no* solution, *one* solution, or *infinitely many* solutions.

15. $x = -3$
$y = 2x + 1$

16. $y = -x - 2$
$y = 2x - 4$

17. $y + x = -2$
$y = -x - 2$

18. $y = 2x + 1$
$y = 2x - 4$

19. $y = -3x + 6$
$y = 2x - 4$

20. $2y - 4x = 2$
$y = 2x - 4$

21. $2y - 4x = 2$
$y = -3x + 6$

22. $2y - 4x = 2$
$y = 2x + 1$

Graph each system of equations. Then determine whether the system has *no solution*, *one* solution, or *infinitely many* solutions. If the system has one solution, name it.

23. $y = -6$
$4x + y = 2$

24. $x = 2$
$3x - y = 8$

25. $y = \frac{1}{2}x$
$2x + y = 10$

26. $y = -x$
$y = 2x - 6$

27. $y = 3x - 4$
$y = -3x - 4$

28. $y = 2x + 6$
$y = -x - 3$

29. $x - 2y = 2$
$3x + y = 6$

30. $x + y = 2$
$2y - x = 10$

31. $3x + 2y = 12$
$3x + 2y = 6$

32. $2x + 3y = 4$
$-4x - 6y = -8$

33. $2x + y = -4$
$5x + 3y = -6$

34. $4x + 3y = 24$
$5x - 8y = -17$

35. $3x + y = 3$
$2y = -6x + 6$

36. $y = x + 3$
$3y + x = 5$

37. $2x + 3y = -17$
$y = x - 4$

38. $y = \frac{2}{3}x - 5$
$3y = 2x$

39. $6 - \frac{3}{8}y = x$
$\frac{2}{3}x + \frac{1}{4}y = 4$

40. $\frac{1}{2}x + \frac{1}{3}y = 6$
$y = \frac{1}{2}x + 2$

41. GEOMETRY The length of the rectangle at the right is 1 meter less than twice its width. What are the dimensions of the rectangle?
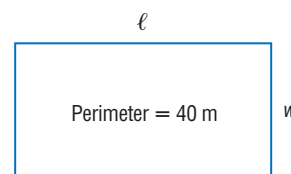
ℓ

Perimeter = 40 m w

GEOMETRY For Exercises 42 and 43, use the graphs of $y = 2x + 6$, $3x + 2y = 19$, and $y = 2$, which contain the sides of a triangle.

42. Find the coordinates of the vertices of the triangle.

43. Find the area of the triangle.

BALLOONING For Exercises 44 and 45, use the information in the graphic at the right.

44. In how many minutes will the balloons be at the same height?

45. How high will the balloons be at that time?

Balloon 1 is 10 meters above the ground, rising 15 meters per minute.

**Balloon 2** is 150 meters above the ground, descending 20 meters per minute.

SAVINGS For Exercises 46 and 47, use the following information.
Monica and Michael Gordon both want to buy a scooter. Monica has already saved $25 and plans to save $5 per week until she can buy the scooter. Michael has $16 and plans to save $8 per week.

46. In how many weeks will Monica and Michael have saved the same amount of money?

47. How much will each person have saved at that time?

BUSINESS For Exercises 48–50, use the graph at the right.

48. Which company had the greater profit during the ten years?

49. Which company had a greater rate of growth?

50. If the profit patterns continue, will the profits of the two companies ever be equal? Explain.

Yearly Profits

Widget Company

Gadget Company

Profit (millions of dollars) vs Year
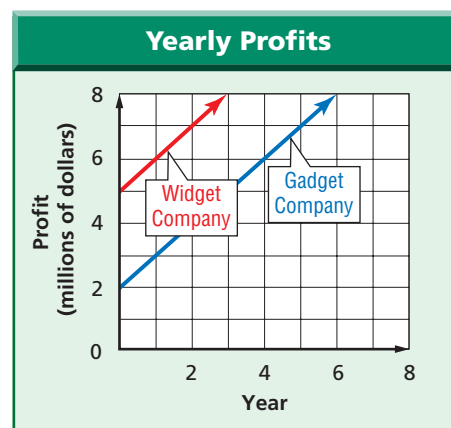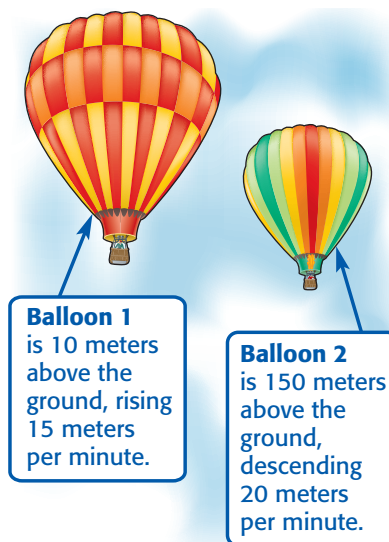
POPULATION For Exercises 51–54, use the following information.
The U.S. Census Bureau divides the country into four sections. They are the Northeast, the Midwest, the South, and the West.

51. In 1990, the population of the Midwest was about 60 million. During the 1990s, the population of this area increased an average of about 0.4 million per year. Write an equation to represent the population of the Midwest for the years since 1990.

52. The population of the West was about 53 million in 1990. The population of this area increased an average of about 1 million per year during the 1990s. Write an equation to represent the population of the West for the years since 1990.

53. Graph the population equations.

54. Assume that the rate of growth of each of these areas remains the same. Estimate when the population of the West would be equal to the population of the Midwest.

55. CRITICAL THINKING The solution of the system of equations $Ax + y = 5$ and $Ax + By = 20$ is $(2, -3)$. What are the values of A and B?

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56. `WRITING IN MATH` Answer the question that was posed at the beginning of the lesson.
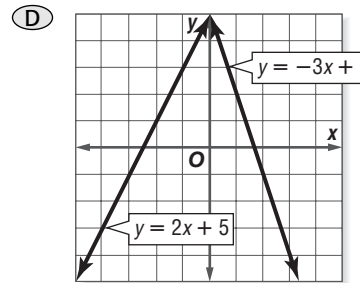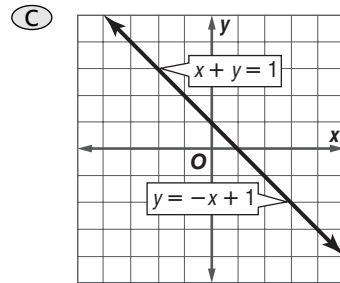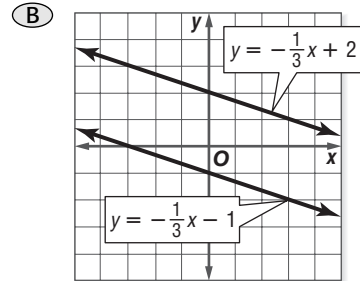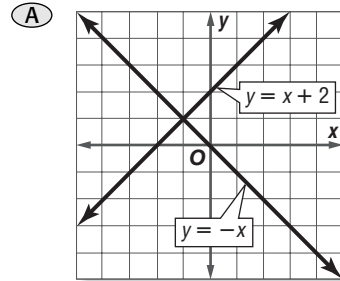
How can you use graphs to compare the sales of two products?

Include the following in your answer:
- an estimate of the year in which the sales of cassette singles equaled the sales of CD singles, and
- an explanation of why graphing works.

57. Which graph represents a system of equations with no solution?

Ⓐ
$y = x + 2$
$y = -x$

Ⓑ
$y = -\frac{1}{3}x + 2$
$y = -\frac{1}{3}x - 1$

Ⓒ
$x + y = 1$
$y = -x + 1$

Ⓓ
$y = -3x + 5$
$y = 2x + 5$

58. How many solutions exist for the system of equations below?

$4x + y = 7$
$3x - y = 0$

  Ⓐ no solution   Ⓑ one solution

  Ⓒ infinitely many solutions   Ⓓ cannot be determined

Maintain Your Skills

Mixed Review **Determine which ordered pairs are part of the solution set for each inequality.** *(Lesson 6-6)*

59. $y \leq 2x$, $\{(1, 4), (-1, 5), (5, -6), (-7, 0)\}$

60. $y < 8 - 3x$, $\{(-4, 2), (-3, 0), (1, 4), (1, 8)\}$

61. MANUFACTURING The inspector at a perfume manufacturer accepts a bottle if it is less than 0.05 ounce above or below 2 ounces. What are the acceptable numbers of ounces for a perfume bottle? *(Lesson 6-5)*

Write each equation in standard form. *(Lesson 5-5)*

62. $y - 1 = 4(x - 5)$ **63.** $y + 2 = \frac{1}{3}(x + 3)$ **64.** $y - 4 = -6(x + 2)$

Getting Ready for the Next Lesson **PREREQUISITE SKILL** **Solve each equation for the variable specified.**
*(To review **solving equations for a specified variable**, see Lesson 3-8.)*

65. $12x - y = 10x$, for y **66.** $6a + b = 2a$, for a

67. $\frac{7m - n}{q} = 10$, for q **68.** $\frac{5tz - s}{2} = 6$, for z

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Graphing Calculator Investigation

Systems of Equations

You can use a TI-83 Plus graphing calculator to solve a system of equations.

Example

Solve the system of equations. State the decimal solution to the nearest hundredth.

$2.93x + y = 6.08$
$8.32x - y = 4.11$

Step 1 Solve each equation for y to enter them into the calculator.

$2.93x + y = 6.08$	First equation
$2.93x + y - 2.93x = 6.08 - 2.93x$	Subtract $2.93x$ from each side.
$y = 6.08 - 2.93x$	Simplify.

|	
$8.32x - y = 4.11$	Second equation
$8.32x - y - 8.32x = 4.11 - 8.32x$	Subtract $8.32x$ from each side.
$-y = 4.11 - 8.32x$	Simplify.
$(-1)(-y) = (-1)(4.11 - 8.32x)$	Multiply each side by -1.
$y = -4.11 + 8.32x$	Simplify.

Step 2 Enter these equations in the **Y=** list and graph.

KEYSTROKES: *Review on pages 224–225.*

Step 3 Use the **CALC** menu to find the point of intersection.

KEYSTROKES: $\boxed{\text{2nd}}$ [CALC] 5 $\boxed{\text{ENTER}}$ $\boxed{\text{ENTER}}$ $\boxed{\text{ENTER}}$

The solution is approximately (0.91, 3.43).
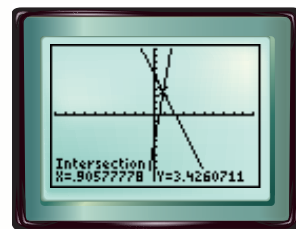
[10, 10] scl: 1 by [−10, 10] scl: 1

Exercises

Use a graphing calculator to solve each system of equations. Write decimal solutions to the nearest hundredth.

1. $y = 3x - 4$
$y = -0.5x + 6$

2. $y = 2x + 5$
$y = -0.2x - 4$

3. $x + y = 5.35$
$3x - y = 3.75$

4. $0.35x - y = 1.12$
$2.25x + y = -4.05$

5. $1.5x + y = 6.7$
$5.2x - y = 4.1$

6. $5.4x - y = 1.8$
$6.2x + y = -3.8$

7. $5x - 4y = 26$
$4x + 2y = 53.3$

8. $2x + 3y = 11$
$4x + y = -6$

9. $0.22x + 0.15y = 0.30$
$-0.33x + y = 6.22$

10. $125x - 200y = 800$
$65x - 20y = 140$

www.algebra1.com/other_calculator_keystrokes

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7-2 Substitution

- Solve systems of equations by using substitution.
- Solve real-world problems involving systems of equations.

