Chapter 12

Rational Expressions and Equations

What You'll Learn

- **Lesson 12-1** Solve problems involving inverse variation.
- **Lessons 12-2, 12-3, 12-4, 12-6, and 12-7** Simplify, add, subtract, multiply, and divide rational expressions.
- **Lesson 12-5** Divide polynomials.
- **Lesson 12-8** Simplify mixed expressions and complex fractions.
- **Lesson 12-9** Solve rational equations.

Key Vocabulary

- inverse variation (p. 642)
- rational expression (p. 648)
- excluded values (p. 648)
- complex fraction (p. 684)
- extraneous solutions (p. 693)

Why It's Important

Performing operations on rational expressions is an important part of working with equations. For example, knowing how to divide rational expressions and polynomials can help you simplify complex expressions. You can use this process to determine the number of flags that a marching band can make from a given amount of material. *You will divide rational expressions and polynomials in Lessons 12-4 and 12-5.*

Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 12.

For Lesson 12-1 — Solve Proportions

Solve each proportion. *(For review, see Lesson 3-6.)*

1. $\frac{y}{9} = \frac{-7}{16}$ **2.** $\frac{4}{x} = \frac{2}{10}$ **3.** $\frac{3}{15} = \frac{1}{n}$ **4.** $\frac{x}{8} = \frac{0.21}{2}$

5. $\frac{1.1}{0.6} = \frac{8.47}{n}$ **6.** $\frac{9}{8} = \frac{y}{6}$ **7.** $\frac{2.7}{3.6} = \frac{8.1}{a}$ **8.** $\frac{0.19}{2} = \frac{x}{24}$

For Lesson 12-2 — Greatest Common Factor

Find the greatest common factor for each pair of monomials. *(For review, see Lesson 9-1.)*

9. 30, 42 **10.** $60r^2, 45r^3$ **11.** $32m^2n^3, 12m^2n$ **12.** $14a^2b^2, 18a^3b$

For Lessons 12-3 through 12-8 — Factor Polynomials

Factor each polynomial. *(For review, see Lessons 9-2 and 9-3.)*

13. $3c^2d - 6c^2d^2$ **14.** $6mn + 15m^2$ **15.** $x^2 + 11x + 24$

16. $x^2 + 4x - 45$ **17.** $2x^2 + x - 21$ **18.** $3x^2 - 12x + 9$

For Lesson 12-9 — Solve Equations

Solve each equation. *(For review, see Lessons 3-4, 3-5, and 9-3.)*

19. $3x - 2 = -5$ **20.** $5x - 8 - 3x = (2x - 3)$ **21.** $\frac{m+9}{5} = \frac{m-10}{11}$ **22.** $\frac{5+x}{x-3} = \frac{14}{10}$

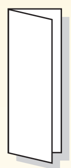
23. $\frac{7n-1}{6} = 5$ **24.** $\frac{4t-5}{-9} = 7$ **25.** $x^2 - x - 56 = 0$ **26.** $x^2 + 2x = 8$

FOLDABLES™ Study Organizer

Rational Expressions and Equations Make this Foldable to help you organize your notes. Begin with a sheet of plain $8\frac{1}{2}$" by 11" paper.

Step 1 **Fold in Half**

Fold in half lengthwise.

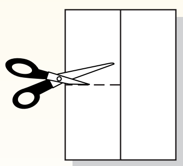


Step 2 **Fold Again**

Fold the top to the bottom.

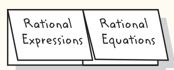
Step 3 **Cut**

Open. Cut along the second fold to the center to make two tabs.



Step 4 **Label**

Label each tab as shown.



Reading and Writing As you read and study the chapter, write notes and examples under each tab. Use this Foldable to apply what you learn about simplifying rational expressions and solving rational equations.

12-1 Inverse Variation

What You'll Learn

- Graph inverse variations.
- Solve problems involving inverse variation.

How is inverse variation related to the gears on a bicycle?

The number of revolutions of the pedals made when riding a bicycle at a constant speed varies inversely as the gear ratio of the bicycle. In other words, as the gear ratio *decreases*, the revolutions per minute (rpm) *increase*. This is why when pedaling up a hill, shifting to a lower gear allows you to pedal with less difficulty.

Pedaling Rates to Maintain Speed of 10 mph

Gear Ratio	Rate
117.8	89.6
108.0	97.8
92.6	114.0
76.2	138.6
61.7	171.2
49.8	212.0
40.5	260.7

Vocabulary

- inverse variation
- product rule

Study Tip

Look Back
To review **direct variation**, see Lesson 5-2.

GRAPH INVERSE VARIATION Recall that some situations in which y increases as x increases are *direct variations*. If y varies directly as x, we can represent this relationship with an equation of the form $y = kx$, where $k \neq 0$. However, in the application above, as one value increases the other value decreases. When the product of two values remains constant, the relationship forms an **inverse variation**. We say *y varies inversely as x* or *y is inversely proportional to x*.

Key Concept — *Inverse Variation*

y varies inversely as x if there is some nonzero constant k such that $xy = k$.

Example 1 Graph an Inverse Variation

DRIVING **The time t it takes to travel a certain distance varies inversely as the rate r at which you travel. The equation $rt = 250$ can be used to represent a person driving 250 miles. Complete the table and draw a graph of the relation.**

r (mph)	5	10	15	20	25	30	35	40	45	50
t (hours)										

Solve for $r = 5$.

$rt = 250$ Original equation

$5t = 250$ Replace r with 5.

$t = \frac{250}{5}$ Divide each side by 5.

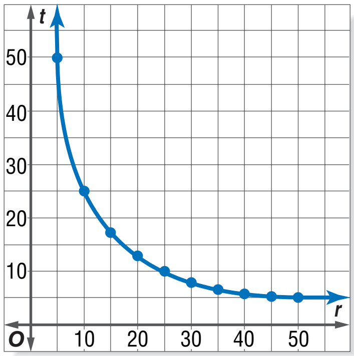
$t = 50$ Simplify.

Solve the equation for the other values of r.

r (mph)	5	10	15	20	25	30	35	40	45	50
t (hours)	50	25	16.67	12.5	10	8.33	7.14	6.25	5.56	5

Next, graph the ordered pairs: (5, 50), (10, 25), (15, 16.67), (20, 12.5), (25, 10), (30, 8.33), (35, 7.14), (40, 6.25), (45, 5.56), and (50, 5).

The graph of an inverse variation is not a straight line like the graph of a direct variation. As the rate r increases, the time t that it takes to travel the same distance decreases.



Graphs of inverse variations can also be drawn using negative values of x.

Example 2 Graph an Inverse Variation

Graph an inverse variation in which y varies inversely as x and $y = 15$ when $x = 6$.

Solve for k.

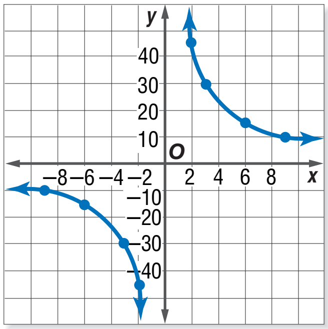
$xy = k$ Inverse variation equation

$(6)(15) = k$ $x = 6, y = 15$

$90 = k$ The constant of variation is 90.

Choose values for x and y whose product is 90.

x	y
−9	−10
−6	−15
−3	−30
−2	−45
0	undefined
2	45
3	30
6	15
9	10



USE INVERSE VARIATION If (x_1, y_1) and (x_2, y_2) are solutions of an inverse variation, then $x_1y_1 = k$ and $x_2y_2 = k$.

$x_1y_1 = k$ and $x_2y_2 = k$

$x_1y_1 = x_2y_2$ Substitute x_2y_2 for k.

The equation $x_1y_1 = x_2y_2$ is called the **product rule** for inverse variations. You can use this equation to form a proportion.

$x_1y_1 = x_2y_2$ Product rule for inverse variations

$\frac{x_1y_1}{x_2y_1} = \frac{x_2y_2}{x_2y_1}$ Divide each side by x_2y_1.

$\frac{x_1}{x_2} = \frac{y_2}{y_1}$ Simplify.

Study Tip

Proportions

Notice that the proportion for inverse variations is different from the proportion for direct variation, $\frac{x_1}{x_2} = \frac{y_1}{y_2}$.

You can use the product rule or a proportion to solve inverse variation problems.

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Example 3 Solve for x

If y varies inversely as x and $y = 4$ when $x = 7$, find x when $y = 14$.

Let $x_1 = 7$, $y_1 = 4$, and $y_2 = 14$. Solve for x_2.

Method 1 Use the product rule.

$$x_1y_1 = x_2y_2 \quad \text{Product rule for inverse variations}$$
$$7 \cdot 4 = x_2 \cdot 14 \quad x_1 = 7,\ y_1 = 4, \text{ and } y_2 = 14$$
$$\frac{28}{14} = x_2 \quad \text{Divide each side by 14.}$$
$$2 = x_2 \quad \text{Simplify.}$$

Method 2 Use a proportion.

$$\frac{x_1}{x_2} = \frac{y_2}{y_1} \quad \text{Proportion for inverse variations}$$
$$\frac{7}{x_2} = \frac{14}{4} \quad x_1 = 7,\ y_1 = 4, \text{ and } y_2 = 14$$
$$28 = 14x_2 \quad \text{Cross multiply.}$$
$$2 = x_2 \quad \text{Divide each side by 14.}$$

Both methods show that $x = 2$ when $y = 14$.

Example 4 Solve for y

If y varies inversely as x and $y = -6$ when $x = 9$, find y when $x = 6$.

Use the product rule.

$$x_1y_1 = x_2y_2 \quad \text{Product rule for inverse variations}$$
$$9 \cdot (-6) = 6y_2 \quad x_1 = 9,\ y_1 = -6, \text{ and } x_2 = 6$$
$$\frac{-54}{6} = y_2 \quad \text{Divide each side by 6.}$$
$$-9 = y_2 \quad \text{Simplify.}$$

Thus, $y = -9$ when $x = 6$.

Inverse variation is often used in real-world situations.

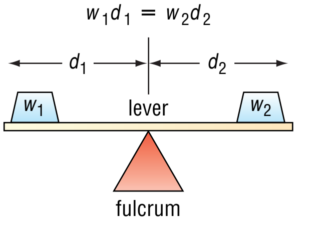
Study Tip

Levers

A lever is a bar with a pivot point called the *fulcrum*. For a lever to balance, the lesser weight must be positioned farther from the fulcrum.

Example 5 Use Inverse Variation to Solve a Problem

PHYSICAL SCIENCE **When two objects are balanced on a lever, their distances from the fulcrum are inversely proportional to their weights. In other words, the greater the weight, the less distance it should be from the fulcrum in order to maintain balance. If an 8-kilogram weight is placed 1.8 meters from the fulcrum, how far should a 12-kilogram weight be placed from the fulcrum in order to balance the lever?**



Let $w_1 = 8$, $d_1 = 1.8$, and $w_2 = 12$. Solve for d_2.

$$w_1d_1 = w_2d_2 \quad \text{Original equation}$$
$$8 \cdot 1.8 = 12d_2 \quad w_1 = 8,\ d_1 = 1.8, \text{ and } w_2 = 12$$
$$\frac{14.4}{12} = d_2 \quad \text{Divide each side by 12.}$$
$$1.2 = d_2 \quad \text{Simplify.}$$

The 12-kilogram weight should be placed 1.2 meters from the fulcrum.

Check for Understanding

Concept Check

1. **OPEN ENDED** **Write** an equation showing an inverse variation with a constant of 8.
2. **Compare and contrast** direct variation and indirect variation equations and graphs.
3. **Determine** which situation is an example of inverse variation. Explain.
   - **a.** Emily spends \$2 each day for snacks on her way home from school. The total amount she spends each week depends on the number of days school was in session.
   - **b.** A business donates \$200 to buy prizes for a school event. The number of prizes that can be purchased depends upon the price of each prize.

Guided Practice

Graph each variation if y varies inversely as x.

4. $y = 24$ when $x = 8$
5. $y = -6$ when $x = -2$

Write an inverse variation equation that relates x and y. Assume that y varies inversely as x. Then solve.

6. If $y = 12$ when $x = 6$, find y when $x = 8$.
7. If $y = -8$ when $x = -3$, find y when $x = 6$.
8. If $y = 2.7$ when $x = 8.1$, find x when $y = 5.4$.
9. If $x = \frac{1}{2}$ when $y = 16$, find x when $y = 32$.

Application

10. **MUSIC** The length of a violin string varies inversely as the frequency of its vibrations. A violin string 10 inches long vibrates at a frequency of 512 cycles per second. Find the frequency of an 8-inch string.

Practice and Apply

Homework Help

For Exercises	See Examples
11–16	1, 2
17–28	3, 4
29–37	5

Extra Practice
See page 846.

Graph each variation if y varies inversely as x.

11. $y = 24$ when $x = -8$
12. $y = 3$ when $x = 4$
13. $y = 5$ when $x = 15$
14. $y = -4$ when $x = -12$
15. $y = 9$ when $x = 8$
16. $y = 2.4$ when $x = 8.1$

Write an inverse variation equation that relates x and y. Assume that y varies inversely as x. Then solve.