Vocabulary
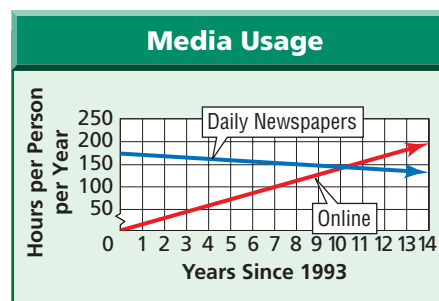
- substitution

How can a system of equations be used to predict media use?

Americans spend more time online than they spend reading daily newspapers. If x represents the number of years since 1993 and y represents the average number of hours per person per year, the following system represents the situation.

reading daily newspapers: $y = -2.8x + 170$
online: $y = 14.4x + 2$

The solution of the system represents the year that the number of hours spent on each activity will be the same. To solve this system, you could graph the equations and find the point of intersection. However, the exact coordinates of the point would be very difficult to determine from the graph. You could find a more accurate solution by using algebraic methods.

Media Usage

SUBSTITUTION The exact solution of a system of equations can be found by using algebraic methods. One such method is called **substitution**.

Algebra Activity

Using Substitution

Use algebra tiles and an equation mat to solve the system of equations.
$3x + y = 8$ and $y = x - 4$

Model and Analyze

Since $y = x - 4$, use 1 positive x tile and 4 negative 1 tiles to represent y. Use algebra tiles to represent $3x + y = 8$.

1. Use what you know about equation mats to solve for x. What is the value of x?
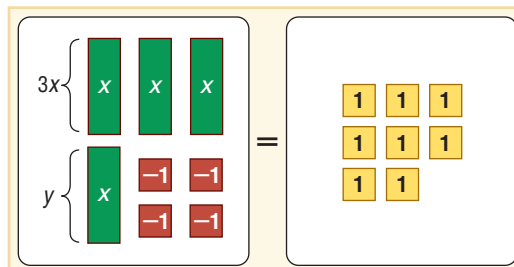2. Use $y = x - 4$ to solve for y.
3. What is the solution of the system of equations?

Make a Conjecture

4. Explain how to solve the following system of equations using algebra tiles.
   $4x + 3y = 10$ and $y = x + 1$
5. Why do you think this method is called substitution?

CONTENTS

Example 1 Solve Using Substitution

Use substitution to solve the system of equations.

$y = 3x$
$x + 2y = -21$

Since $y = 3x$, substitute $3x$ for y in the second equation.

$x + 2y = -21$ Second equation

$x + 2(3x) = -21$ $y = 3x$

$x + 6x = -21$ Simplify.

$7x = -21$ Combine like terms.

$\dfrac{7x}{7} = \dfrac{-21}{7}$ Divide each side by 7.

$x = -3$ Simplify.

Use $y = 3x$ to find the value of y.

$y = 3x$ First equation

$y = 3(-3)$ $x = -3$

$y = -9$ The solution is $(-3, -9)$.

Example 2 Solve for One Variable, Then Substitute

Use substitution to solve the system of equations.

$x + 5y = -3$
$3x - 2y = 8$

Solve the first equation for x since the coefficient of x is 1.

$x + 5y = -3$ First equation

$x + 5y - 5y = -3 - 5y$ Subtract $5y$ from each side.

$x = -3 - 5y$ Simplify.

Find the value of y by substituting $-3 - 5y$ for x in the second equation.

$3x - 2y = 8$ Second equation

$3(-3 - 5y) - 2y = 8$ $x = -3 - 5y$

$-9 - 15y - 2y = 8$ Distributive Property

$-9 - 17y = 8$ Combine like terms.

$-9 - 17y + 9 = 8 + 9$ Add 9 to each side.

$-17y = 17$ Simplify.

$\dfrac{-17y}{-17} = \dfrac{17}{-17}$ Divide each side by -17.

$y = -1$ Simplify.

Substitute -1 for y in either equation to find the value of x.
Choose the equation that is easier to solve.

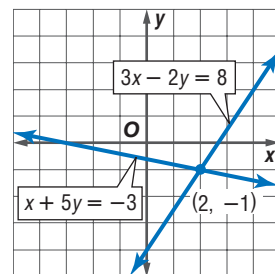$x + 5y = -3$ First equation

$x + 5(-1) = -3$ $y = -1$

$x - 5 = -3$ Simplify.

$x = 2$ Add 5 to each side.

The solution is $(2, -1)$. The graph verifies the solution.

Example 3 *Dependent System*

Use substitution to solve the system of equations.

$6x - 2y = -4$
$y = 3x + 2$

Since $y = 3x + 2$, substitute $3x + 2$ for y in the first equation.

$6x - 2y = -4$ **First equation**

$6x - 2(3x + 2) = -4$ **$y = 3x + 2$**

$6x - 6x - 4 = -4$ **Distributive Property**

$-4 = -4$ **Simplify.**

The statement $-4 = -4$ is true. This means that there are infinitely many solutions of the system of equations. This is true because the slope-intercept form of both equations is $y = 3x + 2$. That is, the equations are equivalent, and they have the same graph.

In general, if you solve a system of linear equations and the result is a true statement (an identity such as $-4 = -4$), the system has an infinite number of solutions. However, if the result is a false statement (for example, $-4 = 5$), the system has no solution.

REAL-WORLD PROBLEMS   Sometimes it is helpful to organize data before solving a problem. Some ways to organize data are to use tables, charts, different types of graphs, or diagrams.

Example 4 *Write and Solve a System of Equations*

METAL ALLOYS   **A metal alloy is 25% copper. Another metal alloy is 50% copper. How much of each alloy should be used to make 1000 grams of a metal alloy that is 45% copper?**

Let a = the number of grams of the 25% copper alloy and b = the number of grams of the 50% copper alloy. Use a table to organize the information.

	25% Copper	50% Copper	45% Copper
Total Grams	a	b	1000
Grams of Copper	$0.25a$	$0.50b$	$0.45(1000)$

The system of equations is $a + b = 1000$ and $0.25a + 0.50b = 0.45(1000)$. Use substitution to solve this system.

$a + b = 1000$ **First equation**

$a + b - b = 1000 - b$ **Subtract b from each side.**

$a = 1000 - b$ **Simplify.**

$0.25a + 0.50b = 0.45(1000)$ **Second equation**

$0.25(1000 - b) + 0.50b = 0.45(1000)$ **$a = 1000 - b$**

$250 - 0.25b + 0.50b = 450$ **Distributive Property**

$250 + 0.25b = 450$ **Combine like terms.**

$250 + 0.25b - 250 = 450 - 250$ **Subtract 250 from each side.**

$0.25b = 200$ **Simplify.**

$\dfrac{0.25b}{0.25} = \dfrac{200}{0.25}$ **Divide each side by 0.25.**

$b = 800$ **Simplify.**

$$a + b = 1000 \qquad \text{First equation}$$
$$a + 800 = 1000 \qquad b = 800$$
$$a + 800 - 800 = 1000 - 800 \qquad \text{Subtract 800 from each side.}$$
$$a = 200 \qquad \text{Simplify.}$$

200 grams of the 25% alloy and 800 grams of the 50% alloy should be used.

Check for Understanding

Concept Check

1. **Explain** why you might choose to use substitution rather than graphing to solve a system of equations.

2. **Describe** the graphs of two equations if solving the system of equations yields the equation $4 = 2$.

3. **OPEN-ENDED** Write a system of equations that has infinitely many solutions.

Guided Practice

Use substitution to solve each system of equations. If the system does *not* have exactly one solution, state whether it has *no* solution or *infinitely many* solutions.

4. $x = 2y$
   $4x + 2y = 15$

5. $y = 3x - 8$
   $y = 4 - x$

6. $2x + 7y = 3$
   $x = 1 - 4y$

7. $6x - 2y = -4$
   $y = 3x + 2$

8. $x + 3y = 12$
   $x - y = 8$

9. $y = \frac{3}{5}x$
   $3x - 5y = 15$

Application

10. **TRANSPORTATION** The Thrust SSC is the world's fastest land vehicle. Suppose the driver of a car whose top speed is 200 miles per hour requests a race against the SSC. The car gets a head start of one-half hour. If there is unlimited space to race, at what distance will the SSC pass the car?

Thrust SSC top speed is 763 mph.

## Practice and Apply

Use substitution to solve each system of equations. If the system does *not* have exactly one solution, state whether it has *no* solution or *infinitely many* solutions.

11. $y = 5x$
    $2x + 3y = 34$

12. $x = 4y$
    $2x + 3y = 44$

13. $x = 4y + 5$
    $x = 3y - 2$

14. $y = 2x + 3$
    $y = 4x - 1$

15. $4c = 3d + 3$
    $c = d - 1$

16. $4x + 5y = 11$
    $y = 3x - 13$

17. $8x + 2y = 13$
    $4x + y = 11$

18. $2x - y = -4$
    $-3x + y = -9$

19. $3x - 5y = 11$
    $x - 3y = 1$

20. $2x + 3y = 1$
    $-3x + y = 15$

21. $c - 5d = 2$
    $2c + d = 4$

22. $5r - s = 5$
    $-4r + 5s = 17$

23. $3x - 2y = 12$
    $x + 2y = 6$

24. $x - 3y = 0$
    $3x + y = 7$

25. $-0.3x + y = 0.5$
    $0.5x - 0.3y = 1.9$

26. $0.5x - 2y = 17$
    $2x + y = 104$

27. $y = \frac{1}{2}x + 3$
    $y = 2x - 1$

28. $x = \frac{1}{2}y + 3$
    $2x - y = 6$

www.algebra1.com/self_check_quiz

CONTENTS

29. GEOMETRY The base of the triangle is 4 inches longer than the length of one of the other sides. Use a system of equations to find the length of each side of the triangle.

x in. *x* in.

Perimeter = 46 in.

y in.
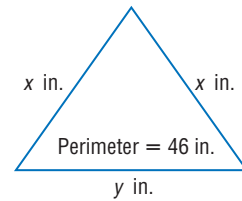
30. FUND-RAISING The Future Teachers of America Club at Paint Branch High School is making a healthy trail mix to sell to students during lunch. The mix will have three times the number of pounds of raisins as sunflower seeds. Sunflower seeds cost $4.00 per pound, and raisins cost $1.50 per pound. If the group has $34.00 to spend on the raisins and sunflower seeds, how many pounds of each should they buy?

31. CHEMISTRY MX Labs needs to make 500 gallons of a 34% acid solution. The only solutions available are a 25% acid solution and a 50% acid solution. How many gallons of each solution should be mixed to make the 34% solution?

32. GEOMETRY Supplementary angles are two angles whose measures have the sum of 180 degrees. Angles X and Y are supplementary, and the measure of angle X is 24 degrees greater than the measure of angle Y. Find the measures of angles X and Y.

33. SPORTS At the end of the 2000 baseball season, the New York Yankees and the Cincinnati Reds had won a total of 31 World Series. The Yankees had won 5.2 times as many World Series as the Reds. How many World Series did each team win?

JOBS **For Exercises 34 and 35, use the following information.**
Shantel Jones has two job offers as a car salesperson. At one dealership, she will receive $600 per month plus a commission of 2% of the total price of the automobiles she sells. At the other dealership, she will receive $1000 per month plus a commission of 1.5% of her total sales.

34. What is the total price of the automobiles that Ms. Jones must sell each month to make the same income from either dealership?

35. Explain which job offer is better.

36. LANDSCAPING A blue spruce grows an average of 6 inches per year. A hemlock grows an average of 4 inches per year. If a blue spruce is 4 feet tall and a hemlock is 6 feet tall, when would you expect the trees to be the same height?

37. TOURISM In 2000, approximately 40.3 million tourists visited South America and the Caribbean. The number of tourists to that area had been increasing at an average rate of 0.8 million tourists per year. In the same year, 17.0 million tourists visited the Middle East. The number of tourists to the Middle East had been increasing at an average rate of 1.8 million tourists per year. If the trend continues, when would you expect the number of tourists to South America and the Caribbean to equal the number of tourists to the Middle East?

38. RESEARCH Use the Internet or other resources to find the pricing plans for various cell phones. Determine the number of minutes you would need to use the phone for two plans to cost the same amount of money. Support your answer with a table, a graph, and/or an equation.

39. CRITICAL THINKING Solve the system of equations. Write the solution as an ordered triple of the form (x, y, z).

$$2x + 3y - z = 17$$
$$y = -3z - 7$$
$$2x = z + 2$$

40. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How can a system of equations be used to predict media use?

Include the following in your answer:
- an explanation of solving a system of equations by using substitution, and
- the year when the number of hours spent reading daily newspapers is the same as the hours spent online.

41. When solving the following system, which expression could be substituted for x?

$$x + 4y = 1$$
$$2x - 3y = -9$$

Ⓐ $4y - 1$　　　Ⓑ $1 - 4y$　　　Ⓒ $3y - 9$　　　Ⓓ $-9 - 3y$

42. If $x - 3y = -9$ and $5x - 2y = 7$, what is the value of x?

Ⓐ 1　　　Ⓑ 2　　　Ⓒ 3　　　Ⓓ 4

Maintain Your Skills

Mixed Review **Graph each system of equations. Then determine whether the system has *no* solution, *one* solution, or *infinitely many* solutions. If the system has one solution, name it.** *(Lesson 7-1)*

43. $x + y = 3$
$x + y = 4$

44. $x + 2y = 1$
$2x + y = 5$

45. $2x + y = 3$
$4x + 2y = 6$

Graph each inequality. *(Lesson 6-6)*

46. $y < -5$　　　**47.** $x \geq 4$　　　**48.** $2x + y > 6$

49. RECYCLING When a pair of blue jeans is made, the leftover denim scraps can be recycled. One pound of denim is left after making every five pair of jeans. How many pounds of denim would be left from 250 pairs of jeans? *(Lesson 3-6)*

Getting Ready for the Next Lesson **PREREQUISITE SKILL** **Simplify each expression.**
*(To review **simplifying expressions**, see Lesson 1-5.)*

50. $6a - 9a$　　　**51.** $8t + 4t$　　　**52.** $-7g - 8g$　　　**53.** $7d - (2d + b)$

𝑃ractice Quiz 1　　　　　　　　　　　　*Lessons 7-1 and 7-2*

Graph each system of equations. Then determine whether the system has *no* solution, *one* solution, or *infinitely many* solutions. If the system has one solution, name it. *(Lesson 7-1)*

1. $x + y = 3$
$x - y = 1$

2. $3x - 2y = -6$
$3x - 2y = 6$

Use substitution to solve each system of equations. If the system does *not* have exactly one solution, state whether it has *no* solution or *infinitely many* solutions. *(Lesson 7-2)*

3. $x + y = 0$
$3x + y = -8$

4. $x - 2y = 5$
$3x - 5y = 8$

5. $x + y = 2$
$y = 2 - x$

CONTENTS

Elimination Using Addition and Subtraction

What You'll Learn

- Solve systems of equations by using elimination with addition.
- Solve systems of equations by using elimination with subtraction.

Vocabulary
- elimination

How can you use a system of equations to solve problems about weather?

On the winter solstice, there are fewer hours of daylight in the Northern Hemisphere than on any other day. On that day in Seward, Alaska, the difference between the number of hours of darkness n and the number of hours of daylight d is 12. The following system of equations represents the situation.

$$n + d = 24$$
$$n - d = 12$$

Notice that if you add these equations, the variable d is eliminated.

$$\begin{array}{r} n + d = 24 \\ (+)\underline{n - d = 12} \\ 2n \quad\;\; = 36 \end{array}$$

ELIMINATION USING ADDITION Sometimes adding two equations together will eliminate one variable. Using this step to solve a system of equations is called **elimination**.

Example **1** *Elimination Using Addition*

Use elimination to solve each system of equations.

$3x - 5y = -16$
$2x + 5y = 31$

Since the coefficients of the y terms, -5 and 5, are additive inverses, you can eliminate the y terms by adding the equations.

$$\begin{array}{r} 3x - 5y = -16 \\ (+)\,\underline{2x + 5y = \quad 31} \\ 5x \qquad\;\; = \quad 15 \end{array}$$ Write the equations in column form and add.

Notice that the y variable is eliminated.

$\dfrac{5x}{5} = \dfrac{15}{5}$ Divide each side by 5.

$x = 3$ Simplify.

Now substitute 3 for x in either equation to find the value of y.

$3x - 5y = -16$ First equation

$3(3) - 5y = -16$ Replace x with 3.

$9 - 5y = -16$ Simplify.

$9 - 5y - 9 = -16 - 9$ Subtract 9 from each side.

$-5y = -25$ Simplify.

$\dfrac{-5y}{-5} = \dfrac{-25}{-5}$ Divide each side by -5.

$y = 5$ Simplify.

The solution is (3, 5).

CONTENTS

Example 2 Write and Solve a System of Equations

Twice one number added to another number is 18. Four times the first number minus the other number is 12. Find the numbers.

Let x represent the first number and y represent the second number.

Twice one number,	added to	another number,	is	18.
$2x$	$+$	y	$=$	18

Four times the first number,	minus	the other number,	is	12.
$4x$	$-$	y	$=$	12

Use elimination to solve the system.

$$\begin{array}{ll} 2x + y = 18 & \text{Write the equations in column form and add.} \\ (+)\ 4x - y = 12 & \\ \hline 6x \quad\ \ = 30 & \text{Notice that the variable } y \text{ is eliminated.} \\ \dfrac{6x}{6} = \dfrac{30}{6} & \text{Divide each side by 6.} \\ x = 5 & \text{Simplify.} \end{array}$$

Now substitute 5 for x in either equation to find the value of y.

$$\begin{array}{ll} 4x - y = 12 & \text{Second equation} \\ 4(5) - y = 12 & \text{Replace } x \text{ with 5.} \\ 20 - y = 12 & \text{Simplify.} \\ 20 - y - 20 = 12 - 20 & \text{Subtract 20 from each side.} \\ -y = -8 & \text{Simplify.} \\ \dfrac{-y}{-1} = \dfrac{-8}{-1} & \text{Divide each side by } -1. \\ y = 8 & \text{The numbers are 5 and 8.} \end{array}$$

ELIMINATION USING SUBTRACTION Sometimes subtracting one equation from another will eliminate one variable.

Example 3 Elimination Using Subtraction

Use elimination to solve the system of equations.

$$5s + 2t = 6$$
$$9s + 2t = 22$$

Since the coefficients of the t terms, 2 and 2, are the same, you can eliminate the t terms by subtracting the equations.

$$\begin{array}{ll} 5s + 2t = \quad 6 & \text{Write the equations in column form and subtract.} \\ (-)\ 9s + 2t = \quad 22 & \\ \hline -4s \quad\quad = -16 & \text{Notice that the variable } t \text{ is eliminated.} \\ \dfrac{-4s}{-4} = \dfrac{-16}{-4} & \text{Divide each side by } -4. \\ s = 4 & \text{Simplify.} \end{array}$$

Now substitute 4 for s in either equation to find the value of t.

$$\begin{array}{ll} 5s + 2t = 6 & \text{First equation} \\ 5(4) + 2t = 6 & s = 4 \\ 20 + 2t = 6 & \text{Simplify.} \\ 20 + 2t - 20 = 6 - 20 & \text{Subtract 20 from each side.} \\ 2t = -14 & \text{Simplify.} \\ \dfrac{2t}{2} = \dfrac{-14}{2} & \text{Divide each side by 2.} \\ t = -7 & \text{The solution is } (4, -7). \end{array}$$

Example 4 **Elimination Using Subtraction**

Multiple-Choice Test Item

If $x - 3y = 7$ and $x + 2y = 2$, what is the value of x?

Ⓐ 4   Ⓑ −1   Ⓒ (−1, 4)   Ⓓ (4, −1)

Read the Test Item

You are given a system of equations, and you are asked to find the value of x.

Solve the Test Item

You can eliminate the x terms by subtracting one equation from the other.

$$x - 3y = 7 \quad \text{Write the equations in column form and subtract.}$$
$$\underline{(-)\ x + 2y = 2}$$
$$-5y = 5 \quad \text{Notice the } x \text{ variable is eliminated.}$$

$$\frac{-5y}{-5} = \frac{5}{-5} \quad \text{Divide each side by } -5.$$
$$y = -1 \quad \text{Simplify.}$$

Now substitute −1 for y in either equation to find the value of x.

$$x + 2y = 2 \quad \text{Second equation}$$
$$x + 2(-1) = 2 \quad y = -1$$
$$x - 2 = 2 \quad \text{Simplify.}$$
$$x - 2 + 2 = 2 + 2 \quad \text{Add 2 to each side.}$$
$$x = 4 \quad \text{Simplify.}$$

Notice that B is the value of y and D is the solution of the system of equations. However, the question asks for the value of x. The answer is A.

Test-Taking Tip

Always read the question carefully. Ask yourself, "What does the question ask?" Then answer that question.

Check for Understanding

Concept Check

1. **OPEN ENDED** Write a system of equations that can be solved by using addition to eliminate one variable.

2. **Describe** a system of equations that can be solved by using subtraction to eliminate one variable.

3. **FIND THE ERROR** Michael and Yoomee are solving a system of equations.

Michael

$$2r + s = 5$$
$$\underline{(+)\ r - s = 1}$$
$$3r \quad = 6$$
$$r = 2$$

$$2r + s = 5$$
$$2(2) + s = 5$$
$$4 + s = 5$$
$$s = 1$$

The solution is (2, 1).

Yoomee

$$2r + s = 5$$
$$\underline{(-)\ r - s = }$$
$$r \quad = 4$$

$$r - s = 1$$
$$4 - s = 1$$
$$-s = -3$$
$$s = 3$$

The solution is (4, 3).

Who is correct? Explain your reasoning.

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Guided Practice

Use elimination to solve each system of equations.

4. $x - y = 14$
$x + y = 20$

5. $2a - 3b = -11$
$a + 3b = 8$

6. $4x + y = -9$
$4x + 2y = -10$

7. $6x + 2y = -10$
$2x + 2y = -10$

8. $2a + 4b = 30$
$-2a - 2b = -21.5$

9. $-4m + 2n = 6$
$-4m + n = 8$

10. The sum of two numbers is 24. Five times the first number minus the second number is 12. What are the two numbers?

11. If $2x + 7y = 17$ and $2x + 5y = 11$, what is the value of $2y$?

Ⓐ -4 Ⓑ -2 Ⓒ 3 Ⓓ 6

Practice and Apply

Use elimination to solve each system of equations.