17. If $y = 12$ when $x = 5$, find y when $x = 3$.
18. If $y = 7$ when $x = -2$, find y when $x = 7$.
19. If $y = 8.5$ when $x = -1$, find x when $y = -1$.
20. If $y = 8$ when $x = 1.55$, find x when $y = -0.62$.
21. If $y = 6.4$ when $x = 4.4$, find x when $y = 3.2$.
22. If $y = 1.6$ when $x = 0.5$, find x when $y = 3.2$.
23. If $y = 4$ when $x = 4$, find y when $x = 7$.
24. If $y = -6$ when $x = -2$, find y when $x = 5$.
25. Find the value of y when $x = 7$ if $y = 7$ when $x = \frac{2}{3}$.
26. Find the value of y when $x = 32$ if $y = 16$ when $x = \frac{1}{2}$.
27. If $x = 6.1$ when $y = 4.4$, find x when $y = 3.2$.
28. If $x = 0.5$ when $y = 2.5$, find x when $y = 20$.

29. **GEOMETRY** A rectangle is 36 inches wide and 20 inches long. How wide is a rectangle of equal area if its length is 90 inches?

30. **MUSIC** The pitch of a musical note varies inversely as its wavelength. If the tone has a pitch of 440 vibrations per second and a wavelength of 2.4 feet, find the pitch of a tone that has a wavelength of 1.6 feet.

31. **COMMUNITY SERVICE** Students at Roosevelt High School are collecting canned goods for a local food pantry. They plan to distribute flyers to homes in the community asking for donations. Last year, 12 students were able to distribute 1000 flyers in nine hours. How long would it take if 15 students hand out the same number of flyers this year?

TRAVEL **For Exercises 32 and 33, use the following information.**
The Zalinski family can drive the 220 miles to their cabin in 4 hours at 55 miles per hour. Son Jeff claims that they could save half an hour if they drove 65 miles per hour, the speed limit.

32. How long will it take the family if they drive 65 miles per hour?

33. How much time would be saved driving at 65 miles per hour?

More About...

Art

American sculptor Alexander Calder was the first artist to use mobiles as an art form.

Source: www.infoplease.com

CHEMISTRY **For Exercises 34–36, use the following information.**
Boyle's Law states that the volume of a gas V varies inversely with applied pressure P.

34. Write an equation to show this relationship.

35. Pressure on 60 cubic meters of a gas is raised from 1 atmosphere to 3 atmospheres. What new volume does the gas occupy?

36. A helium-filled balloon has a volume of 22 cubic meters at sea level where the air pressure is 1 atmosphere. The balloon is released and rises to a point where the air pressure is 0.8 atmosphere. What is the volume of the balloon at this height?

37. **ART** Anna is designing a mobile to suspend from a gallery ceiling. A chain is attached eight inches from the end of a bar that is 20 inches long. On the shorter end of the bar is a sculpture weighing 36 kilograms. She plans to place another piece of artwork on the other end of the bar. How much should the second piece of art weigh if she wants the bar to be balanced?

CRITICAL THINKING **For Exercises 38 and 39, assume that y varies inversely as x.**

38. If the value of x is doubled, what happens to the value of y?

39. If the value of y is tripled, what happens to the value of x?

40. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How is inverse variation related to the gears on a bicycle?

Include the following in your answer:
- an explanation of how shifting to a lower gear ratio affects speed and the pedaling rate on a certain bicycle if a rider is pedaling 73.4 revolutions per minute while traveling at 15 miles per hour, and
- an explanation why the gear ratio affects the pedaling speed of the rider.

Standardized Test Practice

41. Determine the constant of variation if y varies inversely as x and $y = 4.25$ when $x = -1.3$.

(A) -3.269 (B) -5.525 (C) -0.306 (D) -2.950

42. Identify the graph of $xy = k$ if $x = -2$ when $y = -4$.

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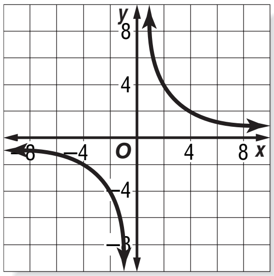

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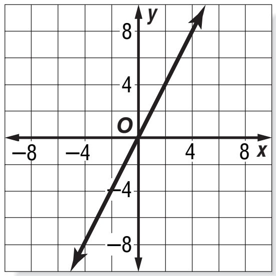

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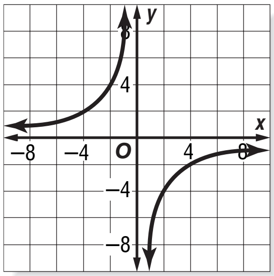

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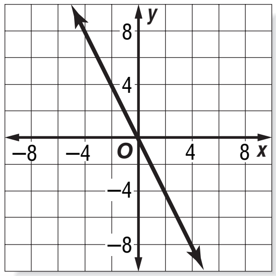

Maintain Your Skills

Mixed Review

For each triangle, find the measure of the indicated angle to the nearest degree. *(Lesson 11-7)*

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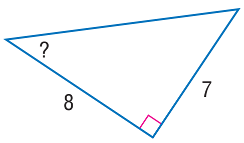

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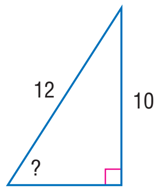

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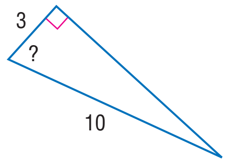


For each set of measures given, find the measures of the missing sides if $\triangle ABC \sim \triangle DEF$. *(Lesson 11-6)*

46. $a = 3, b = 10, c = 9, d = 12$

47. $b = 8, c = 4, d = 21, e = 28$

48. **MUSIC** Two musical notes played at the same time produce harmony. The closest harmony is produced by frequencies with the greatest GCF. A, C, and C sharp have frequencies of 220, 264, and 275, respectively. Which pair of these notes produce the closest harmony? *(Lesson 9-1)*

Solve each equation. *(Lesson 8-6)*

49. $7(2y - 7) = 5(4y + 1)$

50. $w(w + 2) = 2w(w - 3) + 16$

Solve each system of inequalities by graphing. *(Lesson 7-5)*

51. $y \leq 3x - 5$
$y > -x + 1$

52. $y \geq 2x + 3$
$2y \geq -5x - 14$

53. $x + y \leq 1$
$x - y \leq -3$
$y \geq 0$

54. $3x - 2y \geq -16$
$x + 4y < 4$
$5x - 8y < -8$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find the greatest common factor for each set of monomials.** *(To review **greatest common factors**, see Lesson 9-1.)*

55. 36, 15, 45

56. 48, 60, 84

57. 210, 330, 150

58. $17a, 34a^2$

59. $12xy^2, 18x^2y^3$

60. $12pr^2, 40p^4$

12-2 Rational Expressions

What You'll Learn

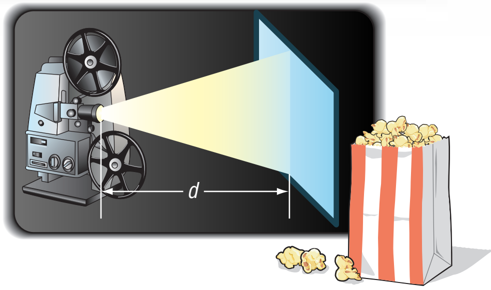
- Identify values excluded from the domain of a rational expression.
- Simplify rational expressions.

Vocabulary

- rational expression
- excluded values

How can a rational expression be used in a movie theater?

The intensity I of an image on a movie screen is inversely proportional to the square of the distance d between the projector and the screen. Recall from Lesson 12-1 that this can be represented by the equation $I = \frac{k}{d^2}$, where k is a constant.



EXCLUDED VALUES OF RATIONAL EXPRESSIONS The expression $\frac{k}{d^2}$ is an example of a rational expression. A **rational expression** is an algebraic fraction whose numerator and denominator are polynomials.

Because a rational expression involves division, the denominator may not have a value of zero. Any values of a variable that result in a denominator of zero must be excluded from the domain of that variable. These are called **excluded values** of the rational expression.

Example 1 One Excluded Value

State the excluded value of $\frac{5m + 3}{m - 6}$.

Exclude the values for which $m - 6 = 0$.

$m - 6 = 0$ The denominator cannot equal 0.

$m = 6$ Add 6 to each side.

Therefore, m cannot equal 6.

To determine the excluded values of a rational expression, you may be able to factor the denominator first.

Example 2 Multiple Excluded Values

State the excluded values of $\frac{x^2 - 5}{x^2 - 5x + 6}$.

Exclude the values for which $x^2 - 5x + 6 = 0$.

$x^2 - 5x + 6 = 0$ The denominator cannot equal zero.

$(x - 2)(x - 3) = 0$ Factor.

Use the Zero Product Property to solve for x.

$x - 2 = 0$ or $x - 3 = 0$

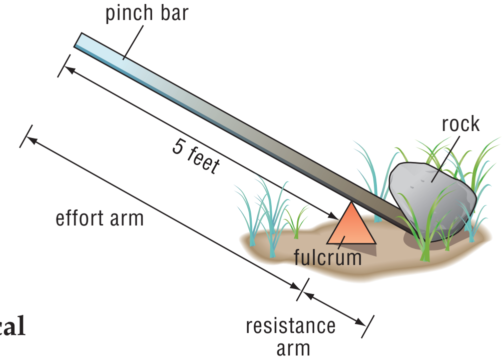
$x = 2$ $\qquad$ $x = 3$

Therefore, x cannot equal 2 or 3.

You can use rational expressions to solve real-world problems.

Example 3 Use Rational Expressions

LANDSCAPING **Kenyi is helping his parents landscape their yard and needs to move some large rocks. He plans to use a 6-foot bar as a lever. He positions it as shown at the right.**



a. The mechanical advantage of a lever is $\frac{L_E}{L_R}$, where L_E is the length of the effort arm and L_R is the length of the resistance arm. Calculate the mechanical advantage of the lever Kenyi is using.

Let b represent the length of the bar and e represent the length of the effort arm. Then $b - e$ represents the length of the resistance arm.

Use the expression for mechanical advantage to write an expression for the mechanical advantage in this situation.

$$\frac{L_E}{L_R} = \frac{e}{b-e} \quad L_E = e,\ L_R = b - e$$

$$= \frac{5}{6-5} \quad e = 5,\ b = 6$$

$$= 5 \quad \text{Simplify.}$$

The mechanical advantage is 5.

b. The force placed on the rock is the product of the mechanical advantage and the force applied to the end of the lever. If Kenyi can apply a force of 180 pounds, what is the greatest weight he can lift with the lever?

Since the mechanical advantage is 5, Kenyi can lift $5 \cdot 180$ or 900 pounds with this lever.

Career Choices

Landscape Architect

Landscape architects plan the location of structures, roads, and walkways as well as the arrangement of flowers, trees, and shrubs in a variety of settings.

Source: U.S. Bureau of Labor and Statistics

Online Research For information about a career as a landscape architect, visit: www.algebra1.com/careers

SIMPLIFY RATIONAL EXPRESSIONS Simplifying rational expressions is similar to simplifying fractions with numbers. To simplify a rational expression, you must eliminate any common factors of the numerator and denominator. To do this, use their greatest common factor (GCF). Remember that $\frac{ab}{ac} = \frac{a}{a} \cdot \frac{b}{c}$ and $\frac{a}{a} = 1$. So, $\frac{ab}{ac} = 1 \cdot \frac{b}{c}$ or $\frac{b}{c}$.

Example 4 Expression Involving Monomials

Simplify $\frac{-7a^2b^3}{21a^5b}$.

$$\frac{-7a^2b^3}{21a^5b} = \frac{(7a^2b)(-b^2)}{(7a^2b)(3a^3)} \quad \text{The GCF of the numerator and denominator is } 7a^2b.$$

$$= \frac{\cancel{(7a^2b)}^{1}(-b^2)}{\cancel{(7a^2b)}_{1}(3a^3)} \quad \text{Divide the numerator and denominator by } 7a^2b.$$

$$= \frac{-b^2}{3a^3} \quad \text{Simplify.}$$

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You can use the same procedure to simplify a rational expression in which the numerator and denominator are polynomials.

Study Tip

Simplest Form
When a rational expression is in simplest form, the numerator and denominator have no common factors other than 1 or −1.

Example 5 Expressions Involving Polynomials

Simplify $\frac{x^2 - 2x - 15}{x^2 - x - 12}$.

$\frac{x^2 - 2x - 15}{x^2 - x - 12} = \frac{(x + 3)(x - 5)}{(x + 3)(x - 4)}$ Factor.

$= \frac{\cancel{(x + 3)}^{1}(x - 5)}{\cancel{(x + 3)}_{1}(x - 4)}$ Divide the numerator and denominator by the GCF, $x + 3$.

$= \frac{x - 5}{x - 4}$ Simplify.

It is important to determine the excluded values of a rational expression using the original expression rather than the simplified expression.

Example 6 Excluded Values

Simplify $\frac{3x - 15}{x^2 - 7x + 10}$. State the excluded values of x.

$\frac{3x - 15}{x^2 - 7x + 10} = \frac{3(x - 5)}{(x - 2)(x - 5)}$ Factor.

$= \frac{3\cancel{(x - 5)}^{1}}{(x - 2)\cancel{(x - 5)}_{1}}$ Divide the numerator and denominator by the GCF, $x - 5$.

$= \frac{3}{x - 2}$ Simplify.

Exclude the values for which $x^2 - 7x + 10$ equals 0.

$x^2 - 7x + 10 = 0$ The denominator cannot equal zero.

$(x - 5)(x - 2) = 0$ Factor.

$x = 5$ or $x = 2$ Zero Product Property

CHECK Verify the excluded values by substituting them into the original expression.

$\frac{3x - 15}{x^2 - 7x + 10} = \frac{3(5) - 15}{5^2 - 7(5) + 10}$ $x = 5$

$= \frac{15 - 15}{25 - 35 + 10}$ Evaluate.

$= \cancel{\frac{0}{0}}$ Simplify.

$\frac{3x - 15}{x^2 - 7x + 10} = \frac{3(2) - 15}{2^2 - 7(2) + 10}$ $x = 2$

$= \frac{6 - 15}{4 - 14 + 10}$ Evaluate.

$= \cancel{\frac{-9}{0}}$ Simplify.

The expression is undefined when $x = 5$ and $x = 2$. Therefore, $x \neq 5$ and $x \neq 2$.

Check for Understanding

Concept Check

1. **Describe** how you would determine the values to be excluded from the expression $\frac{x + 3}{x^2 + 5x + 6}$.
2. **OPEN ENDED** Write a rational expression involving one variable for which the excluded values are -4 and -7.
3. **Explain** why -2 may not be the only excluded value of a rational expression that simplifies to $\frac{x - 3}{x + 2}$.

Guided Practice

State the excluded values for each rational expression.

4. $\frac{4a}{3 + a}$
5. $\frac{x^2 - 9}{2x + 6}$
6. $\frac{n + 5}{n^2 + n - 20}$

Simplify each expression. State the excluded values of the variables.

7. $\frac{56x^2y}{70x^3y^2}$
8. $\frac{x^2 - 49}{x + 7}$
9. $\frac{x + 4}{x^2 + 8x + 16}$
10. $\frac{x^2 - 2x - 3}{x^2 - 7x + 12}$
11. $\frac{a^2 + 4a - 12}{a^2 + 2a - 8}$
12. $\frac{2x^2 - x - 21}{2x^2 - 15x + 28}$
13. Simplify $\frac{b^2 - 3b - 4}{b^2 - 13b + 36}$. State the excluded values of b.

Application

AQUARIUMS **For Exercises 14 and 15, use the following information.**
Jenna has guppies in her aquarium. One week later, she adds four neon fish.

14. Write an expression that represents the fraction of neon fish in the aquarium.
15. Suppose that two months later the guppy population doubles, she still has four neons, and she buys 5 different tropical fish. Write an expression that shows the fraction of neons in the aquarium after the other fish have been added.

Practice and Apply

Homework Help

For Exercises	See Examples
16–23	1, 2
24–27	4, 6
28–41	5, 6
42–54	3

Extra Practice
See page 846.

State the excluded values for each rational expression.

16. $\frac{m + 3}{m - 2}$
17. $\frac{3b}{b + 5}$
18. $\frac{3n + 18}{n^2 - 36}$
19. $\frac{2x - 10}{x^2 - 25}$
20. $\frac{a^2 - 2a + 1}{a^2 + 2a - 3}$
21. $\frac{x^2 - 6x + 9}{x^2 + 2x - 15}$
22. $\frac{n^2 - 36}{n^2 + n - 30}$
23. $\frac{25 - x^2}{x^2 + 12x + 35}$

Simplify each expression. State the excluded values of the variables.

24. $\frac{35yz^2}{14y^2z}$
25. $\frac{14a^3b^2}{42ab^3}$
26. $\frac{64qr^2s}{16q^2rs}$
27. $\frac{9x^2yz}{24xyz^2}$
28. $\frac{7a^3b^2}{21a^2b + 49ab^3}$
29. $\frac{3m^2n^3}{36mn^3 - 12m^2n^2}$
30. $\frac{x^2 + x - 20}{x + 5}$
31. $\frac{z^2 + 10z + 16}{z + 2}$
32. $\frac{4x + 8}{x^2 + 6x + 8}$
33. $\frac{2y - 4}{y^2 + 3y - 10}$
34. $\frac{m^2 - 36}{m^2 - 5m - 6}$
35. $\frac{a^2 - 9}{a^2 + 6a - 27}$
36. $\frac{x^2 + x - 2}{x^2 - 3x + 2}$
37. $\frac{b^2 + 2b - 8}{b^2 - 20b + 64}$
38. $\frac{x^2 - x - 20}{x^3 + 10x^2 + 24x}$
39. $\frac{n^2 - 8n + 12}{n^3 - 12n^2 + 36n}$
40. $\frac{4x^2 - 6x - 4}{2x^2 - 8x + 8}$
41. $\frac{3m^2 + 9m + 6}{4m^2 + 12m + 8}$

COOKING **For Exercises 42–45, use the following information.**

The formula $t = \frac{40(25 + 1.85a)}{50 - 1.85a}$ relates the time t in minutes that it takes to cook an average-size potato in an oven that is at an altitude of a thousands of feet.

42. What is the value of a for an altitude of 4500 feet?

43. Calculate the time it takes to cook a potato at an altitude of 3500 feet.

44. About how long will it take to cook a potato at an altitude of 7000 feet?

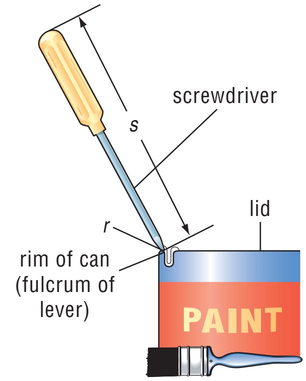
45. The altitude in Exercise 44 is twice that of Exercise 43. How do your cooking times compare for those two altitudes?

WebQuest

You can use a rational expression to determine how an amusement park can finance a new roller coaster. Visit www.algebra1.com/webquest to continue work on your WebQuest project.

PHYSICAL SCIENCE **For Exercises 46–48, use the following information.**

To pry the lid off a paint can, a screwdriver that is 17.5 centimeters long is used as a lever. It is placed so that 0.4 centimeter of its length extends inward from the rim of the can.



46. Write an equation that can be used to calculate the mechanical advantage.

47. What is the mechanical advantage?

48. If a force of 6 pounds is applied to the end of the screwdriver, what is the force placed on the lid?

FIELD TRIPS **For Exercises 49–52, use the following information.**

Mrs. Hoffman's art class is taking a trip to the museum. A bus that can seat up to 56 people costs $450 for the day, and group rate tickets at the museum cost $4 each.

49. If there are no more than 56 students going on the field trip, write an expression for the total cost for n students to go to the museum.

50. Write a rational expression that could be used to calculate the cost of the trip per student.

51. How many students must attend in order to keep the cost under $15 per student?

52. How would you change the expression for cost per student if the school were to cover the cost of two adult chaperones?

More About. . .

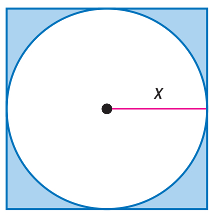
Farming

Although the amount of farmland in the United States is declining, crop production has increased steadily due in part to better irrigation practices.

Source: U.S. Department of Agriculture

FARMING **For Exercises 53 and 54, use the following information.**

Some farmers use an irrigation system that waters a circular region in a field. Suppose a square field with sides of length $2x$ is irrigated from the center of the square. The irrigation system can reach a radius of x.



53. Write an expression that represents the fraction of the field that is irrigated.

54. Calculate the percent of the field that is irrigated to the nearest whole percent.

55. **CRITICAL THINKING** Two students graphed the following equations on their calculators.

$$y = \frac{x^2 - 16}{x - 4} \qquad y = x + 4$$

They were surprised to see that the graphs appeared to be identical.

a. Explain why the graphs appear to be the same.

b. Explain how and why the graphs are different.

56. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can a rational expression be used in a movie theater?

Include the following in your answer:

- a description of how you determine the excluded values of a rational expression, and
- an example of another real-world situation that could be described using a rational expression.

Standardized Test Practice

57. Which expression is written in simplest form?

(A) $\dfrac{x^2 + 3x + 2}{x^2 + x - 2}$ (B) $\dfrac{3x - 3}{2x^2 - 2}$

(C) $\dfrac{x^2 + 7x}{x^2 + 3x - 4}$ (D) $\dfrac{2x^2 - 5x - 3}{x^2 + x - 12}$

58. In which expression are 1 and 5 excluded values?

(A) $\dfrac{x^2 + 6x + 5}{x^2 - 3x + 2}$ (B) $\dfrac{x^2 - 3x + 2}{x^2 - 6x + 5}$

(C) $\dfrac{x^2 - 6x + 5}{x^2 - 3x + 2}$ (D) $\dfrac{x^2 - 3x + 2}{x^2 + 6x + 5}$

Maintain Your Skills

Mixed Review

Write an inverse variation equation that relates x and y. Assume that y varies inversely as x. Then solve. *(Lesson 12-1)*

59. If $y = 6$ when $x = 10$, find y when $x = -12$.

60. If $y = 16$ when $x = \frac{1}{2}$, find x when $y = 32$.

61. If $y = -2.5$ when $x = 3$, find y when $x = -8$.

Use a calculator to find the measure of each angle to the nearest degree. *(Lesson 11-7)*

62. $\sin N = 0.2347$

63. $\cos B = 0.3218$

64. $\tan V = 0.0765$

65. $\sin A = 0.7011$

Solve each equation. Check your solution. *(Lesson 11-3)*

66. $\sqrt{a + 3} = 2$

67. $\sqrt{2z + 2} = z - 3$

68. $\sqrt{13 - 4p} - p = 8$

69. $\sqrt{3r^2 + 61} = 2r + 1$

Find the next three terms in each geometric sequence. *(Lesson 10-7)*

70. 1, 3, 9, 27, …

71. 6, 24, 96, 384, …

72. $\frac{1}{4}, -\frac{1}{2}, 1, -2, \ldots$

73. $4, 3, \frac{9}{4}, \frac{27}{16}, \ldots$

74. **GEOMETRY** Find the area of a rectangle if the length is $2x + y$ units and the width is $x + y$ units. *(Lesson 8-7)*

Getting Ready for the Next Lesson

BASIC SKILL **Complete.**

75. 84 in. = ____ ft

76. 4.5 m = ____ cm

77. 4 h 15 min = ____ s

78. 18 mi = ____ ft

79. 3 days = ____ h

80. 220 mL = ____ L

Graphing Calculator Investigation

A Follow-Up of Lesson 12-2

Rational Expressions

When simplifying rational expressions, you can use a TI-83 Plus graphing calculator to support your answer. If the graphs of the original expression and the simplified expression coincide, they are equivalent. You can also use the graphs to see excluded values.

Simplify $\frac{x^2 - 25}{x^2 + 10x + 25}$.

Step 1 *Factor the numerator and denominator.*

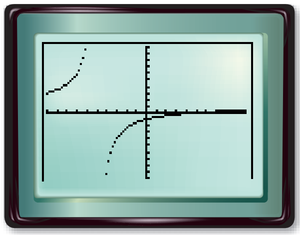
- $\frac{x^2 - 25}{x^2 + 10x + 25} = \frac{(x - 5)(x + 5)}{(x + 5)(x + 5)}$

  $= \frac{(x - 5)}{(x + 5)}$ When $x = -5$, $x + 5 = 0$. Therefore, x cannot equal -5 because you cannot divide by zero.

Step 2 *Graph the original expression.*

- Set the calculator to Dot mode.
- Enter $\frac{x^2 - 25}{x^2 + 10x + 25}$ as Y1 and graph.

KEYSTROKES: [MODE] [▼] [▼] [▼] [▼] [▶] [ENTER] [Y=] [(] [X,T,θ,n] [x^2] [–] 25 [)] [÷] [(] [X,T,θ,n] [x^2] [+] 10 [X,T,θ,n] [+] 25 [)] [ZOOM] 6



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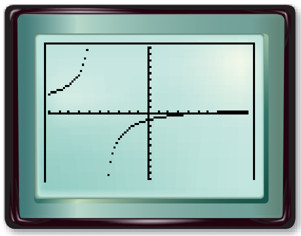
Step 3 *Graph the simplified expression.*

- Enter $\frac{(x - 5)}{(x + 5)}$ as Y2 and graph.

KEYSTROKES: [Y=] [▼] [▼] [(] [X,T,θ,n] [–] 5 [)] [÷] [(] [X,T,θ,n] [+] 5 [)] [GRAPH]



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Since the graphs overlap, the two expressions are equivalent.

Exercises

Simplify each expression. Then verify your answer graphically. Name the excluded values.

1. $\frac{3x + 6}{x^2 + 7x + 10}$ **2.** $\frac{x^2 - 9x + 8}{x^2 - 16x + 64}$ **3.** $\frac{5x^2 + 10x + 5}{3x^2 + 6x + 3}$

4. Simplify the rational expression $\frac{2x - 9}{4x^2 - 18x}$ and answer the following questions using the TABLE menu on your calculator.

a. How can you use the TABLE function to verify that the original expression and the simplified expression are equivalent?

b. How does the TABLE function show you that an x value is an excluded value?

www.algebra1.com/other_calculator_keystrokes

CONTENTS

12-3 Multiplying Rational Expressions

What You'll Learn

- Multiply rational expressions.
- Use dimensional analysis with multiplication.

How can you multiply rational expressions to determine the cost of electricity?