12. $x + y = -3$
$x - y = 1$

13. $s - t = 4$
$s + t = 2$

14. $3m - 2n = 13$
$m + 2n = 7$

15. $-4x + 2y = 8$
$4x - 3y = -10$

16. $3a + b = 5$
$2a + b = 10$

17. $2m - 5n = -6$
$2m - 7n = -14$

18. $3r - 5s = -35$
$2r - 5s = -30$

19. $13a + 5b = -11$
$13a + 11b = 7$

20. $3x - 5y = 16$
$-3x + 2y = -10$

21. $6s + 5t = 1$
$6s - 5t = 11$

22. $4x - 3y = 12$
$4x + 3y = 24$

23. $a - 2b = 5$
$3a - 2b = 9$

24. $4x + 5y = 7$
$8x + 5y = 9$

25. $8a + b = 1$
$8a - 3b = 3$

26. $1.44x - 3.24y = -5.58$
$1.08x + 3.24y = 9.99$

27. $7.2m + 4.5n = 129.06$
$7.2m + 6.7n = 136.54$

28. $\frac{3}{5}c - \frac{1}{5}d = 9$
$\frac{7}{5}c + \frac{1}{5}d = 11$

29. $\frac{2}{3}x - \frac{1}{2}y = 14$
$\frac{5}{6}x - \frac{1}{2}y = 18$

30. The sum of two numbers is 48, and their difference is 24. What are the numbers?

31. Find the two numbers whose sum is 51 and whose difference is 13.

32. Three times one number added to another number is 18. Twice the first number minus the other number is 12. Find the numbers.

33. One number added to twice another number is 23. Four times the first number added to twice the other number is 38. What are the numbers?

34. BUSINESS In 1999, the United States produced about 2 million more motor vehicles than Japan. Together, the two countries produced about 22 million motor vehicles. How many vehicles were produced in each country?
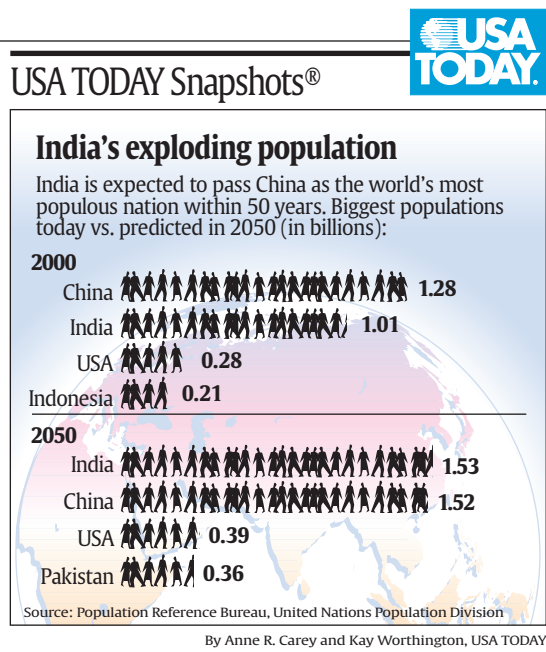
35. PARKS A youth group and their leaders visited Mammoth Cave. Two adults and 5 students in one van paid $77 for the Grand Avenue Tour of the cave. Two adults and 7 students in a second van paid $95 for the same tour. Find the adult price and the student price of the tour.

36. FOOTBALL During the National Football League's 1999 season, Troy Aikman, the quarterback for the Dallas Cowboys, earned $0.467 million more than Deion Sanders, the Cowboys cornerback. Together they cost the Cowboys $12.867 million. How much did each player make?

POPULATIONS For Exercises 37–39, use the information in the graph at the right.

37. Let x represent the number of years since 2000 and y represent population in billions. Write an equation to represent the population of China.

38. Write an equation to represent the population of India.

39. Use elimination to find the year when the populations of China and India are predicted to be the same. What is the predicted population at that time?

USA TODAY Snapshots®

India's exploding population

India is expected to pass China as the world's most populous nation within 50 years. Biggest populations today vs. predicted in 2050 (in billions):

2000
China 1.28
India 1.01
USA 0.28
Indonesia 0.21

2050
India 1.53
China 1.52
USA 0.39
Pakistan 0.36

Source: Population Reference Bureau, United Nations Population Division

By Anne R. Carey and Kay Worthington, USA TODAY

40. **CRITICAL THINKING** The graphs of $Ax + By = 15$ and $Ax - By = 9$ intersect at (2, 1). Find A and B.

41. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can you use a system of equations to solve problems about weather?

Include the following in your answer:
- an explanation of how to use elimination to solve a system of equations, and
- a step-by-step solution of the Seward daylight problem.

42. If $2x - 3y = -9$ and $3x - 3y = -12$, what is the value of y?
    Ⓐ -3 Ⓑ 1 Ⓒ $(-3, 1)$ Ⓓ $(1, -3)$

43. What is the solution of $4x + 2y = 8$ and $2x + 2y = 2$?
    Ⓐ $(-2, 3)$ Ⓑ $(3, 2)$ Ⓒ $(3, -2)$ Ⓓ $(12, -3)$

Maintain Your Skills

Mixed Review

Use substitution to solve each system of equations. If the system does *not* have exactly one solution, state whether it has *no* solution or *infinitely many* solutions. *(Lesson 7-2)*

44. $y = 5x$
    $x + 2y = 22$

45. $x = 2y + 3$
    $3x + 4y = -1$

46. $2y - x = -5$
    $4y - 3x = -1$

Graph each system of equations. Then determine whether the system has *no* solution, *one* solution, or *infinitely many* solutions. If the system has one solution, name it. *(Lesson 7-1)*

47. $x - y = 3$
    $3x + y = 1$

48. $2x - 3y = 7$
    $3y = 7 + 2x$

49. $4x + y = 12$
    $x = 3 - \frac{1}{4}y$

50. Write an equation of a line that is parallel to the graph of $y = \frac{5}{4}x - 3$ and passes through the origin. *(Lesson 5-6)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL Use the Distributive Property to rewrite each expression without parentheses. *(To review the **Distributive Property**, see Lesson 1-5.)*

51. $2(3x + 4y)$ 52. $6(2a - 5b)$ 53. $-3(-2m + 3n)$ 54. $-5(4t - 2s)$

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7-4 Elimination Using Multiplication
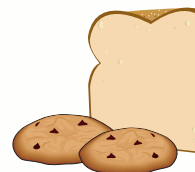
What You'll Learn

- Solve systems of equations by using elimination with multiplication.
- Determine the best method for solving systems of equations.

How can a manager use a system of equations to plan employee time?

The Finneytown Bakery is making peanut butter cookies and loaves of quick bread. The preparation and baking times for each are given in the table below.

For these two items, the management has allotted 800 minutes of employee time and 900 minutes of oven time. If c represents the number of batches of cookies and b represents the number of loaves of bread, the following system of equations can be used to determine how many of each to bake.

$$20c + 10b = 800$$
$$10c + 30b = 900$$

	Cookies (per batch)	Bread (per loaf)
Preparation	20 min	10 min
Baking	10 min	30 min

ELIMINATION USING MULTIPLICATION Neither variable in the system above can be eliminated by simply adding or subtracting the equations. However, you can use the Multiplication Property of Equality so that adding or subtracting eliminates one of the variables.

Example 1 Multiply One Equation to Eliminate

Use elimination to solve the system of equations.

$$3x + 4y = 6$$
$$5x + 2y = -4$$

Multiply the second equation by -2 so the coefficients of the y terms are additive inverses. Then add the equations.

$$3x + 4y = 6 \qquad\qquad\qquad 3x + 4y = 6$$
$$5x + 2y = -4 \quad \text{Multiply by } -2. \quad (+) \; -10x - 4y = 8$$
$$\overline{\qquad\qquad\qquad\qquad -7x \qquad = 14} \quad \text{Add the equations.}$$
$$\frac{-7x}{-7} = \frac{14}{-7} \quad \text{Divide each side by } -7.$$
$$x = -2 \quad \text{Simplify.}$$

Now substitute -2 for x in either equation to find the value of y.

$$3x + 4y = 6 \qquad \text{First equation}$$
$$3(-2) + 4y = 6 \qquad x = -2$$
$$-6 + 4y = 6 \qquad \text{Simplify.}$$
$$-6 + 4y + 6 = 6 + 6 \qquad \text{Add 6 to each side.}$$
$$4y = 12 \qquad \text{Simplify.}$$
$$\frac{4y}{4} = \frac{12}{4} \qquad \text{Divide each side by 4.}$$
$$y = 3 \qquad \text{The solution is } (-2, 3).$$

CONTENTS

For some systems of equations, it is necessary to multiply each equation by a different number in order to solve the system by elimination. You can choose to eliminate either variable.

Example 2 Multiply Both Equations to Eliminate

Use elimination to solve the system of equations.

$3x + 4y = -25$
$2x - 3y = 6$

Method 1 Eliminate x.

$3x + 4y = -25$ Multiply by 2.
$2x - 3y = 6$ Multiply by -3.

$$6x + 8y = -50$$
$$(+) -6x + 9y = -18$$
$$\overline{ 17y = -68}$$ Add the equations.
$$\frac{17y}{17} = \frac{-68}{17}$$ Divide each side by 17.
$$y = -4$$ Simplify.

Now substitute -4 for y in either equation to find the value of x.

$2x - 3y = 6$ Second equation
$2x - 3(-4) = 6$ $y = -4$
$2x + 12 = 6$ Simplify.
$2x + 12 - 12 = 6 - 12$ Subtract 12 from each side.
$2x = -6$ Simplify.
$\dfrac{2x}{2} = \dfrac{-6}{2}$ Divide each side by 2.
$x = -3$ Simplify.

The solution is $(-3, -4)$.

Method 2 Eliminate y.

$3x + 4y = -25$ Multiply by 3.
$2x - 3y = 6$ Multiply by 4.

$$9x + 12y = -75$$
$$(+) 8x - 12y = 24$$
$$\overline{17x = -51}$$ Add the equations.
$$\frac{17x}{17} = \frac{-51}{17}$$ Divide each side by 17.
$$x = -3$$ Simplify.

Now substitute -3 for x in either equation to find the value of y.

$2x - 3y = 6$ Second equation
$2(-3) - 3y = 6$ $x = -3$
$-6 - 3y = 6$ Simplify.
$-6 - 3y + 6 = 6 + 6$ Add 6 to each side.
$-3y = 12$ Simplify.
$\dfrac{-3y}{-3} = \dfrac{12}{-3}$ Divide each side by -3.
$y = -4$ Simplify.

The solution is $(-3, -4)$, which matches the result obtained with Method 1.

Study Tip

Using Multiplication
There are many other combinations of multipliers that could be used to solve the system in Example 2. For instance, the first equation could be multiplied by -2 and the second by 3.

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DETERMINE THE BEST METHOD You have learned five methods for solving systems of linear equations.

Concept Summary	Solving Systems of Equations
Method	**The Best Time to Use**
Graphing	to estimate the solution, since graphing usually does not give an exact solution
Substitution	if one of the variables in either equation has a coefficient of 1 or -1
Elimination Using Addition	if one of the variables has opposite coefficients in the two equations
Elimination Using Subtraction	if one of the variables has the same coefficient in the two equations
Elimination Using Multiplication	if none of the coefficients are 1 or -1 and neither of the variables can be eliminated by simply adding or subtracting the equations

Example 3 *Determine the Best Method*

Determine the best method to solve the system of equations. Then solve the system.

$4x - 3y = 12$
$x + 2y = 14$

- For an exact solution, an algebraic method is best.
- Since neither the coefficients of x nor the coefficients of y are the same or additive inverses, you cannot use elimination using addition or subtraction.
- Since the coefficient of x in the second equation is 1, you can use the substitution method. You could also use elimination using multiplication.