There are 25 lights around a patio. Each light is 40 watts, and the cost of electricity is 15 cents per kilowatt-hour. You can use the expression below to calculate the cost of using the lights for h hours.

$$25 \cancel{\text{lights}} \cdot h \cancel{\text{hours}} \cdot \frac{40 \cancel{\text{watts}}}{\cancel{\text{light}}} \cdot \frac{1 \cancel{\text{kilowatt}}}{1000 \cancel{\text{watts}}} \cdot \frac{15 \cancel{\text{cents}}}{1 \cancel{\text{kilowatt}} \cdot \cancel{\text{hour}}} \cdot \frac{1 \text{ dollar}}{100 \cancel{\text{cents}}}$$

From this point on, you may assume that no denominator of a rational expression has a value of zero.

MULTIPLY RATIONAL EXPRESSIONS The multiplication expression above is similar to the multiplication of rational expressions. Recall that to multiply rational numbers expressed as fractions, you multiply numerators and multiply denominators. You can use this same method to multiply rational expressions.

Example 1 Expressions Involving Monomials

a. Find $\frac{5ab^3}{8c^2} \cdot \frac{16c^3}{15a^2b}$.

Method 1 Divide by the greatest common factor after multiplying.

$\frac{5ab^3}{8c^2} \cdot \frac{16c^3}{15a^2b} = \frac{80ab^3c^3}{120a^2bc^2}$ ← Multiply the numerators. ← Multiply the denominators.

$= \frac{\overset{1}{\cancel{40abc^2}}(2b^2c)}{\underset{1}{\cancel{40abc^2}}(3a)}$ The GCF is $40abc^2$.

$= \frac{2b^2c}{3a}$ Simplify.

Method 2 Divide by the common factors before multiplying.

$\frac{5ab^3}{8c^2} \cdot \frac{16c^3}{15a^2b} = \frac{\overset{1}{\cancel{5}}\overset{1}{\cancel{a}}\overset{b^2}{\cancel{b^3}}}{\underset{1}{\cancel{8}}\underset{1}{\cancel{c^2}}} \cdot \frac{\overset{2}{\cancel{16}}\overset{c}{\cancel{c^3}}}{\underset{3}{\cancel{15}}\underset{a}{\cancel{a^2}}\underset{1}{\cancel{b}}}$ Divide by common factors 5, 8, a, b, and c^2.

$= \frac{2b^2c}{3a}$ Multiply.

b. Find $\frac{12xy^2}{45mp^2} \cdot \frac{27m^3p}{40x^3y}$.

$\frac{12xy^2}{45mp^2} \cdot \frac{27m^3p}{40x^3y} = \frac{\overset{3}{\cancel{12}}\overset{1}{\cancel{x}}\overset{y}{\cancel{y^2}}}{\underset{5}{\cancel{45}}\underset{1}{\cancel{m}}\underset{p}{\cancel{p^2}}} \cdot \frac{\overset{3}{\cancel{27}}\overset{m^2}{\cancel{m^3}}\overset{1}{\cancel{p}}}{\underset{10}{\cancel{40}}\underset{x^2}{\cancel{x^3}}\underset{1}{\cancel{y}}}$ Divide by common factors 4, 9, x, y, m, and p.

$= \frac{9m^2y}{50x^2p}$ Multiply.

Sometimes you must factor a quadratic expression before you can simplify a product of rational expressions.

Example 2 Expressions Involving Polynomials

a. Find $\frac{x-5}{x} \cdot \frac{x^2}{x^2 - 2x - 15}$.

$\frac{x-5}{x} \cdot \frac{x^2}{x^2-2x-15} = \frac{x-5}{x} \cdot \frac{x^2}{(x-5)(x+3)}$ Factor the denominator.

$= \frac{\overset{x}{\cancel{x^2}}\overset{1}{\cancel{(x-5)}}}{\underset{1}{\cancel{x}}\underset{1}{\cancel{(x-5)}}(x+3)}$ The GCF is $x(x-5)$.

$= \frac{x}{x+3}$ Simplify.

b. Find $\frac{a^2+7a+10}{a+1} \cdot \frac{3a+3}{a+2}$.

$\frac{a^2+7a+10}{a+1} \cdot \frac{3a+3}{a+2} = \frac{(a+5)(a+2)}{a+1} \cdot \frac{3(a+1)}{a+2}$ Factor the numerators.

$= \frac{3(a+5)\overset{1}{\cancel{(a+2)}}\overset{1}{\cancel{(a+1)}}}{\underset{1}{\cancel{(a+1)}}\underset{1}{\cancel{(a+2)}}}$ The GCF is $(a+1)(a+2)$.

$= \frac{3(a+5)}{1}$ Multiply.

$= 3a + 15$ Simplify.

Study Tip

Look Back
To review **dimensional analysis**, see Lesson 3-8.

DIMENSIONAL ANALYSIS When you multiply fractions that involve units of measure, you can divide by the units in the same way that you divide by variables. Recall that this process is called dimensional analysis.

Example 3 Dimensional Analysis

OLYMPICS In the 2000 Summer Olympics in Sydney, Australia, Maurice Green of the United States won the gold medal for the 100-meter sprint. His winning time was 9.87 seconds. What was his speed in kilometers per hour? Round to the nearest hundredth.

$\frac{100 \text{ meters}}{9.87 \text{ seconds}} \cdot \frac{1 \text{ kilometer}}{1000 \text{ meters}} \cdot \frac{60 \text{ seconds}}{1 \text{ minute}} \cdot \frac{60 \text{ minutes}}{1 \text{ hour}}$

$= \frac{100 \cancel{\text{ meters}}}{9.87 \cancel{\text{ seconds}}} \cdot \frac{1 \text{ kilometer}}{1000 \cancel{\text{ meters}}} \cdot \frac{60 \cancel{\text{ seconds}}}{1 \cancel{\text{ minute}}} \cdot \frac{60 \cancel{\text{ minutes}}}{1 \text{ hour}}$

$= \frac{\overset{1}{\cancel{100}} \cdot 1 \cdot 60 \cdot 60 \text{ kilometers}}{9.87 \cdot \underset{10}{\cancel{1000}} \cdot 1 \cdot 1 \text{ hours}}$

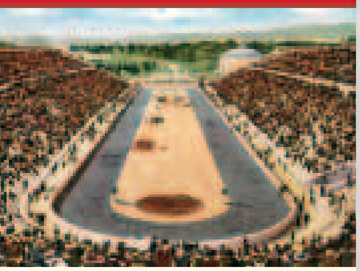
$= \frac{60 \cdot 60 \text{ kilometers}}{9.87 \cdot 10 \text{ hours}}$ Simplify.

$= \frac{3600 \text{ kilometers}}{98.7 \text{ hours}}$ Multiply.

$= \frac{36.47 \text{ kilometers}}{1 \text{ hour}}$ Divide numerator and denominator by 98.7.

His speed was 36.47 kilometers per hour.

More About...



Olympics

American sprinter Thomas Burke won the 100-meter dash at the first modern Olympics in Athens, Greece, in 1896 in 12.0 seconds.

Source: www.olympics.org

Check for Understanding

Concept Check

1. **OPEN ENDED** Write two rational expressions whose product is $\frac{2}{x}$.
2. **Explain** why $-\frac{x+6}{x-5}$ is not equivalent to $\frac{-x+6}{x-5}$.
3. **FIND THE ERROR** Amiri and Hoshi multiplied $\frac{x-3}{x+3}$ and $\frac{4x}{x^2-4x+3}$.

Amiri

$$\frac{x-3}{x+3}\cdot\frac{4x}{x^2-4x+3}$$
$$=\frac{\cancel{(x-3)}4x}{(x+3)\cancel{(x-3)}(x-1)}$$
$$=\frac{4x}{(x+3)(x-1)}$$

Hoshi

$$\frac{x-3}{x+3}\cdot\frac{4x}{x^2-4x+3}$$
$$=\frac{\cancel{x-3}}{\cancel{x+3}}\cdot\frac{\cancel{4x}}{x^2-\cancel{4x}+3}$$
$$=\frac{1}{x^2+3}$$

Who is correct? Explain your reasoning.

Guided Practice

Find each product.

4. $\frac{64y^2}{5y}\cdot\frac{5y}{8y}$
5. $\frac{15s^2t^3}{12st}\cdot\frac{16st^2}{10s^3t^3}$
6. $\frac{m+4}{3m}\cdot\frac{4m^2}{(m+4)(m+5)}$
7. $\frac{x^2-4}{2}\cdot\frac{4}{x-2}$
8. $\frac{n^2-16}{n+4}\cdot\frac{n+2}{n^2-8n+16}$
9. $\frac{x-5}{x^2-7x+10}\cdot\frac{x^2+x-6}{5}$

10. Find $\frac{24 \text{ feet}}{1 \text{ second}}\cdot\frac{60 \text{ seconds}}{1 \text{ minute}}\cdot\frac{60 \text{ minutes}}{1 \text{ hour}}\cdot\frac{1 \text{ mile}}{5280 \text{ feet}}$.

Application

11. **SPACE** The moon is about 240,000 miles from Earth. How many days would it take a spacecraft to reach the moon if it travels at an average of 100 miles per minute?

Practice and Apply

Homework Help

For Exercises	See Examples
12–15	1
16–27	2
28–37	3

Extra Practice
See page 847.

Find each product.

12. $\frac{8}{x^2}\cdot\frac{x^4}{4x}$
13. $\frac{10r^3}{6n^3}\cdot\frac{42n^2}{35r^3}$
14. $\frac{10y^3z^2}{6wx^3}\cdot\frac{12w^2x^2}{25y^2z^4}$
15. $\frac{3a^2b}{2gh}\cdot\frac{24g^2h}{15ab^2}$
16. $\frac{(x-8)}{(x+8)(x-3)}\cdot\frac{(x+4)(x-3)}{(x-8)}$
17. $\frac{(n-1)(n+1)}{(n+1)}\cdot\frac{(n-4)}{(n-1)(n+4)}$
18. $\frac{(z+4)(z+6)}{(z-6)(z+1)}\cdot\frac{(z+1)(z-5)}{(z+3)(z+4)}$
19. $\frac{(x-1)(x+7)}{(x-7)(x-4)}\cdot\frac{(x-4)(x+10)}{(x+1)(x+10)}$
20. $\frac{x^2-25}{9}\cdot\frac{x+5}{x-5}$
21. $\frac{y^2-4}{y^2-1}\cdot\frac{y+1}{y+2}$
22. $\frac{1}{x^2+x-12}\cdot\frac{x-3}{x+5}$
23. $\frac{x-6}{x^2+4x-32}\cdot\frac{x-4}{x+2}$
24. $\frac{x+3}{x+4}\cdot\frac{x}{x^2+7x+12}$
25. $\frac{n}{n^2+8n+15}\cdot\frac{2n+10}{n^2}$
26. $\frac{b^2+12b+11}{b^2-9}\cdot\frac{b+9}{b^2+20b+99}$
27. $\frac{a^2-a-6}{a^2-16}\cdot\frac{a^2+7a+12}{a^2+4a+4}$

Find each product.

28. $\dfrac{2.54 \text{ centimeters}}{1 \text{ inch}} \cdot \dfrac{12 \text{ inches}}{1 \text{ foot}} \cdot \dfrac{3 \text{ feet}}{1 \text{ yard}}$

29. $\dfrac{60 \text{ kilometers}}{1 \text{ hour}} \cdot \dfrac{1000 \text{ meters}}{1 \text{ kilometer}} \cdot \dfrac{1 \text{ hour}}{60 \text{ minutes}} \cdot \dfrac{1 \text{ minutes}}{60 \text{ seconds}}$

30. $\dfrac{32 \text{ feet}}{1 \text{ second}} \cdot \dfrac{60 \text{ seconds}}{1 \text{ minute}} \cdot \dfrac{60 \text{ minutes}}{1 \text{ hour}} \cdot \dfrac{1 \text{ mile}}{5280 \text{ feet}}$

31. $10 \text{ feet} \cdot 18 \text{ feet} \cdot 3 \text{ feet} \cdot \dfrac{1 \text{ yard}^3}{27 \text{ feet}^3}$

32. **DECORATING** Alani's bedroom is 12 feet wide and 14 feet long. What will it cost to carpet her room if the carpet costs $18 per square yard?

33. **EXCHANGE RATES** While traveling in Canada, Johanna bought some gifts to bring home. She bought 2 T-shirts that cost $21.95 (Canadian). If the exchange rate at the time was 1 U.S. dollar for 1.37 Canadian dollars, how much did Johanna spend in U.S. dollars?

Online Research **Data Update** Visit www.algebra1.com/data_update to find the most recent exchange rate between the United States and Canadian currency. How much does a $21.95 (Canadian) purchase cost in U.S. dollars?

More About. . .

Exchange Rates

A system of floating exchange rates among international currencies was established in 1976. It was needed because the old system of basing a currency's value on gold had become obsolete.

Source: www.infoplease.com

34. **CITY MAINTENANCE** Street sweepers can clean 3 miles of streets per hour. A city owns 2 street sweepers, and each sweeper can be used for three hours before it comes in for an hour to refuel. How many miles of streets can be cleaned in 18 hours on the road?