The following solution uses substitution. *Which method would you prefer?*

$$x + 2y = 14 \qquad \text{Second equation}$$
$$x + 2y - 2y = 14 - 2y \qquad \text{Subtract } 2y \text{ from each side.}$$
$$x = 14 - 2y \qquad \text{Simplify.}$$

$$4x - 3y = 12 \qquad \text{First equation}$$
$$4(14 - 2y) - 3y = 12 \qquad x = 14 - 2y$$
$$56 - 8y - 3y = 12 \qquad \text{Distributive Property}$$
$$56 - 11y = 12 \qquad \text{Combine like terms.}$$
$$56 - 11y - 56 = 12 - 56 \qquad \text{Subtract 56 from each side.}$$
$$-11y = -44 \qquad \text{Simplify.}$$
$$\frac{-11y}{-11} = \frac{-44}{-11} \qquad \text{Divide each side by } -11.$$
$$y = 4 \qquad \text{Simplify.}$$

$$x + 2y = 14 \qquad \text{Second equation}$$
$$x + 2(4) = 14 \qquad y = 4$$
$$x + 8 = 14 \qquad \text{Simplify.}$$
$$x + 8 - 8 = 14 - 8 \qquad \text{Subtract 8 from each side.}$$
$$x = 6 \qquad \text{Simplify.}$$

The solution is (6, 4).

Example 4 Write and Solve a System of Equations

TRANSPORTATION A coal barge on the Ohio River travels 24 miles upstream in 3 hours. The return trip takes the barge only 2 hours. Find the rate of the barge in still water.

Let b = the rate of the barge in still water and c = the rate of the current. Use the formula rate × time = distance, or $rt = d$.

	r	t	d	$rt = d$
Downstream	$b + c$	2	24	$2b + 2c = 24$
Upstream	$b - c$	3	24	$3b - 3c = 24$

This system cannot easily be solved using substitution. It cannot be solved by just adding or subtracting the equations.

The best way to solve this system is to use elimination using multiplication. Since the problem asks for b, eliminate c.

$2b + 2c = 24$ Multiply by 3.
$3b - 3c = 24$ Multiply by 2.

$$6b + 6c = 72$$
$$(+)\ 6b - 6c = 48$$
$$12b \qquad = 120 \quad \text{Add the equations.}$$

$$\frac{12b}{12} = \frac{120}{12} \quad \text{Divide each side by 12.}$$

$$b = 10 \quad \text{Simplify.}$$

The rate of the barge in still water is 10 miles per hour.

Transportation
About 203 million tons of freight are transported on the Ohio River each year making it the second most used commercial river in the United States.
Source: *World Book Encyclopedia*

Check for Understanding

Concept Check

1. **Explain** why multiplication is sometimes needed to solve a system of equations by elimination.

2. **OPEN ENDED**   Write a system of equations that could be solved by multiplying one equation by 5 and then adding the two equations together to eliminate one variable.

3. **Describe** two methods that could be used to solve the following system of equations. Which method do you prefer? Explain.
   $a - b = 5$
   $2a + 3b = 15$

Guided Practice   Use elimination to solve each system of equations.

4. $2x - y = 6$
   $3x + 4y = -2$

5. $x + 5y = 4$
   $3x - 7y = -10$

6. $4x + 7y = 6$
   $6x + 5y = 20$

7. $4x + 2y = 10.5$
   $2x + 3y = 10.75$

Determine the best method to solve each system of equations. Then solve the system.

8. $4x + 3y = 19$
   $3x - 4y = 8$

9. $3x - 7y = 6$
   $2x + 7y = 4$

10. $y = 4x + 11$
    $3x - 2y = -7$

11. $5x - 2y = 12$
    $3x - 2y = -2$

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12. **BUSINESS**   The owners of the River View Restaurant have hired enough servers to handle 17 tables of customers, and the fire marshal has approved the restaurant for a limit of 56 customers. How many two-seat tables and how many four-seat tables should the owners purchase?

Practice and Apply

Use elimination to solve each system of equations.

13. $-5x + 3y = 6$
    $x - y = 4$

14. $x + y = 3$
    $2x - 3y = 16$

15. $2x + y = 5$
    $3x - 2y = 4$

16. $4x - 3y = 12$
    $x + 2y = 14$

17. $5x - 2y = -15$
    $3x + 8y = 37$

18. $8x - 3y = -11$
    $2x - 5y = 27$

19. $4x - 7y = 10$
    $3x + 2y = -7$

20. $2x - 3y = 2$
    $5x + 4y = 28$

21. $1.8x - 0.3y = 14.4$
    $x - 0.6y = 2.8$

22. $0.4x + 0.5y = 2.5$
    $1.2x - 3.5y = 2.5$

23. $3x - \frac{1}{2}y = 10$
    $5x + \frac{1}{4}y = 8$

24. $2x + \frac{2}{3}y = 4$
    $x - \frac{1}{2}y = 7$

25. Seven times a number plus three times another number equals negative one. The sum of the two numbers is negative three. What are the numbers?

26. Five times a number minus twice another number equals twenty-two. The sum of the numbers is three. Find the numbers.

Determine the best method to solve each system of equations. Then solve the system.

27. $3x - 4y = -10$
    $5x + 8y = -2$

28. $9x - 8y = 42$
    $4x + 8y = -16$

29. $y = 3x$
    $3x + 4y = 30$

30. $x = 4y + 8$
    $2x - 8y = -3$

31. $2x - 3y = 12$
    $x + 3y = 12$

32. $4x - 2y = 14$
    $y = x$

33. $x - y = 2$
    $5x + 3y = 18$

34. $y = 2x + 9$
    $2x - y = -9$

35. $6x - y = 9$
    $6x - y = 11$

36. $x = 8y$
    $2x + 3y = 38$

37. $\frac{2}{3}x - \frac{1}{2}y = 14$
    $\frac{5}{6}x - \frac{1}{2}y = 18$

38. $\frac{1}{2}x - \frac{2}{3}y = \frac{7}{3}$
    $\frac{3}{2}x + 2y = -25$

39. **BASKETBALL**   In basketball, a free throw is 1 point and a field goal is either 2 points or 3 points. Suppose a professional basketball player scored a total of 1938 points in one season. The total number of 2-point field goals and 3-point field goals was 701, and he made 475 of the 557 free throws that he attempted. Find the number of 2-point field goals and 3-point field goals the player made that season.

Online Research **Data Update** What are the current statistics for your favorite basketball player? Visit www.algebra1.com/data_update to learn more.

40. **CRITICAL THINKING**   The solution of the system $4x + 5y = 2$ and $6x - 2y = b$ is $(3, a)$. Find the values of a and b.

41. **CAREERS**   Mrs. Henderson discovered that she had accidentally reversed the digits of a test and shorted a student 36 points. Mrs. Henderson told the student that the sum of the digits was 14 and agreed to give the student his correct score plus extra credit if he could determine his actual score without looking at his test. What was his actual score on the test?

www.algebra1.com/self_check_quiz

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42. NUMBER THEORY The sum of the digits of a two-digit number is 14. If the digits are reversed, the new number is 18 less than the original number. Find the original number.

43. TRANSPORTATION Traveling against the wind, a plane flies 2100 miles from Chicago to San Diego in 4 hours and 40 minutes. The return trip, traveling with a wind that is twice as fast, takes 4 hours. Find the rate of the plane in still air.

44. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How can a manager use a system of equations to plan employee time?

Include the following in your answer:
- a demonstration of how to solve the system of equations concerning the cookies and bread, and
- an explanation of how a restaurant manager would schedule oven and employee time.

45. If $5x + 3y = 12$ and $4x - 5y = 17$, what is the value of y?
Ⓐ −1 Ⓑ 3 Ⓒ (−1, 3) Ⓓ (3, −1)

46. Determine the number of solutions of the system $x + 2y = -1$ and $2x + 4y = -2$.
Ⓐ 0 Ⓑ 1 Ⓒ 2 Ⓓ infinitely many

Maintain Your Skills

Mixed Review **Use elimination to solve each system of equations.** *(Lesson 7-3)*

47. $x + y = 8$
$x - y = 4$

48. $2r + s = 5$
$r - s = 1$

49. $x + y = 18$
$x + 2y = 25$

Use substitution to solve each system of equations. If the system does *not* have exactly one solution, state whether it has *no* solution or *infinitely many* solutions. *(Lesson 7-2)*

50. $2x + 3y = 3$
$x = -3y$

51. $x + y = 0$
$3x + y = -8$

52. $x - 2y = 7$
$-3x + 6y = -21$

53. CAREERS A store manager is paid $32,000 a year plus 4% of the revenue the store makes above quota. What is the amount of revenue above quota needed for the manager to have an annual income greater than $45,000? *(Lesson 6-3)*

Getting Ready for the Next Lesson **PREREQUISITE SKILL Graph each inequality.**
*(To review **graphing inequalities**, see Lesson 6-6.)*

54. $y \geq x - 7$ **55.** $x + 3y \geq 9$ **56.** $-y \leq x$ **57.** $-3x + y \geq -1$

Practice Quiz 2 — Lessons 7-3 and 7-4

**Use elimination to solve each system of equations.** *(Lessons 7-3 and 7-4)*

1. $5x + 4y = 2$
$3x - 4y = 14$

2. $2x - 3y = 13$
$2x + 2y = -2$

3. $6x - 2y = 24$
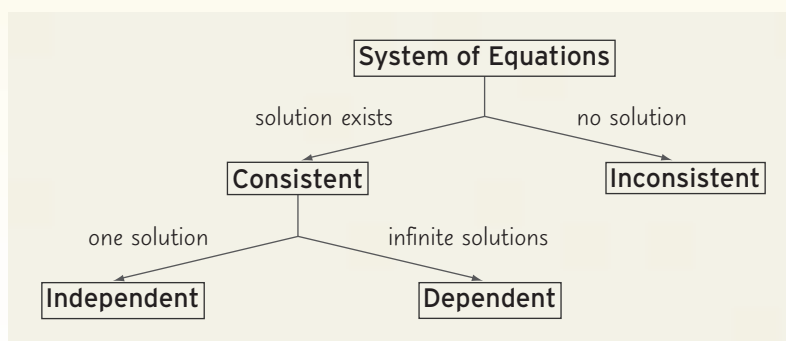$3x + 4y = 27$

4. $5x + 2y = 4$
$10x + 4y = 9$

5. The price of a cellular telephone plan is based on peak and nonpeak service. Kelsey used 45 peak minutes and 50 nonpeak minutes and was charged $27.75. That same month, Mitch used 70 peak minutes and 30 nonpeak minutes for a total charge of $36. What are the rates per minute for peak and nonpeak time? *(Lesson 7-4)*

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Reading Mathematics
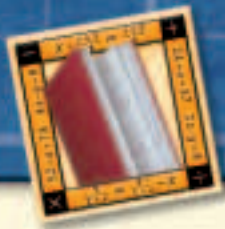
Making Concept Maps

After completing a chapter, it is wise to review each lesson's main topics and vocabulary. In Lesson 7-1, the new vocabulary words were *system of equations*, *consistent*, *inconsistent*, *independent*, and *dependent*. They are all related in that they explain how many and what kind of solutions a system of equations has.

A graphic organizer called a *concept map* is a convenient way to show these relationships. A concept map is shown below for the vocabulary words for Lesson 7-1. The main ideas are placed in boxes. Any information that describes how to move from one box to the next is placed along the arrows.

```
              System of Equations
     solution exists            no solution
              Consistent          Inconsistent
  one solution        infinite solutions
   Independent            Dependent
```

Concept maps are used to organize information. They clearly show how ideas are related to one another. They also show the flow of mental processes needed to solve problems.

Reading to Learn

Review Lessons 7-2, 7-3, and 7-4.

1. Write a couple of sentences describing the information in the concept map above.

2. How do you decide whether to use substitution or elimination? Give an example of a system that you would solve using each method.

3. How do you decide whether to multiply an equation by a factor?

4. How do you decide whether to add or subtract two equations?

5. Copy and complete the concept map below for solving systems of equations by using either substitution or elimination.

```
                  System of Equations
  one variable has a                variables have
  coefficient of 1 or -1            any coefficients
       [     ]                      [          ]
                               [        ]    [       ]
                      opposite signs   same sign
                         [      ]      [       ]
```
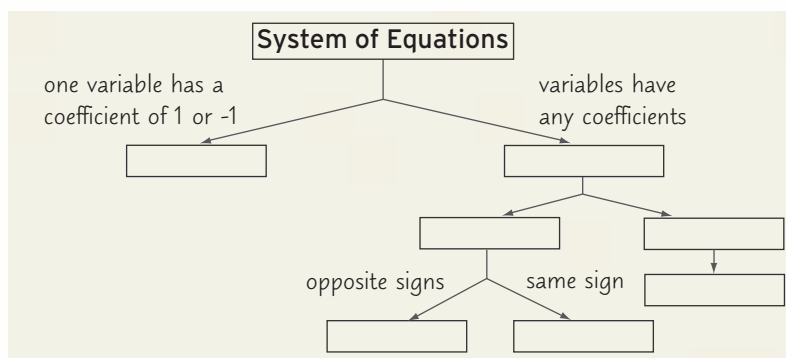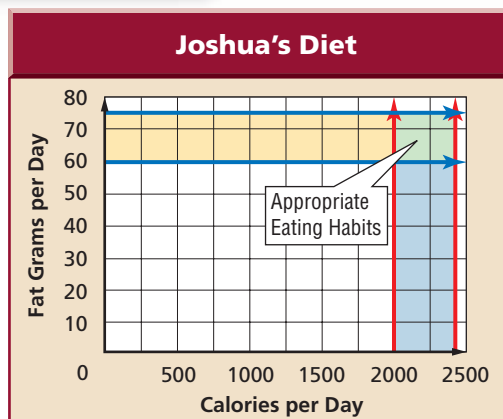
Graphing Systems of Inequalities

- Solve systems of inequalities by graphing.
- Solve real-world problems involving systems of inequalities.

Vocabulary

- system of inequalities

How can you use a system of inequalities to plan a sensible diet?

Joshua watches what he eats. His doctor told him to eat between 2000 and 2400 Calories per day. The doctor also wants him to keep his daily fat intake between 60 and 75 grams. The graph indicates the appropriate amounts of Calories and fat for Joshua. The graph is of a system of inequalities. He should try to keep his Calorie and fat intake to amounts represented in the green section.

Joshua's Diet

SYSTEMS OF INEQUALITIES To solve a **system of inequalities**, you need to find the ordered pairs that satisfy all the inequalities involved. One way to do this is to graph the inequalities on the same coordinate plane. The solution set is represented by the intersection, or overlap, of the graphs.

Example 1 Solve by Graphing

Solve the system of inequalities by graphing.

$$y < -x + 1$$
$$y \leq 2x + 3$$

The solution includes the ordered pairs in the intersection of the graphs of $y < -x + 1$ and $y \leq 2x + 3$. This region is shaded in green at the right. The graphs of $y = -x + 1$ and $y = 2x + 3$ are boundaries of this region. The graph of $y = -x + 1$ is dashed and is *not* included in the graph of $y < -x + 1$. The graph of $y = 2x + 3$ is included in the graph of $y \leq 2x + 3$.
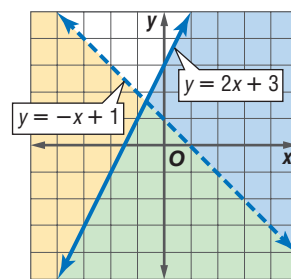
Example 2 No Solution

Solve the system of inequalities by graphing.

$$x - y < -1$$
$$x - y > 3$$

The graphs of $x - y = -1$ and $x - y = 3$ are parallel lines. Because the two regions have no points in common, the system of inequalities has no solution.
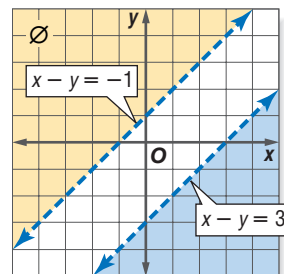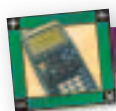
CONTENTS

You can use a TI-83 Plus to solve systems of inequalities.

## Graphing Calculator Investigation
Graphing Systems of Inequalities

To graph the system $y \geq 4x - 3$ and $y \leq -2x + 9$ on a TI-83 Plus, select the SHADE feature in the DRAW menu. Enter the function that is the lower boundary of the region to be shaded, followed by the upper boundary. (Note that inequalities that have $>$ or $\geq$ are lower boundaries and inequalities that have $<$ or $\leq$ are upper boundaries.)

```
Shade(4X-3, -2X+9
)
```
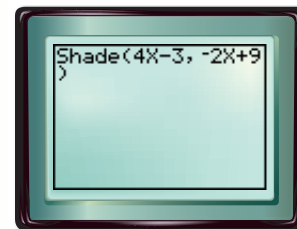
Think and Discuss

1. To graph the system $y \leq 3x + 1$ and $y \geq -2x - 5$ on a graphing calculator, which function should you enter first?
2. Use a graphing calculator to graph the system $y \leq 3x + 1$ and $y \geq -2x - 5$.
3. Explain how you could use a graphing calculator to graph the system $2x + y \geq 7$ and $x - 2y \geq 5$.
4. Use a graphing calculator to graph the system $2x + y \geq 7$ and $x - 2y \geq 5$.
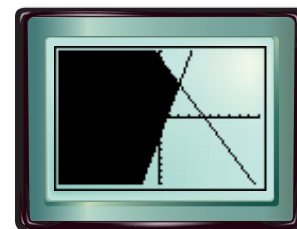
$[-10, 10]$ scl: 1 by $[-10, 10]$ scl: 1

REAL-WORLD PROBLEMS   In real-life problems involving systems of inequalities, sometimes only whole-number solutions make sense.

Example 3 Use a System of Inequalities to Solve a Problem

COLLEGE   **The middle 50% of first-year students attending Florida State University score between 520 and 620, inclusive, on the verbal portion of the SAT and between 530 and 630, inclusive, on the math portion. Graph the scores that a student would need to be in the middle 50% of FSU freshmen.**

Words   The verbal score is between 520 and 620, inclusive. The math score is between 530 and 630, inclusive.

Variables   If $v =$ the verbal score and $m =$ the math score, the following inequalities represent the middle 50% of Florida State University freshmen.
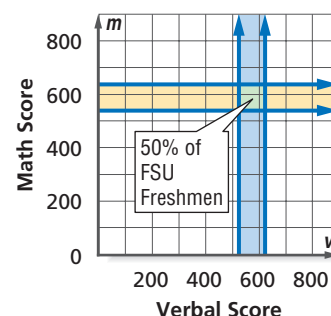
Inequalities

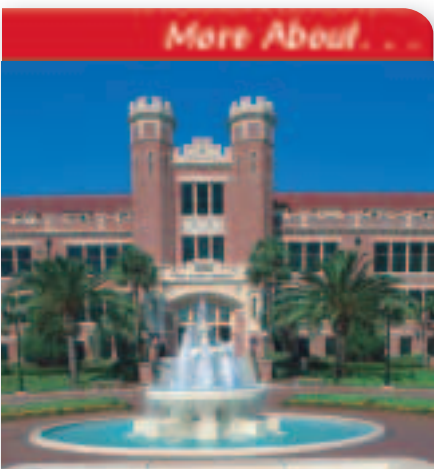The verbal score is between 520 and 620, inclusive.

$$520 \leq v \leq 620$$

The math score is between 530 and 630, inclusive.

$$530 \leq m \leq 630$$

The solution is the set of all ordered pairs whose graphs are in the intersection of the graphs of these inequalities. However, since SAT scores are whole numbers, only whole-number solutions make sense in this problem.

www.algebra1.com/extra_examples

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Example **4** **Use a System of Inequalities**

AGRICULTURE To ensure a growing season of sufficient length, Mr. Hobson has at most 16 days left to plant his corn and soybean crops. He can plant corn at a rate of 250 acres per day and soybeans at a rate of 200 acres per day. If he has at most 3500 acres available, how many acres of each type of crop can he plant?

Let c = the number of days that corn will be planted and s = the number of days that soybeans will be planted. Since both c and s represent a number of days, neither can be a negative number. The following system of inequalities can be used to represent the conditions of this problem.

$c \geq 0$

$s \geq 0$

$c + s \leq 16$

$250c + 200s \leq 3500$

The solution is the set of all ordered pairs whose graphs are in the intersection of the graphs of these inequalities. This region is shown in green at the right. Only the portion of the region in the first quadrant is used since $c \geq 0$ and $s \geq 0$.
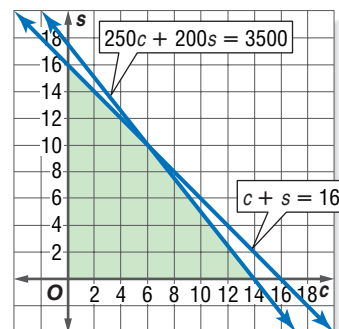
Any point in this region is a possible solution. For example, since (7, 8) is a point in the region, Mr. Hobson could plant corn for 7 days and soybeans for 8 days. In this case, he would use 15 days to plant 250(7) or 1750 acres of corn and 200(8) or 1600 acres of soybeans.
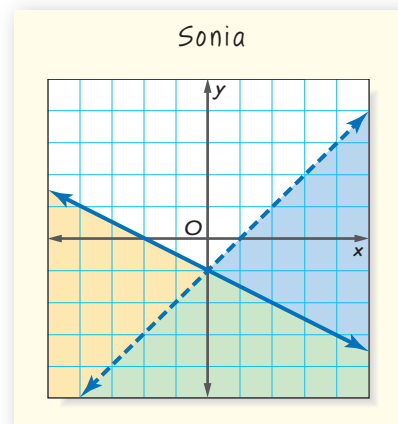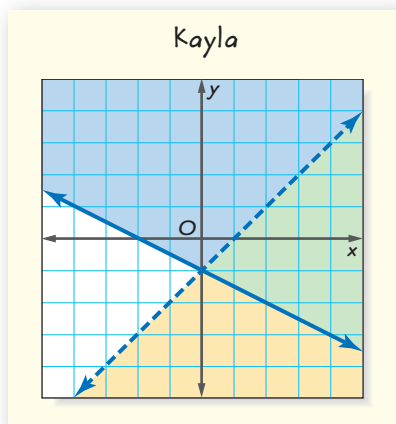
Check for Understanding

Concept Check 1. **OPEN ENDED** Draw the graph of a system of inequalities that has no solution.

2. **Determine** which of the following ordered pairs represent a solution of the system of inequalities graphed at the right.

   a. (3, 1)　　　　　　**b.** (−1, −3)

   c. (2, 3)　　　　　　**d.** (4, −2)

   e. (3, −2)　　　　　 **f.** (1, 4)
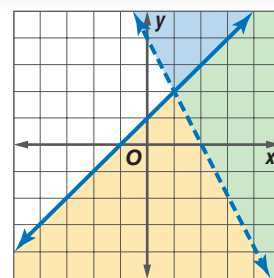
3. **FIND THE ERROR** Kayla and Sonia are solving the system of inequalities $x + 2y \geq -2$ and $x - y > 1$.

Kayla

Sonia

Who is correct? Explain your reasoning.

CONTENTS

Solve each system of inequalities by graphing.

4. $x > 5$
$y \leq 4$

5. $y > 3$
$y > -x + 4$

6. $y \leq -x + 3$
$y \leq x + 3$

7. $2x + y \geq 4$
$y \leq -2x - 1$

8. $2y + x < 6$
$3x - y > 4$

9. $x - 2y \leq 2$
$3x + 4y \leq 12$
$x \geq 0$

HEALTH **For Exercises 10 and 11, use the following information.**
Natasha walks and jogs at least 3 miles every day. Natasha walks 4 miles per hour and jogs 8 miles per hour. She only has a half-hour to exercise.