TRAFFIC **For Exercises 35–37, use the following information.**
During rush hour one evening, traffic was backed up for 13 miles along a particular stretch of freeway. Assume that each vehicle occupied an average of 30 feet of space in a lane and that the freeway has three lanes.

35. Write an expression that could be used to determine the number of vehicles involved in the backup.

36. How many vehicles are involved in the backup?

37. Suppose that there are 8 exits along this stretch of freeway, and it takes each vehicle an average of 24 seconds to exit the freeway. Approximately how many hours will it take for all the vehicles in the backup to exit?

38. **CRITICAL THINKING** Identify the expressions that are equivalent to $\frac{x}{y}$. Explain why the expressions are equivalent.

a. $\dfrac{x+3}{y+3}$ **b.** $\dfrac{3-x}{3-y}$ **c.** $\dfrac{3x}{3y}$ **d.** $\dfrac{x^3}{y^3}$ **e.** $\dfrac{n^3x}{n^3y}$

39. ***WRITING IN MATH*** Answer the question that was posed at the beginning of the lesson.

How can you multiply rational expressions to determine the cost of electricity?

Include the following in your answer:

- an expression that you could use to determine the cost of using 60-watt light bulbs instead of 40-watt bulbs, and
- an example of a real-world situation in which you must multiply rational expressions.

Standardized Test Practice

40. Which expression is the product of $\frac{13xyz}{4x^2y}$ and $\frac{8x^2z^2}{2y^3}$?

(A) $\frac{13xy^3}{z^3}$ (B) $\frac{13xz^2}{y^3}$ (C) $\frac{13xyz}{z^3}$ (D) $\frac{13xz^3}{y^3}$

41. Identify the product of $\frac{4a+4}{a^2+a}$ and $\frac{a^2}{3a-3}$.

(A) $\frac{4a}{3(a-1)}$ (B) $\frac{4a}{3}$ (C) $\frac{4a}{3(a+1)}$ (D) $\frac{4a^2}{3(a-1)}$

Maintain Your Skills

Mixed Review

State the excluded values for each rational expression. *(Lesson 12-2)*

42. $\frac{s+6}{s^2-36}$ 43. $\frac{a^2-25}{a^2+3a-10}$ 44. $\frac{x+3}{x^2+6x+9}$

Write an inverse variation equation that relates x and y. Assume that y varies inversely as x. Then solve. *(Lesson 12-1)*

45. If $y = 9$ when $x = 8$, find x when $y = 6$.
46. If $y = 2.4$ when $x = 8.1$, find y when $x = 3.6$.
47. If $y = 24$ when $x = -8$, find y when $x = 4$.
48. If $y = 6.4$ when $x = 4.4$, find x when $y = 3.2$.

Simplify. Assume that no denominator is equal to zero. *(Lesson 8-2)*

49. $\frac{-7^{12}}{7^9}$ 50. $\frac{20p^6}{8p^8}$ 51. $\frac{24a^3b^4c^7}{6a^6c^2}$

Solve each inequality. Then check your solution. *(Lesson 6-2)*

52. $\frac{g}{8} < \frac{7}{2}$ 53. $3.5r \geq 7.35$ 54. $\frac{9k}{4} > \frac{3}{5}$

55. **FINANCE** The total amount of money Antonio earns mowing lawns and doing yard work varies directly with the number of days he works. At one point, he earned \$340 in 4 days. At this rate, how long will it take him to earn \$935? *(Lesson 5-2)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Factor each polynomial.**
*(To review **factoring polynomials**, see Lessons 9-3 through 9-6.)*

56. $x^2 - 3x - 40$ 57. $n^2 - 64$ 58. $x^2 - 12x + 36$
59. $a^2 + 2a - 35$ 60. $2x^2 - 5x - 3$ 61. $3x^3 - 24x^2 + 36x$

Practice Quiz 1 — *Lessons 12-1 through 12-3*

Graph each variation if y varies inversely as x. *(Lesson 12-1)*

1. $y = 28$ when $x = 7$ 2. $y = -6$ when $x = 9$

Simplify each expression. *(Lesson 12-2)*

3. $\frac{28a^2}{49ab}$ 4. $\frac{y+3y^2}{3y+1}$ 5. $\frac{b^2-3b-4}{b^2-13b+36}$ 6. $\frac{3n^2+5n-2}{3n^2-13n+4}$

Find each product. *(Lesson 12-3)*

7. $\frac{3m^2}{2m} \cdot \frac{18m^2}{9m}$ 8. $\frac{5a+10}{10x^2} \cdot \frac{4x^3}{a^2+11a+18}$ 9. $\frac{4n+8}{n^2-25} \cdot \frac{n-5}{5n+10}$ 10. $\frac{x^2-x-6}{x^2-9} \cdot \frac{x^2+7x+12}{x^2+4x+4}$

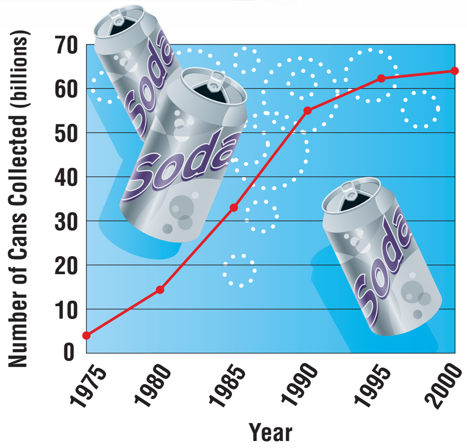
12-4 Dividing Rational Expressions

What You'll Learn

- Divide rational expressions.
- Use dimensional analysis with division.

How can you determine the number of aluminum soft drink cans made each year?

Most soft drinks come in aluminum cans. Although more cans are used today than in the 1970s, the demand for new aluminum has declined. This is due in large part to the great number of cans that are recycled. In recent years, approximately 63.9 billion cans were recycled annually. This represents $\frac{5}{8}$ of all cans produced.



DIVIDE RATIONAL EXPRESSIONS Recall that to divide rational numbers expressed as fractions you multiply by the reciprocal of the divisor. You can use this same method to divide rational expressions.

Example 1 Expression Involving Monomials

Find $\frac{5x^2}{7} \div \frac{10x^3}{21}$.

$$\frac{5x^2}{7} \div \frac{10x^3}{21} = \frac{5x^2}{7} \cdot \frac{21}{10x^3}$$ Multiply by $\frac{21}{10x^3}$, the reciprocal of $\frac{10x^3}{21}$.

$$= \frac{\overset{1}{\cancel{5x^2}}}{\underset{1}{\cancel{7}}} \cdot \frac{\overset{3}{\cancel{21}}}{\underset{2x}{\cancel{10x^3}}}$$ Divide by common factors 5, 7, and x^2.

$$= \frac{3}{2x}$$ Simplify.

Example 2 Expression Involving Binomials

Find $\frac{n+1}{n+3} \div \frac{2n+2}{n+4}$.

$$\frac{n+1}{n+3} \div \frac{2n+2}{n+4} = \frac{n+1}{n+3} \cdot \frac{n+4}{2n+2}$$ Multiply by $\frac{n+4}{2n+2}$, the reciprocal of $\frac{2n+2}{n+4}$.

$$= \frac{n+1}{n+3} \cdot \frac{n+4}{2(n+1)}$$ Factor $2n + 2$.

$$= \frac{\overset{1}{\cancel{n+1}}}{n+3} \cdot \frac{n+4}{2\underset{1}{\cancel{(n+1)}}}$$ The GCF is $n + 1$.

$$= \frac{n+4}{2(n+3)} \text{ or } \frac{n+4}{2n+6}$$ Simplify.

Often the quotient of rational expressions involves a divisor that is a binomial.

Study Tip

Multiplicative Inverse

As with rational numbers, dividing rational expressions involves multiplying by the inverse. Remember that the inverse of $a + 2$ is $\frac{1}{a+2}$.

Example 3 Divide by a Binomial

Find $\frac{5a + 10}{a + 5} \div (a + 2)$.

$$\frac{5a+10}{a+5} \div (a+2) = \frac{5a+10}{a+5} \cdot \frac{1}{(a+2)} \quad \text{Multiply by } \frac{1}{(a+2)}\text{, the reciprocal of } (a+2).$$

$$= \frac{5(a+2)}{a+5} \cdot \frac{1}{(a+2)} \quad \text{Factor } 5a + 10.$$

$$= \frac{5\cancel{(a+2)}^{1}}{a+5} \cdot \frac{1}{\cancel{(a+2)}_{1}} \quad \text{The GCF is } a + 2.$$

$$= \frac{5}{a+5} \quad \text{Simplify.}$$

Sometimes you must factor a quadratic expression before you can simplify the quotient of rational expressions.

Example 4 Expression Involving Polynomials

Find $\frac{m^2 + 3m + 2}{4} \div \frac{m+2}{m+1}$.

$$\frac{m^2+3m+2}{4} \div \frac{m+2}{m+1} = \frac{m^2+3m+2}{4} \cdot \frac{m+1}{m+2} \quad \text{Multiply by the reciprocal, } \frac{m+1}{m+2}.$$

$$= \frac{(m+1)(m+2)}{4} \cdot \frac{m+1}{m+2} \quad \text{Factor } m^2 + 3m + 2.$$

$$= \frac{(m+1)\cancel{(m+2)}^{1}}{4} \cdot \frac{m+1}{\cancel{m+2}_{1}} \quad \text{The GCF is } m + 2.$$

$$= \frac{(m+1)^2}{4} \quad \text{Simplify.}$$

DIMENSIONAL ANALYSIS You can divide rational expressions that involve units of measure by using dimensional analysis.

Example 5 Dimensional Analysis

SPACE In November, 1996, NASA launched the Mars Global Surveyor. It took 309 days for the orbiter to travel 466,000,000 miles from Earth to Mars. What was the speed of the spacecraft in miles per hour? Round to the nearest hundredth.

Use the formula for rate, time, and distance.

$$rt = d \quad \text{rate} \cdot \text{time} = \text{distance}$$

$$r \cdot 309 \text{ days} = 466{,}000{,}000 \text{ mi} \quad t = 309 \text{ days}, d = 466{,}000{,}000$$

$$r = \frac{466{,}000{,}000 \text{ mi}}{309 \text{ days}} \quad \text{Divide each side by 309 days.}$$

$$= \frac{466{,}000{,}000 \text{ miles}}{309 \cancel{\text{ days}}} \cdot \frac{1 \cancel{\text{ day}}}{24 \text{ hours}} \quad \text{Convert days to hours.}$$

$$= \frac{466{,}000{,}000 \text{ miles}}{7416 \text{ hours}} \text{ or about } \frac{62{,}837.11 \text{ miles}}{1 \text{ hour}}$$

Thus, the spacecraft traveled at a rate of about 62,837.11 miles per hour.

More About . . .

Space

The first successful Mars probe was the Mariner 4, which arrived at Mars on July 14, 1965.

Source: NASA

Check for Understanding

Concept Check

1. **OPEN ENDED** Write two rational expressions whose quotient is $\frac{5z}{xy}$.
2. **Tell** whether the following statement is *always*, *sometimes*, or *never* true. Explain your reasoning.
   Every real number has a reciprocal.
3. **Explain** how to calculate the mass in kilograms of one cubic meter of a substance whose density is 2.16 grams per cubic centimeter.

Guided Practice

Find each quotient.

4. $\frac{10n^3}{7} \div \frac{5n^2}{21}$
5. $\frac{2a}{a+3} \div \frac{a+7}{a+3}$
6. $\frac{3m-15}{m+4} \div \frac{m-5}{6m+24}$
7. $\frac{2x+6}{x+5} \div (x+3)$
8. $\frac{k+3}{k^2+4k+4} \div \frac{2k+6}{k+2}$
9. $\frac{2x-4}{x^2+11x+18} \div \frac{x+1}{x^2+5x+6}$
10. Express 85 kilometers per hour in meters per second.
11. Express 32 pounds per square foot as pounds per square inch.

Application

12. **COOKING** Latisha was making candy using a two-quart pan. As she stirred the mixture, she noticed that the pan was about $\frac{2}{3}$ full. If each piece of candy has a volume of about $\frac{3}{4}$ ounce, approximately how many pieces of candy will Latisha make?

Practice and Apply

Homework Help

For Exercises	See Examples
13–18	1
19–22	3
23, 24	2
29–36	4
25–28, 37–41	5

Extra Practice
See page 847.

Find each quotient.

13. $\frac{a^2}{b^2} \div \frac{a}{b^3}$
14. $\frac{n^4}{p^2} \div \frac{n^2}{p^3}$
15. $\frac{4x^3}{y^4} \div \frac{8x^2}{y^2}$
16. $\frac{10m^2}{7n^2} \div \frac{25m^4}{14n^3}$
17. $\frac{x^2y^3z}{s^2t^2} \div \frac{x^2yz^3}{s^3t^2}$
18. $\frac{a^4bc^3}{g^2h^3} \div \frac{ab^2c^2}{g^3h^3}$
19. $\frac{b^2-9}{4b} \div (b-3)$
20. $\frac{m^2-16}{5m} \div (m+4)$
21. $\frac{3k}{k+1} \div (k-2)$
22. $\frac{5d}{d-3} \div (d+1)$
23. $\frac{3x+12}{4x-18} \div \frac{2x+8}{x+4}$
24. $\frac{4a-8}{2a-6} \div \frac{2a-4}{a-4}$

Complete.