10. Draw a graph of the possible amounts of time she can spend walking and jogging.

11. List three possible solutions.

Practice and Apply

Solve each system of inequalities by graphing.

12. $y < 0$
$x \geq 0$

13. $x > -4$
$y \leq -1$

14. $y \geq -2$
$y - x < 1$

15. $x \geq 2$
$y + x \leq 5$

16. $x \leq 3$
$x + y > 2$

17. $y \geq 2x + 1$
$y \leq -x + 1$

18. $y < 2x + 1$
$y \geq -x + 3$

19. $y - x < 1$
$y - x > 3$

20. $y - x < 3$
$y - x \geq 2$

21. $2x + y \leq 4$
$3x - y \geq 6$

22. $3x - 4y < 1$
$x + 2y \leq 7$

23. $x + y > 4$
$-2x + 3y < -12$

24. $2x + y \geq -4$
$-5x + 2y < 1$

25. $y \leq x + 3$
$2x - 7y \leq 4$
$3x + 2y \leq 6$

26. $x < 2$
$4y > x$
$2x - y > -9$
$x + 3y < 9$

Write a system of inequalities for each graph.

27.

28.
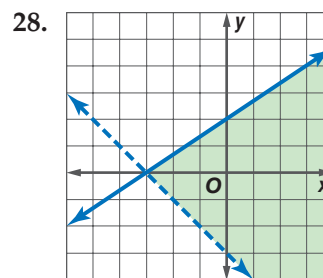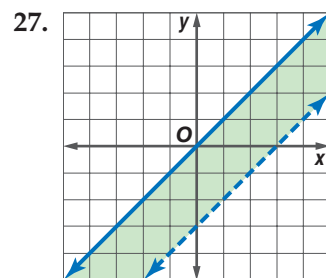
ART **For Exercises 29 and 30, use the following information.**
A painter has exactly 32 units of yellow dye and 54 units of blue dye. She plans to mix the dyes to make two shades of green. Each gallon of the lighter shade of green requires 4 units of yellow dye and 1 unit of blue dye. Each gallon of the darker shade of green requires 1 unit of yellow dye and 6 units of blue dye.

29. Make a graph showing the numbers of gallons of the two greens she can make.

30. List three possible solutions.

31. **HEALTH** The LDL or "bad" cholesterol of a teenager should be less than 110. The HDL or "good" cholesterol of a teenager should be between 35 and 59. Make a graph showing appropriate levels of cholesterol for a teenager.

32. **CRITICAL THINKING** Write a system of inequalities equivalent to $|x| \leq 4$.

MANUFACTURING For Exercises 33 and 34, use the following information.
The Natural Wood Company has machines that sand and varnish desks and tables. The table gives the time requirements of the machines.

Machine	Hours per Desk	Hours per Table	Total Hours Available Each Week
Sanding	2	1.5	31
Varnishing	1.5	1	22

33. Make a graph showing the number of desks and the number of tables that can be made in a week.

34. List three possible solutions.

35. WRITING IN MATH   Answer the question that was posed at the beginning of the lesson.

How can you use a system of inequalities to plan a sensible diet?

Include the following in your answer:
- two appropriate Calorie and fat intakes for a day, and
- the system of inequalities that is represented by the graph.

Graphing Calculator

GRAPHING SYSTEMS OF INEQUALITIES   Use a graphing calculator to solve each system of inequalities. Sketch the results.

36. $y \leq x + 9$
$y \geq -x - 4$

37. $y \leq 2x + 10$
$y \geq 7x + 15$

38. $3x - y \leq 6$
$x - y \geq -1$

Standardized Test Practice
Ⓐ Ⓑ Ⓒ Ⓓ

39. Which ordered pair does *not* satisfy the system $x + 2y > 5$ and $3x - y < -2$?
Ⓐ $(-3, 7)$　　　Ⓑ $(0, 5)$　　　Ⓒ $(-1, 4)$　　　Ⓓ $(0, 2.5)$

40. Which system of inequalities is represented by the graph?
Ⓐ $y \leq 2x + 2$
$y > -x - 1$
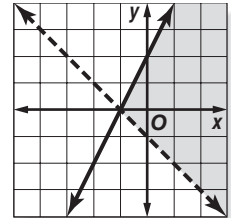
Ⓑ $y \geq 2x + 2$
$y < -x - 1$

Ⓒ $y < 2x + 2$
$y \leq -x - 1$

Ⓓ $y > 2x + 2$
$y \leq -x - 1$

Maintain Your Skills

Mixed Review   Use elimination to solve each system of equations.　*(Lessons 7-3 and 7-4)*

41. $2x + 3y = 1$
$4x - 5y = 13$

42. $5x - 2y = -3$
$3x + 6y = -9$

43. $-3x + 2y = 12$
$2x - 3y = -13$

44. $6x - 2y = 4$
$5x - 3y = -2$

45. $2x + 5y = 13$
$3x - 5y = -18$

46. $3x - y = 6$
$3x + 2y = 15$

Write an equation of the line that passes through each point with the given slope.
(Lesson 5-4)

47. $(4, -1)$, $m = 2$

48. $(1, 0)$, $m = -6$

49. $(5, -2)$, $m = \frac{1}{3}$

Web Quest   **Internet Project**

The Spirit of the Games

It's time to complete your project. Use the information and data you have gathered about the Olympics to prepare a portfolio or Web page. Be sure to include graphs and/or tables in your project.

www.algebra1.com/webquest

◄ **CONTENTS** ►

Vocabulary and Concept Check

| consistent (p. 369) | elimination (p. 382) | independent (p. 369) | system of equations (p. 369) |
| dependent (p. 369) | inconsistent (p. 369) | substitution (p. 376) | system of inequalities (p. 394) |

Choose the correct term to complete each statement.

1. If a system of equations has exactly one solution, it is *(dependent, independent)*.

2. If the graph of a system of equations is parallel lines, the system is *(consistent, inconsistent)*.

3. A system of equations that has infinitely many solutions is *(dependent, independent)*.

4. If the graphs of the equations in a system have the same slope and different y-intercepts, the graph of the system is a pair of *(intersecting lines, parallel lines)*.

5. If the graphs of the equations in a system have the same slope and y-intercept(s), the system has *(exactly one, infinitely many)* solution(s).

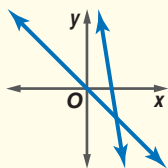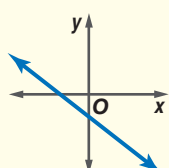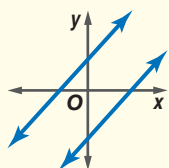6. The solution of a system of equations is $(3, -5)$. The system is *(consistent , inconsistent)*.

Lesson-by-Lesson Review

### 7-1 Graphing Systems of Inequalities

Concept Summary

	Intersecting Lines	Same Line	Parallel Lines
Graph of a System			
Number of Solutions	exactly one solution	infinitely many	no solutions
Terminology	consistent and independent	consistent and dependent	inconsistent

Example   **Graph the system of equations. Then determine whether the system has *no* solution, *one* solution, or *infinitely many* solutions. If the system has one solution, name it.**

$3x + y = -4$
$6x + 2y = -8$
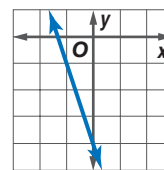
When the lines are graphed, they coincide. There are infinitely many solutions.

Exercises   Graph each system of equations. Then determine whether the system of equations has *one* solution, *no* solution, or *infinitely many* solutions. If the system has one solution, name it.   *See Example 2 on page 370.*

7. $x - y = 9$
   $x + y = 11$

8. $9x + 2 = 3y$
   $y - 3x = 8$

9. $2x - 3y = 4$
   $6y = 4x - 8$

10. $3x - y = 8$
    $3x = 4 - y$

7-2 *Substitution*

Concept Summary

- In a system of equations, solve one equation for a variable, and then substitute that expression into the second equation to solve.

Example **Use substitution to solve the system of equations.**

$y = x - 1$
$4x - y = 19$

Since $y = x - 1$, substitute $x - 1$ for y in the second equation.

$$4x - y = 19 \quad \text{Second equation}$$
$$4x - (x - 1) = 19 \quad y = x - 1$$
$$4x - x + 1 = 19 \quad \text{Distributive Property}$$
$$3x + 1 = 19 \quad \text{Combine like terms.}$$
$$3x = 18 \quad \text{Subtract 1 from each side.}$$
$$x = 6 \quad \text{Divide each side by 3.}$$

Use $y = x - 1$ to find the value of y.

$$y = x - 1 \quad \text{First equation}$$
$$y = 6 - 1 \quad x = 6$$
$$y = 5 \quad \text{The solution is (6, 5).}$$

Exercises Use substitution to solve each system of equations. If the system does *not* have exactly one solution, state whether it has *no* solutions or *infinitely many* solutions. *See Examples 1–3 on pages 377 and 378.*

11. $2m + n = 1$
  $m - n = 8$

12. $x = 3 - 2y$
  $2x + 4y = 6$

13. $3x - y = 1$
  $2x + 4y = 3$

14. $0.6m - 0.2n = 0.9$
  $n = 4.5 - 3m$

7-3 *Elimination Using Addition and Subtraction*

Concept Summary

- Sometimes adding or subtracting two equations will eliminate one variable.

Example **Use elimination to solve the system of equations.**

$2m - n = 4$
$m + n = 2$

You can eliminate the n terms by adding the equations.

$$2m - n = 4 \quad \text{Write the equations in column form and add.}$$
$$\underline{(+) \; m + n = 2}$$
$$3m \quad\quad = 6 \quad \text{Notice the variable } n \text{ is eliminated.}$$
$$m = 2 \quad \text{Divide each side by 3.}$$

Now substitute 2 for m in either equation to find n.

$$m + n = 2 \qquad \text{Second equation}$$
$$2 + n = 2 \qquad m = 2$$
$$2 + n - 2 = 2 - 2 \quad \text{Subtract 2 from each side.}$$
$$n = 0 \qquad \text{Simplify.}$$

The solution is (2, 0).

Exercises   Use elimination to solve each system of equations.
See Examples 1–3 on pages 382 and 383.

15. $x + 2y = 6$
$\quad\; x - 3y = -4$

16. $2m - n = 5$
$\quad\;\; 2m + n = 3$

17. $3x - y = 11$
$\quad\;\; x + y = 5$

18. $3x + 1 = -7y$
$\quad\;\; 6x + 7y = 0$

7-4 Elimination Using Multiplication

Concept Summary

- Multiplying one equation by a number or multiplying each equation by a different number is a strategy that can be used to solve a system of equations by elimination.
- There are five methods for solving systems of equations.

Method	The Best Time to Use
Graphing	to estimate the solution, since graphing usually does not give an exact solution
Substitution	if one of the variables in either equation has a coefficient of 1 or -1
Elimination Using Addition	if one of the variables has opposite coefficients in the two equations
Elimination Using Subtraction	if one of the variables has the same coefficient in the two equations
Elimination Using Multiplication	if none of the coefficients are 1 or -1 and neither of the variables can be eliminated by simply adding or subtracting the equations

Example   Use elimination to solve the system of equations.

$x + 2y = 8$
$3x + y = 1.5$

Multiply the second equation by -2 so the coefficients of the y terms are additive inverses. Then add the equations.