25. $24 \text{ yd}^3 =$ ____ ft^3
26. $0.35 \text{ m}^3 =$ ____ cm^3
27. 330 ft/s = ____ mi/h
28. $1730 \text{ plants/km}^2 =$ ____ plants/m^2
29. What is the quotient when $\frac{2x+6}{x+5}$ is divided by $\frac{2}{x+5}$?
30. Find the quotient when $\frac{m-8}{m+7}$ is divided by $m^2 - 7m - 8$.

Find each quotient.

31. $\dfrac{x^2 + 2x + 1}{2} \div \dfrac{x + 1}{x - 1}$

32. $\dfrac{n^2 + 3n + 2}{4} \div \dfrac{n + 1}{n + 2}$

33. $\dfrac{a^2 + 8a + 16}{a^2 - 6a + 9} \div \dfrac{2a + 8}{3a - 9}$

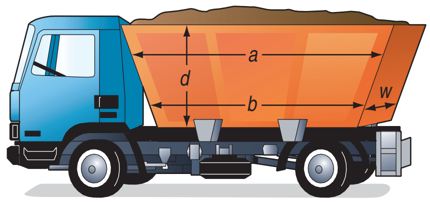
34. $\dfrac{b + 2}{b^2 + 4b + 4} \div \dfrac{2b + 4}{b + 4}$

35. $\dfrac{x^2 + x - 2}{x^2 + 5x + 6} \div \dfrac{x^2 + 2x - 3}{x^2 + 7x + 12}$

36. $\dfrac{x^2 + 2x - 15}{x^2 - x - 30} \div \dfrac{x^2 - 3x - 18}{x^2 - 2x - 24}$

37. **TRIATHLONS** Irena is training for an upcoming triathlon and plans to run 12 miles today. Jorge offered to ride his bicycle to help her maintain her pace. If Irena wants to keep a steady pace of 6.5 minutes per mile, how fast should Jorge ride in miles per hour?

CONSTRUCTION **For Exercises 38 and 39, use the following information.**
A construction supervisor needs to determine how many truckloads of earth must be removed from a site before a foundation can be poured. The bed of the truck has the shape shown at the right.



38. Use the formula $V = \dfrac{d(a + b)}{2} \cdot w$ to write an equation involving units that represents the volume of the truck bed in cubic yards if $a = 18$ feet, $b = 15$ feet, $w = 9$ feet, and $d = 5$ feet.

39. There are 20,000 cubic yards of earth that must be removed from the excavation site. Write an equation involving units that represents the number of truckloads that will be required to remove all of the earth. Then solve the equation.

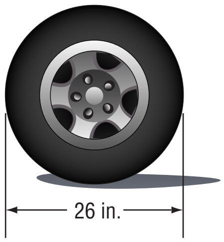
More About. . .

Triathlons

The Ironman Championship Triathlon held in Hawaii consists of a 2.4-mile swim, a 112-mile bicycle ride, and a 26.2-mile run.

Source: www.infoplease.com

TRUCKS **For Exercises 40 and 41, use the following information.**
The speedometer of John's truck uses the revolutions of his tires to calculate the speed of the truck.



40. How many revolutions per minute do the tires make when the truck is traveling at 55 miles per hour?

41. Suppose John buys tires with a diameter of 30 inches. When the speedometer reads 55 miles per hour, the tires would still revolve at the same rate as before. However, with the new tires, the truck travels a different distance in each revolution. Calculate the actual speed when the speedometer reads 55 miles per hour.

42. **CRITICAL THINKING** Which expression is *not* equivalent to the reciprocal of $\dfrac{x^2 - 4y^2}{x + 2y}$? Justify your answer.

a. $\dfrac{1}{\dfrac{x^2 - 4y^2}{x + 2y}}$ **b.** $\dfrac{-1}{2y - x}$ **c.** $\dfrac{1}{x - 2y}$ **d.** $\dfrac{1}{x} - \dfrac{1}{2y}$

SCULPTURE **For Exercises 43 and 44, use the following information.**
A sculptor had a block of marble in the shape of a cube with sides x feet long. A piece that was $\frac{1}{2}$ foot thick was chiseled from the bottom of the block. Later, the sculptor removed a piece $\frac{3}{4}$ foot wide from the side of the marble block.

43. Write a rational expression that represents the volume of the block of marble that remained.

44. If the remaining marble was cut into ten pieces weighing 85 pounds each, write an expression that represents the weight of the original block of marble.

www.algebra1.com/self_check_quiz

45. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can you determine the number of aluminum soft drink cans made each year?

Include the following in your answer:

- a rational expression that will give the amount of new aluminum needed to produce x aluminum cans today when $\frac{5}{8}$ of the cans are recycled and 33 cans are produced from a pound of aluminum.

Standardized Test Practice

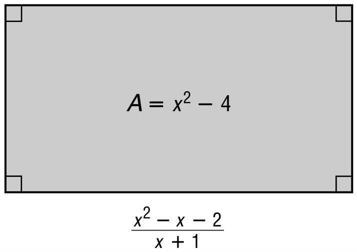
46. Which expression is the quotient of $\frac{3b}{5c}$ and $\frac{18b}{15c}$?

Ⓐ $\frac{18b^2}{15c^2}$ Ⓑ $\frac{1}{2}$ Ⓒ $\frac{18b}{15c}$ Ⓓ 2

47. Which expression could be used for the width of the rectangle?

Ⓐ $x - 2$ Ⓑ $(x + 2)(x - 2)^2$

Ⓒ $x + 2$ Ⓓ $(x + 2)(x - 2)$



Maintain Your Skills

Mixed Review

Find each product. *(Lesson 12-3)*

48. $\frac{x - 5}{x^2 - 7x + 10} \cdot \frac{x - 2}{1}$

49. $\frac{x^2 + 3x - 10}{x^2 + 8x + 15} \cdot \frac{x^2 + 5x + 6}{x^2 + 4x + 4}$

50. $\frac{x + 4}{4y} \cdot \frac{16y}{x^2 + 7x + 12}$

51. $\frac{x^2 + 8x + 15}{x + y} \cdot \frac{7x + 14y}{x + 3}$

Simplify each expression. *(Lesson 12-2)*

52. $\frac{c - 6}{c^2 - 12c + 36}$

53. $\frac{25 - x^2}{x^2 + x - 30}$

54. $\frac{a + 3}{a^2 + 4a + 3}$

55. $\frac{n^2 - 16}{n^2 - 8n + 16}$

Solve each equation. Check your solutions. *(Lesson 9-6)*

56. $3y^2 = 147$

57. $9x^2 - 24x = -16$

58. $a^2 + 225 = 30a$

59. $(n + 6)^2 = 14$

Find the degree of each polynomial. *(Lesson 8-4)*

60. $13 + \frac{1}{8}$

61. $z^3 - 2z^2 + 3z - 4$

62. $a^5b^2c^3 + 6a^3b^3c^2$

Solve each inequality. Then check your solution. *(Lesson 6-2)*

63. $6 \leq 0.8g$

64. $-15b < -28$

65. $-0.049 \leq 0.07x$

66. $\frac{3}{7}h < \frac{3}{49}$

67. $\frac{12r}{-4} > \frac{3}{20}$

68. $\frac{y}{6} \geq \frac{1}{2}$

69. **MANUFACTURING** Tanisha's Sporting Equipment manufactures tennis racket covers at the rate of 3250 each month. How many tennis racket covers will the company manufacture in one year? *(Lesson 5-3)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Simplify.** *(To review **dividing monomials**, see Lesson 8-2.)*

70. $\frac{6x^2}{x^4}$

71. $\frac{5m^4}{25m}$

72. $\frac{18a^3}{45a^5}$

73. $\frac{b^6c^3}{b^3c^6}$

74. $\frac{12x^3y^2}{28x^4y}$

75. $\frac{7x^4z^2}{z^3}$

Reading Mathematics

Rational Expressions

Several concepts need to be applied when reading rational expressions.

- A fraction bar acts as a grouping symbol, where the entire numerator is divided by the entire denominator.

Example 1 $\dfrac{6x + 4}{10}$

It is correct to read the expression as *the quantity six x plus four divided by ten*.

It is incorrect to read the expression as *six x divided by ten plus four*, or *six x plus four divided by ten*.

- If a fraction consists of two or more terms divided by a one-term denominator, the denominator divides each term.

Example 2 $\dfrac{6x + 4}{10}$

It is correct to write $\dfrac{6x + 4}{10} = \dfrac{6x}{10} + \dfrac{4}{10}$.

$$= \frac{3x}{5} + \frac{2}{5} \quad \text{or} \quad \frac{3x + 2}{5}$$

It is also correct to write $\dfrac{6x + 4}{10} = \dfrac{2(3x + 2)}{2 \cdot 5}$.

$$= \frac{\cancel{2}(3x + 2)}{\cancel{2} \cdot 5} \quad \text{or} \quad \frac{3x + 2}{5}$$

It is incorrect to write $\dfrac{6x + 4}{10} = \dfrac{\overset{3x}{\cancel{6x}} + 4}{\underset{5}{\cancel{10}}} = \dfrac{3x + 4}{5}$.

Reading to Learn

Write the verbal translation of each rational expression.

1. $\dfrac{m + 2}{4}$ **2.** $\dfrac{3x}{x - 1}$ **3.** $\dfrac{a + 2}{a^2 + 8}$

4. $\dfrac{x^2 - 25}{x + 5}$ **5.** $\dfrac{x^2 - 3x + 18}{x - 2}$ **6.** $\dfrac{x^2 + 2x - 35}{x^2 - x - 20}$

Simplify each expression.

7. $\dfrac{3x + 6}{9}$ **8.** $\dfrac{4n - 12}{8}$ **9.** $\dfrac{5x^2 - 25x}{10x}$

10. $\dfrac{x + 3}{x^2 + 7x + 12}$ **11.** $\dfrac{x + y}{x^2 + 2xy + y^2}$ **12.** $\dfrac{x^2 - 16}{x^2 - 8x + 16}$

12-5 Dividing Polynomials

What You'll Learn

- Divide a polynomial by a monomial.
- Divide a polynomial by a binomial.

How is division used in sewing?

Marching bands often use intricate marching routines and colorful flags to add interest to their shows. Suppose a partial roll of fabric is used to make flags. The original roll was 36 yards long, and $7\frac{1}{2}$ yards of the fabric were used to make a banner for the band. Each flag requires $1\frac{1}{2}$ yards of fabric. The expression $\dfrac{36 \text{ yards} - 7\frac{1}{2} \text{ yards}}{1\frac{1}{2} \text{ yards}}$ can be used to represent the number of flags that can be made using the roll of fabric.

DIVIDE POLYNOMIALS BY MONOMIALS To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.

Example 1 Divide a Binomial by a Monomial

Find $(3r^2 - 15r) \div 3r$.

$$
\begin{aligned}
(3r^2 - 15r) \div 3r &= \frac{3r^2 - 15r}{3r} && \text{Write as a rational expression.} \\
&= \frac{3r^2}{3r} - \frac{15r}{3r} && \text{Divide each term by } 3r. \\
&= \frac{\overset{r}{\cancel{3r^2}}}{\underset{1}{\cancel{3r}}} - \frac{\overset{5}{\cancel{15r}}}{\underset{1}{\cancel{3r}}} && \text{Simplify each term.} \\
&= r - 5 && \text{Simplify.}
\end{aligned}
$$

Example 2 Divide a Polynomial by a Monomial

Find $(n^2 + 10n + 12) \div 5n$.

$$
\begin{aligned}
(n^2 + 10n + 12) \div 5n &= \frac{n^2 + 10n + 12}{5n} && \text{Write as a rational expression.} \\
&= \frac{n^2}{5n} + \frac{10n}{5n} + \frac{12}{5n} && \text{Divide each term by } 5n. \\
&= \frac{\overset{n}{\cancel{n^2}}}{\underset{5}{\cancel{5n}}} + \frac{\overset{2}{\cancel{10n}}}{\underset{1}{\cancel{5n}}} + \frac{12}{5n} && \text{Simplify each term.} \\
&= \frac{n}{5} + 2 + \frac{12}{5n} && \text{Simplify.}
\end{aligned}
$$

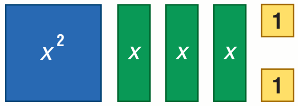
DIVIDE POLYNOMIALS BY BINOMIALS You can use algebra tiles to model some quotients of polynomials.

Algebra Activity

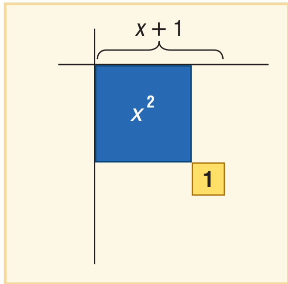
Dividing Polynomials

Use algebra tiles to find $(x^2 + 3x + 2) \div (x + 1)$.

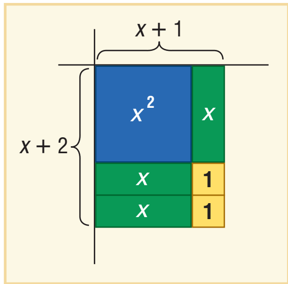
Step 1 Model the polynomial $x^2 + 3x + 2$.



Step 2 Place the x^2 tile at the corner of the product mat. Place one of the 1 tiles as shown to make a length of $x + 1$.



Step 3 Use the remaining tiles to make a rectangular array.



The width of the array, $x + 2$, is the quotient.

Model and Analyze

Use algebra tiles to find each quotient.

1. $(x^2 + 3x - 4) \div (x - 1)$

2. $(x^2 - 5x + 6) \div (x - 2)$

3. $(x^2 - 16) \div (x + 4)$

4. $(2x^2 - 4x - 6) \div (x - 3)$

5. Describe what happens when you try to model $(3x^2 - 4x + 3) \div (x + 2)$. What do you think the result means?

Recall from Lesson 12-4 that when you factor, some divisions can be performed easily.

Example 3 Divide a Polynomial by a Binomial

Find $(s^2 + 6s - 7) \div (s + 7)$.

$$(s^2 + 6s - 7) \div (s + 7) = \frac{s^2 + 6s - 7}{(s + 7)}$$ Write as a rational expression.

$$= \frac{(s + 7)(s - 1)}{(s + 7)}$$ Factor the numerator.

$$= \frac{\cancel{(s + 7)}^{1}(s - 1)}{\cancel{(s + 7)}_{1}}$$ Divide by the GCF.

$$= s - 1$$ Simplify.

www.algebra1.com/extra_examples

In Example 3 the division could be performed easily by dividing by common factors. However, when you cannot factor, you can use a long division process similar to the one you use in arithmetic.

Example 4 Long Division

Find $(x^2 + 3x - 24) \div (x - 4)$.

The expression $x^2 + 3x - 24$ cannot be factored, so use long division.

Step 1 Divide the first term of the dividend, x^2, by the first term of the divisor, x.

$$
\begin{array}{rll}
 & x & x^2 \div x = x \\
x - 4 \enclose{longdiv}{} & x^2 + 3x - 24 & \\
 & \underline{(-)\ x^2 - 4x} & \text{Multiply } x \text{ and } x - 4. \\
 & 7x & \text{Subtract.}
\end{array}
$$

Step 2 Divide the first term of the partial dividend, $7x - 24$, by the first term of the divisor, x.

$$
\begin{array}{rll}
 & x + 7 & 7x \div x = 7 \\
x - 4 \enclose{longdiv}{} & x^2 + 3x - 24 & \\
 & \underline{(-)\ x^2 - 4x} & \\
 & 7x - 24 & \text{Subtract and bring down the 24.} \\
 & \underline{(-)\ 7x - 28} & \text{Multiply 7 and } x - 4. \\
 & 4 & \text{Subtract.}
\end{array}
$$

The quotient of $(x^2 + 3x - 24) \div (x - 4)$ is $x + 7$ with a remainder of 4, which can be written as $x + 7 + \frac{4}{x - 4}$. Since there is a nonzero remainder, $x - 4$ is not a factor of $x^2 + 3x - 24$.

When the dividend is an expression like $a^3 + 8a - 21$, there is no a^2 term. In such situations, you must rename the dividend using 0 as the coefficient of the missing terms.

Example 5 Polynomial with Missing Terms

Find $(a^3 + 8a - 24) \div (a - 2)$.

Rename the a^2 term using a coefficient of 0.

$(a^3 + 8a - 24) \div (a - 2) = (a^3 + 0a^2 + 8a - 24) \div (a - 2)$

$$
\begin{array}{rll}
 & a^2 + 2a + 12 & \\
a - 2 \enclose{longdiv}{} & a^3 + 0a^2 + 8a - 24 & \\
 & \underline{(-)\ a^3 - 2a^2} & \text{Multiply } a^2 \text{ and } a - 2. \\
 & 2a^2 + 8a & \text{Subtract and bring down } 8a. \\
 & \underline{(-)\ 2a^2 - 4a} & \text{Multiply } 2a \text{ and } a - 2. \\
 & 12a - 24 & \text{Subtract and bring down 24.} \\
 & \underline{(-)\ 12a - 24} & \text{Multiply 12 and } a - 2. \\
 & 0 & \text{Subtract.}
\end{array}
$$

Therefore, $(a^3 + 8a - 24) \div (a - 2) = a^2 + 2a + 12$.

Study Tip

Factors
When the remainder in a division problem is 0, the divisor is a factor of the dividend.

Check for Understanding

Concept Check

1. **Choose** the divisors of $2x^2 - 9x + 9$ that result in a remainder of 0.
   a. $x + 3$ **b.** $x - 3$ **c.** $2x - 3$ **d.** $2x + 3$
2. **Explain** the meaning of a remainder of zero in a long division of a polynomial by a binomial.
3. **OPEN ENDED** Write a third-degree polynomial that includes a zero term. Rewrite the polynomial so that it can be divided by $x + 5$ using long division.

Guided Practice

Find each quotient.

4. $(4x^3 + 2x^2 - 5) \div 2x$
5. $\dfrac{14a^2b^2 + 35ab^2 + 2a^2}{7a^2b^2}$
6. $(n^2 + 7n + 12) \div (n + 3)$
7. $(r^2 + 12r + 36) \div (r + 9)$
8. $\dfrac{4m^3 + 5m - 21}{2m - 3}$
9. $(2b^2 + 3b - 5) \div (2b - 1)$

Application

10. **ENVIRONMENT** The equation $C = \dfrac{120{,}000p}{1 - p}$ models the cost C in dollars for a manufacturer to reduce the pollutants by a given percent, written as p in decimal form. How much will the company have to pay to remove 75% of the pollutants it emits?

Practice and Apply

Homework Help

For Exercises	See Examples
11–14	1, 2
15–18, 23, 24	3
19–22, 25, 26	4
27–30	5

Extra Practice
See page 847.

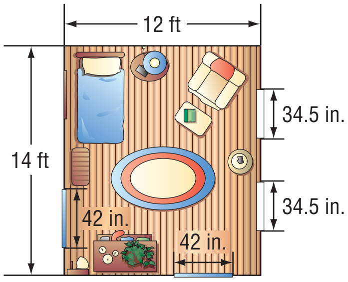
Find each quotient.

11. $(x^2 + 9x - 7) \div 3x$
12. $(a^2 + 7a - 28) \div 7a$
13. $\dfrac{9s^3t^2 - 15s^2t + 24t^3}{3s^2t^2}$
14. $\dfrac{12a^3b + 16ab^3 - 8ab}{4ab}$
15. $(x^2 + 9x + 20) \div (x + 5)$
16. $(x^2 + 6x - 16) \div (x - 2)$
17. $(n^2 - 2n - 35) \div (n + 5)$
18. $(s^2 + 11s + 18) \div (s + 9)$
19. $(z^2 - 2z - 30) \div (z + 7)$
20. $(a^2 + 4a - 22) \div (a - 3)$
21. $(2r^2 - 3r - 35) \div (r - 5)$
22. $(3p^2 + 20p + 11) \div (p + 6)$
23. $\dfrac{3t^2 + 14t - 24}{3t - 4}$
24. $\dfrac{12n^2 + 36n + 15}{2n + 5}$
25. $\dfrac{3x^3 + 8x^2 + x - 7}{x + 2}$
26. $\dfrac{20b^3 - 27b^2 + 13b - 3}{4b - 3}$
27. $\dfrac{6x^3 - 9x^2 + 6}{2x - 3}$
28. $\dfrac{9g^3 + 5g - 8}{3g - 2}$
29. Determine the quotient when $6n^3 + 5n^2 + 12$ is divided by $2n + 3$.
30. What is the quotient when $4t^3 + 17t^2 - 1$ is divided by $4t + 1$?

LANDSCAPING **For Exercises 31 and 32, use the following information.**
A heavy object can be lifted more easily using a lever and fulcrum. The amount that can be lifted depends upon the length of the lever, the placement of the fulcrum, and the force applied. The expression $\dfrac{W(L - x)}{x}$ represents the weight of an object that can be lifted if W pounds of force are applied to a lever L inches long with the fulcrum placed x inches from the object.

31. Suppose Leyati, who weighs 150 pounds, uses all of his weight to lift a rock using a 60-inch lever. Write an expression that could be used to determine the heaviest rock he could lift if the fulcrum is x inches from the rock.
32. Use the expression to find the weight of a rock that could be lifted by a 210-pound man using a six-foot lever placed 20 inches from the rock.

33. **DECORATING** Anoki wants to put a decorative border 3 feet above the floor around his bedroom walls. If the border comes in 5-yard rolls, how many rolls of border should Anoki buy?



PIZZA **For Exercises 34 and 35, use the following information.**
The expression $\frac{\pi d^2}{64}$ can be used to determine the number of slices of a round pizza with diameter d.

34. Write a formula to calculate the cost per slice s of a pizza that costs C dollars.

35. Copy and complete the table below. Which size pizza offers the best price per slice?

Size	10-inch	14-inch	18-inch
Price	\$4.99	\$8.99	\$12.99
Number of slices			
Cost per slice			

SCIENCE **For Exercises 36–38, use the following information.**
The *density* of a material is its mass per unit volume.

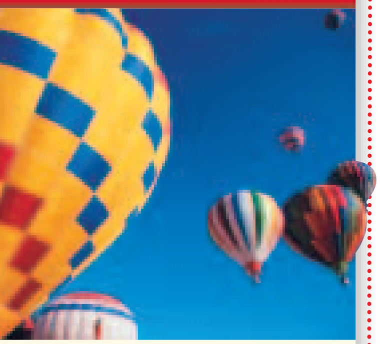
36. Determine the densities for the materials listed in the table. Round to the nearest hundredth.

37. Make a graph of the densities computed in Exercise 36.

38. Interpret the line plot made in Exercise 37.

Material	Mass (g)	Volume (cm^3)
aluminum	4.15	1.54
gold	2.32	0.12
silver	6.30	0.60
steel	7.80	1.00
iron	15.20	1.95
copper	2.48	0.28
blood	4.35	4.10
lead	11.30	1.00
brass	17.90	2.08
concrete	40.00	20.00

More About. . .

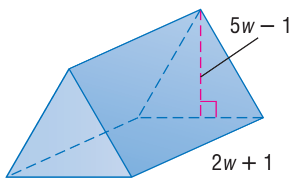


Science

When air is heated it is less dense than the air surrounding it, and the heated air rises. This is why a hot air balloon is able to fly.

Source: www.howstuffworks.com

39. **GEOMETRY** The volume of a prism with a triangular base is $10w^3 + 23w^2 + 5w - 2$. The height of the prism is $2w + 1$, and the height of the triangle is $5w - 1$. What is the measure of the base of the triangle? $\left(\textit{Hint}: V = Bh\right)$



CRITICAL THINKING **Find the value of k in each situation.**

40. k is an integer and there is no remainder when $x^2 + 7x + 12$ is divided by $x + k$.

41. When $x^2 + 7x + k$ is divided by $x + 2$, there is a remainder of 2.

42. $x + 7$ is a factor of $x^2 - 2x - k$.

43. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

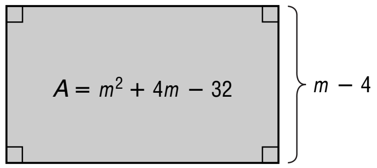
How is division used in sewing?

Include the following in your answer:

- a description showing that $\dfrac{36 \text{ yards} - 7\frac{1}{2} \text{ yards}}{1\frac{1}{2} \text{ yards}}$ and $\dfrac{36 \text{ yards}}{1\frac{1}{2} \text{ yards}} - \dfrac{7\frac{1}{2} \text{ yards}}{1\frac{1}{2} \text{ yards}}$ result in the same answer, and
- a convincing explanation to show that $\dfrac{a - b}{c} = \dfrac{a}{c} - \dfrac{b}{c}$.