$x + 2y = 8$
$3x + y = 1.5$ **Multiply by -2.**

$$\begin{aligned} x + 2y &= \;\;\;8 \\ (+) -6x - 2y &= -3 \\ \hline -5x &= \;\;\;5 \end{aligned} \quad \text{Add the equations.}$$

$$\frac{-5x}{-5} = \frac{5}{-5} \quad \text{Divide each side by } -5.$$
$$x = -1 \quad \text{Simplify.}$$

(continued on the next page)

Chapter

7   For More ....

• Extra Practice, see pages 835–836.
• Mixed Problem Solving, see page 859.

$$x + 2y = 8 \qquad \text{First equation}$$
$$-1 + 2y = 8 \qquad x = -1$$
$$-1 + 2y + 1 = 8 + 1 \qquad \text{Add 1 to each side.}$$
$$2y = 9 \qquad \text{Simplify.}$$
$$\frac{2y}{2} = \frac{9}{2} \qquad \text{Divide each side by 2.}$$
$$y = 4.5 \qquad \text{Simplify.}$$

The solution is $(-1, 4.5)$.

Exercises   Use elimination to solve each system of equations.
See Examples 1 and 2 on pages 387 and 388.

19. $x - 5y = 0$
      $2x - 3y = 7$

20. $x - 2y = 5$
      $3x - 5y = 8$

21. $2x + 3y = 8$
      $x - y = 2$

22. $-5x + 8y = 21$
      $10x + 3y = 15$

Determine the best method to solve each system of equations. Then solve the system.   *See Example 3 on page 389.*

23. $y = 2x$
      $x + 2y = 8$

24. $9x + 8y = 7$
      $18x - 15y = 14$

25. $3x + 5y = 2x$
      $x + 3y = y$

26. $2x + y = 3x - 15$
      $x + 5 = 4y + 2x$

7-5 *Graphing Systems of Inequalities*

See pages 394–398.

Concept Summary

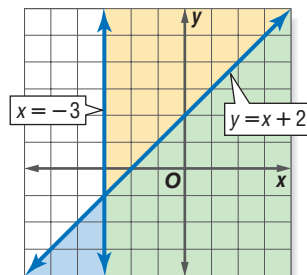• Graph each inequality on a coordinate plane to determine the intersection of the graphs.

Example   **Solve the system of inequalities.**

$x \geq -3$
$y \leq x + 2$

The solution includes the ordered pairs in the intersection of the graphs $x \geq -3$ and $y \leq x + 2$. This region is shaded in green. The graphs of $x \geq -3$ and $y \leq x + 2$ are boundaries of this region.

Exercises   Solve each system of inequalities by graphing.
See Examples 1 and 2 on page 394.

27. $y < 3x$
      $x + 2y \geq -21$

28. $y > -x - 1$
      $y \leq 2x + 1$

29. $2x + y < 9$
      $x + 11y < -6$

30. $x \geq 1$
      $y + x \leq 3$

CONTENTS

Vocabulary and Concepts

Choose the letter that best matches each description.

1. a system of equations with two parallel lines
2. a system of equations with at least one ordered pair that satisfies both equations
3. a system of equations may be solved using this method

   a. consistent
   b. elimination
   c. inconsistent

Skills and Applications

Graph each system of equations. Then determine whether the system has
no **solution,** *one* **solution, or** *infinitely many* **solutions. If the system has one solution, name it.**

4. $y = x + 2$
   $y = 2x + 7$

5. $x + 2y = 11$
   $x = 14 - 2y$

6. $3x + y = 5$
   $2y - 10 = -6x$

Use substitution or elimination to solve each system of equations.

7. $2x + 5y = 16$
   $5x - 2y = 11$

8. $y + 2x = -1$
   $y - 4 = -2x$

9. $2x + y = -4$
   $5x + 3y = -6$

10. $y = 7 - x$
    $x - y = -3$

11. $x = 2y - 7$
    $y - 3x = -9$

12. $x + y = 10$
    $x - y = 2$

13. $3x - y = 11$
    $x + 2y = -36$

14. $3x + y = 10$
    $3x - 2y = 16$

15. $5x - 3y = 12$
    $-2x + 3y = -3$

16. $2x + 5y = 12$
    $x - 6y = -11$

17. $x + y = 6$
    $3x - 3y = 13$

18. $3x + \frac{1}{3}y = 10$

    $2x - \frac{5}{3}y = 35$

19. **NUMBER THEORY** The units digit of a two-digit number exceeds twice the tens digit by 1. Find the number if the sum of its digits is 10.

20. **GEOMETRY** The difference between the length and width of a rectangle is 7 centimeters. Find the dimensions of the rectangle if its perimeter is 50 centimeters.

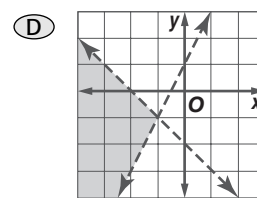**Solve each system of inequalities by graphing.**
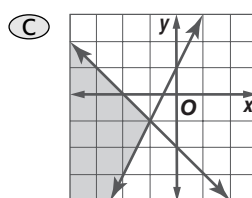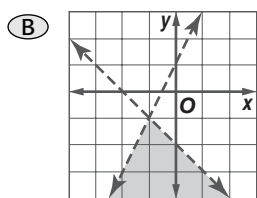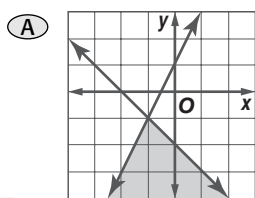
21. $y > -4$
    $y < -1$

22. $y \le 3$
    $y > -x + 2$

23. $x \le 2y$
    $2x + 3y \le 7$

24. **FINANCE** Last year, Jodi invested \$10,000, part at 6% annual interest and the rest at 8% annual interest. If she received \$760 in interest at the end of the year, how much did she invest at each rate?

25. **STANDARDIZED TEST PRACTICE** Which graph represents the system of inequalities $y > 2x + 1$ and $y < -x - 2$?

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. What is the solution of $4x - 2(x - 2) - 8 = 0$? (Lesson 3-4)

   (A) -2 (B) 2

   (C) 5 (D) 6

2. Noah paid $17.11 for a CD, including tax. If the tax rate is 7%, then what was the price of the CD before tax? (Lesson 3-5)

   (A) $10.06   (B) $11.98

   (C) $15.99   (D) $17.04

3. What is the range of $f(x) = 2x - 3$ when the domain is $\{3, 4, 5\}$? (Lesson 4-3)

   (A) $\{0, 1, 2\}$ (B) $\{3, 5, 7\}$

   (C) $\{6, 8, 10\}$ (D) $\{9, 11, 13\}$

4. Jolene kept a log of the numbers of birds that visited a birdfeeder over periods of several hours. In the table below, she recorded the number of hours she watched and the cumulative number of birds that she saw each session. Which equation best represents the data set shown in the table? (Lesson 4-8)

Number of hours, x	1	3	4	6
Number of birds, y	6	14	18	26

   (A) $y = x + 5$ (B) $y = 3x + 3$

   (C) $y = 3x + 5$ (D) $y = 4x + 2$

5. Which equation describes the graph? (Lesson 5-3)

   (A) $3y - 4x = -12$

   (B) $4y + 3x = -16$

   (C) $3y + 4x = -12$

   (D) $3y + 4x = -9$
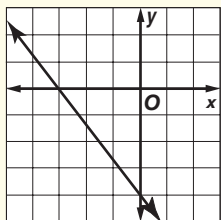
6. Which equation represents a line parallel to the line given by $y - 3x = 6$? (Lesson 5-6)

   (A) $y = -3x + 4$ (B) $y = 3x - 2$

   (C) $y = \frac{1}{3}x + 6$ (D) $y = -\frac{1}{3}x + 4$

7. Tamika has $185 in her bank account. She needs to deposit enough money so that she can withdraw $230 for her car payment and still have at least $200 left in the account. Which inequality describes d, the amount she needs to deposit? (Lesson 6-1)

   (A) $d(185 - 230) \geq 200$

   (B) $185 - 230d \geq 200$

   (C) $185 + 230 + d \geq 200$

   (D) $185 + d - 230 \geq 200$

8. The perimeter of a rectangular garden is 68 feet. The length of the garden is 4 more than twice the width. Which system of equations will determine the length ℓ and the width w of the garden? (Lesson 7-2)

   (A) $2\ell + 2w = 68$ (B) $2\ell + 2w = 68$
   $\ell = 4 - 2w$ $\ell = 2w + 4$

   (C) $2 + 2w = 68$ (D) $2\ell + 2w = 68$
   $2\ell - w = 4$ $w = 2\ell + 4$

9. Ernesto spent a total of $64 for a pair of jeans and a shirt. The jeans cost $6 more than the shirt. What was the cost of the jeans? (Lesson 7-2)

   (A) $26   (B) $29

   (C) $35   (D) $58

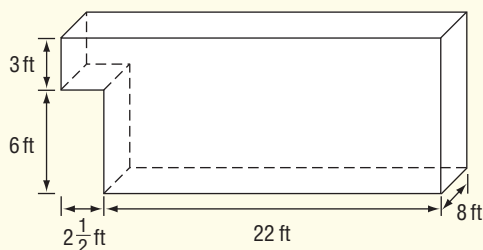10. What is the value of y in the following system of equations? (Lesson 7-3)

    $3x + 4y = 8$
    $3x + 2y = -2$

    (A) -2 (B) 4

    (C) 5 (D) 6

**CONTENTS**

Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

11. The diagram shows the dimensions of the cargo area of a delivery truck.

What is the maximum volume of cargo, in cubic feet, that can fit in the truck? (Prerequisite Skill)

12. The perimeter of the square below is 204 feet. What is the value of x? (Lesson 3-4)

$(5x + 6)$ ft
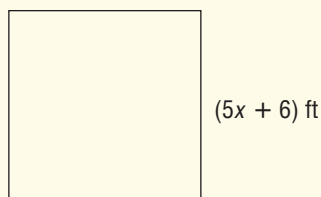
13. What is the x-intercept of the graph of $4x + 3y = 12$? (Lesson 4-5)

14. What are the slope and the y-intercept of the graph of the equation $4x - 2y = 5$? (Lesson 5-4)

**15.** Solve the following system of equations. (Lesson 7-2)

$$5x - y = 10$$
$$7x - 2y = 11$$

Test-Taking Tip

Ⓐ Ⓑ Ⓒ Ⓓ

Questions 11 and 12
To prepare for a standardized test, make flash cards of key mathematical terms, such as "perimeter" and "volume." Use the glossary of your textbook to determine the important terms and their correct definitions.

16. Two times one number minus three times another number is -11. The sum of the first number and three times the second number is 8. What are the two numbers? (Lesson 7-4)

17. Write a system of inequalities for the graph. (Lesson 7-5)
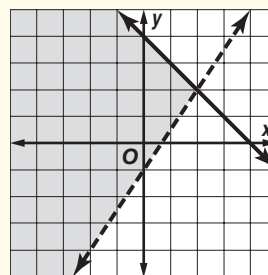
Part 3 Extended Response

Record your answers on a sheet of paper. Show your work.

18. During a work-out session, Mark either ran at a speed of 7 miles per hour or walked at a speed of 3 miles per hour. He completed 20 miles in 3 hours. (Lesson 7-1)

a. Let r represent the number of miles Mark ran and w represent the number of miles Mark walked. Write a system of equations that represents the situation.

b. Solve the system of equations to find how much time Mark spent running.

19. The manager of a movie theater found that Saturday's sales were $3675. He knew that a total of 650 tickets were sold Saturday. Adult tickets cost $7.50, and children's tickets cost $4.50. (Lesson 7-2)

a. Write equations to represent the number of tickets sold and the amount of money collected.

b. How many of each kind of ticket were sold? Show your work. Include all steps.