Standardized Test Practice

44. Which expression represents the length of the rectangle?



Ⓐ $m + 7$ Ⓑ $m - 8$

Ⓒ $m - 7$ Ⓓ $m + 8$

45. What is the quotient of $x^3 + 5x - 20$ divided by $x - 3$?

Ⓐ $x^2 - 3x + 14 + \dfrac{22}{x - 3}$ Ⓑ $x^2 + 3x + 14 + \dfrac{22}{x - 3}$

Ⓒ $x^2 + 8x + \dfrac{4}{x - 3}$ Ⓓ $x^2 + 3x - 14 + \dfrac{22}{x - 3}$

Maintain Your Skills

Mixed Review

Find each quotient. *(Lesson 12-4)*

46. $\dfrac{x^2 + 5x + 6}{x^2 - x - 12} \div \dfrac{x + 2}{x^2 + x - 20}$

47. $\dfrac{m^2 + m - 6}{m^2 + 8m + 15} \div \dfrac{m^2 - m - 2}{m^2 + 9m + 20}$

Find each product. *(Lesson 12-3)*

48. $\dfrac{b^2 + 19b + 84}{b - 3} \cdot \dfrac{b^2 - 9}{b^2 + 15b + 36}$

49. $\dfrac{z^2 + 16z + 39}{z^2 + 9z + 18} \cdot \dfrac{z + 5}{z^2 + 18z + 65}$

Simplify. Then use a calculator to verify your answer. *(Lesson 11-2)*

50. $3\sqrt{7} - \sqrt{7}$

51. $\sqrt{72} + \sqrt{32}$

52. $\sqrt{12} - \sqrt{18} + \sqrt{48}$

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*. *(Lesson 9-6)*

53. $d^2 - 3d - 40$

54. $x^2 + 8x + 16$

55. $t^2 + t + 1$

56. **BUSINESS** Jorge Martinez has budgeted \$150 to have business cards printed. A card printer charges \$11 to set up each job and an additional \$6 per box of 100 cards printed. What is the greatest number of cards Mr. Martinez can have printed? *(Lesson 6-3)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find each sum.**

*(To review **addition of polynomials**, see Lesson 8-5.)*

57. $(6n^2 - 6n + 10m^3) + (5n - 6m^3)$

58. $(3x^2 + 4xy - 2y^2) + (x^2 + 9xy + 4y^2)$

59. $(a^3 - b^3) + (-3a^3 - 2a^2b + b^2 - 2b^3)$

60. $(2g^3 + 6h) + (-4g^2 - 8h)$

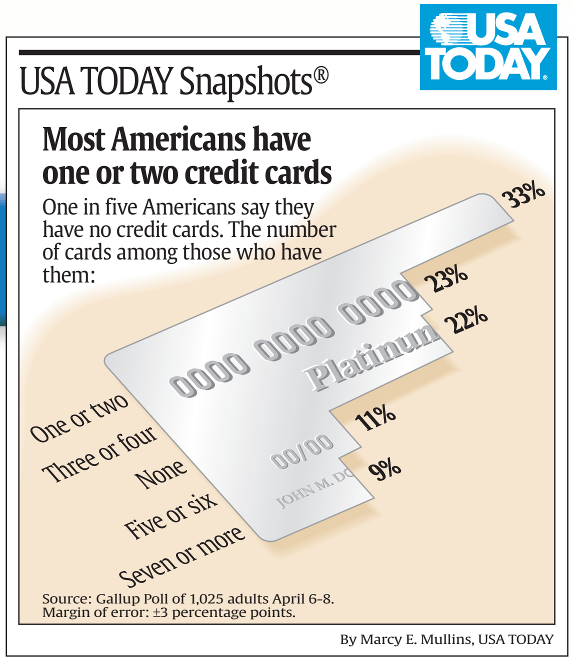
12-6 Rational Expressions with Like Denominators

What You'll Learn

- Add rational expressions with like denominators.
- Subtract rational expressions with like denominators.

How can you use rational expressions to interpret graphics?

The graphic at the right shows the number of credit cards Americans have. To determine what fraction of those surveyed have no more than two credit cards, you can use addition. Remember that percents can be written as fractions with denominators of 100.



$$\underbrace{\frac{22}{100}}_{\text{No credit cards}} \underbrace{+}_{\text{plus}} \underbrace{\frac{33}{100}}_{\text{one or two credit cards}} \underbrace{=}_{\text{equals}} \underbrace{\frac{55}{100}}_{\text{no more than two credit cards.}}$$

Thus, $\frac{55}{100}$ or 55% of those surveyed have no more than two credit cards.

ADD RATIONAL EXPRESSIONS Recall that to add fractions with like denominators you add the numerators and then write the sum over the common denominator. You can add rational expressions with like denominators in the same way.

Example 1 Numbers in Denominator

Find $\frac{3n}{12} + \frac{7n}{12}$.

$\frac{3n}{12} + \frac{7n}{12} = \frac{3n + 7n}{12}$ — The common denominator is 12.

$= \frac{10n}{12}$ — Add the numerators.

$= \frac{\cancelto{5}{10}n}{\cancelto{6}{12}}$ — Divide by the common factor, 2.

$= \frac{5n}{6}$ — Simplify.

Sometimes the denominators of rational expressions are binomials. As long as each rational expression has exactly the same binomial as its denominator, the process of adding is the same.

Example 2 Binomials in Denominator

Find $\frac{2x}{x+1} + \frac{2}{x+1}$.

$\frac{2x}{x+1} + \frac{2}{x+1} = \frac{2x+2}{x+1}$ — The common denominator is $x + 1$.

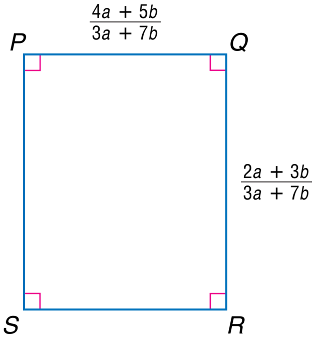
$= \frac{2(x+1)}{x+1}$ — Factor the numerator.

$= \frac{2\overset{1}{\cancel{(x+1)}}}{\underset{1}{\cancel{x+1}}}$ — Divide by the common factor, $x + 1$.

$= \frac{2}{1}$ or 2 — Simplify.

Example 3 Find a Perimeter

GEOMETRY **Find an expression for the perimeter of rectangle *PQRS*.**



$P = 2\ell + 2w$ — Perimeter formula

$= 2\left(\frac{4a+5b}{3a+7b}\right) + 2\left(\frac{2a+3b}{3a+7b}\right)$ — $\ell = \frac{4a+5b}{3a+7b}$, $w = \frac{2a+3b}{3a+7b}$

$= \frac{2(4a+5b) + 2(2a+3b)}{3a+7b}$ — The common denominator is $3a + 7b$.

$= \frac{8a + 10b + 4a + 6b}{3a+7b}$ — Distributive Property

$= \frac{12a+16b}{3a+7b}$ — Combine like terms.

$= \frac{4(3a+4b)}{3a+7b}$ — Factor.

The perimeter can be represented by the expression $\frac{4(3a+4b)}{3a+7b}$.

SUBTRACT RATIONAL EXPRESSIONS To subtract rational expressions with like denominators, subtract the numerators and write the difference over the common denominator. Recall that to subtract an expression, you add its additive inverse.

Example 4 Subtract Rational Expressions

Find $\frac{3x+4}{x-2} - \frac{x-1}{x-2}$.

$\frac{3x+4}{x-2} - \frac{x-1}{x-2} = \frac{(3x+4)-(x-1)}{x-2}$ — The common denominator is $x - 2$.

$= \frac{(3x+4)+[-(x-1)]}{x-2}$ — The additive inverse of $(x - 1)$ is $-(x - 1)$.

$= \frac{3x+4-x+1}{x-2}$ — Distributive Property

$= \frac{2x+5}{x-2}$ — Simplify.

Study Tip

Common Misconception
Adding the additive inverse will help you avoid the following error in the numerator.
$\cancel{(3x + 4) - (x - 1) =}$
$\cancel{3x + 4 - x - 1.}$

Sometimes you must express a denominator as its additive inverse to have like denominators.

Example 5 Inverse Denominators

Find $\frac{2m}{m-9} + \frac{4m}{9-m}$.

The denominator $9 - m$ is the same as $-(-9 + m)$ or $-(m - 9)$. Rewrite the second expression so that it has the same denominator as the first.

$$\frac{2m}{m-9} + \frac{4m}{9-m} = \frac{2m}{m-9} + \frac{4m}{-(m-9)} \qquad 9 - m = -(m-9)$$

$$= \frac{2m}{m-9} - \frac{4m}{m-9} \qquad \text{Rewrite using like denominators.}$$

$$= \frac{2m-4m}{m-9} \qquad \text{The common denominator is } m-9.$$

$$= \frac{-2m}{m-9} \qquad \text{Subtract.}$$

Check for Understanding

Concept Check

1. **OPEN ENDED** Write two rational expressions with a denominator of $x + 2$ that have a sum of 1.
2. **Describe** how adding rational expressions with like denominators is similar to adding fractions with like denominators.
3. **Compare and contrast** two rational expressions whose sum is 0 with two rational expressions whose difference is 0.
4. **FIND THE ERROR** Russell and Ginger are finding the difference of $\frac{7x+2}{4x-3}$ and $\frac{x-8}{3-4x}$.

Russell

$$\frac{7x+2}{4x-3} - \frac{x-8}{3-4x} = \frac{7x+2}{4x-3} + \frac{x-8}{4x-3}$$
$$= \frac{7x+x+2-8}{4x-3}$$
$$= \frac{8x-6}{4x-3}$$
$$= \frac{2(4x-3)}{4x-3}$$
$$= 2$$

Ginger

$$\frac{7x+2}{4x-3} - \frac{x-8}{3-4x} = \frac{-2-7x}{3-4x} - \frac{x-8}{3-4x}$$
$$= \frac{-2+8-7x-x}{3-4x}$$
$$= \frac{-6-8x}{3-4x}$$
$$= \frac{-2(3-4x)}{3-4x}$$

Who is correct? Explain your reasoning.

Guided Practice

Find each sum.

5. $\frac{a+2}{4} + \frac{a-2}{4}$
6. $\frac{3x}{x+1} + \frac{3}{x+1}$
7. $\frac{2-n}{n-1} + \frac{1}{n-1}$
8. $\frac{4t-1}{1-4t} + \frac{2t+3}{1-4t}$

Find each difference.

9. $\frac{5a}{12} - \frac{7a}{12}$
10. $\frac{7}{n-3} - \frac{4}{n-3}$
11. $\frac{3m}{m-2} - \frac{6}{2-m}$
12. $\frac{x^2}{x-y} - \frac{y^2}{x-y}$

Application

13. **SCHOOL** Most schools create daily attendance reports to keep track of their students. Suppose that one day, out of 960 students, 45 were absent due to illness, 29 were participating in a wrestling tournament, 10 were excused to go to their doctors, and 12 were at a music competition. What fraction of the students were absent from school on this day?

Practice and Apply

Homework Help

For Exercises	See Examples
14–17	1
18–25, 27, 42, 43, 45, 46	2, 3
28–35, 38, 39, 44, 47, 48	4
26, 36, 37	5

Extra Practice
See page 848.

Find each sum.

14. $\frac{m}{3} + \frac{2m}{3}$

15. $\frac{12z}{7} + \frac{-5z}{7}$

16. $\frac{x+3}{5} + \frac{x+2}{5}$

17. $\frac{n-7}{2} + \frac{n+5}{2}$

18. $\frac{2y}{y+3} + \frac{6}{y+3}$

19. $\frac{3r}{r+5} + \frac{15}{r+5}$

20. $\frac{k-5}{k-1} + \frac{4}{k-1}$

21. $\frac{n-2}{n+3} + \frac{-1}{n+3}$

22. $\frac{4x-5}{x-2} + \frac{x+3}{x-2}$

23. $\frac{2a+3}{a-4} + \frac{a-2}{a-4}$

24. $\frac{5s+1}{2s+1} + \frac{3s-2}{2s+1}$

25. $\frac{9b+3}{2b+6} + \frac{5b+4}{2b+6}$

26. What is the sum of $\frac{12x-7}{3x-2}$ and $\frac{9x-5}{2-3x}$?

27. Find the sum of $\frac{11x-5}{2x+5}$ and $\frac{11x+12}{2x+5}$.

Find each difference.

28. $\frac{5x}{7} - \frac{3x}{7}$

29. $\frac{4n}{3} - \frac{2n}{3}$

30. $\frac{x+4}{5} - \frac{x+2}{5}$

31. $\frac{a+5}{6} - \frac{a+3}{6}$

32. $\frac{2}{x+7} - \frac{-5}{x+7}$

33. $\frac{4}{z-2} - \frac{-6}{z-2}$

34. $\frac{5}{3x-5} - \frac{3x}{3x-5}$

35. $\frac{4}{7m-2} - \frac{7m}{7m-2}$

36. $\frac{2x}{x-2} - \frac{2x}{2-x}$

37. $\frac{5y}{y-3} - \frac{5y}{3-y}$

38. $\frac{8}{3t-4} - \frac{6t}{3t-4}$

39. $\frac{15x}{5x+1} - \frac{-3}{5x+1}$

40. Find the difference of $\frac{10a-12}{2a-6}$ and $\frac{6a}{6-2a}$.

41. What is the difference of $\frac{b-15}{2b+12}$ and $\frac{-3b+8}{2b+12}$?

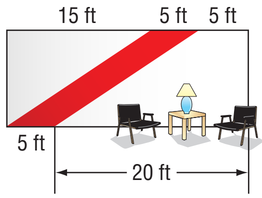
42. **POPULATION** The United States population in 1998 is described in the table. Use this information to write the fraction of the population that is 80 years or older.

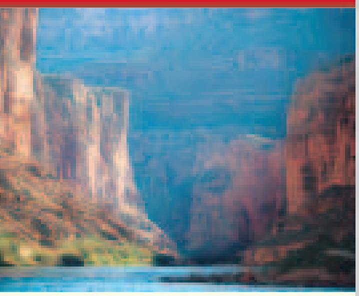
Age	Number of People
0–19	77,525,000
20–39	79,112,000
40–59	68,699,000
60–79	35,786,000
80–99	8,634,000
100+	61,000

Source: *Statistical Abstract of the United States*

43. **CONSERVATION** The freshman class chose to plant spruce and pine trees at a wildlife sanctuary for a service project. Some students can plant 140 trees on Saturday, and others can plant 20 trees after school on Monday and again on Tuesday. Write an expression for the fraction of the trees that could be planted on these days if n represents the number of spruce trees and there are twice as many pine trees.

44. **GEOMETRIC DESIGN** A student center is a square room that is 25 feet wide and 25 feet long. The walls are 10 feet high and each wall is painted white with a red diagonal stripe as shown. What fraction of the walls are painted red?





More About. . .

Hiking

Due to its popularity, the Grand Canyon is one of the most threatened natural areas in the United States.

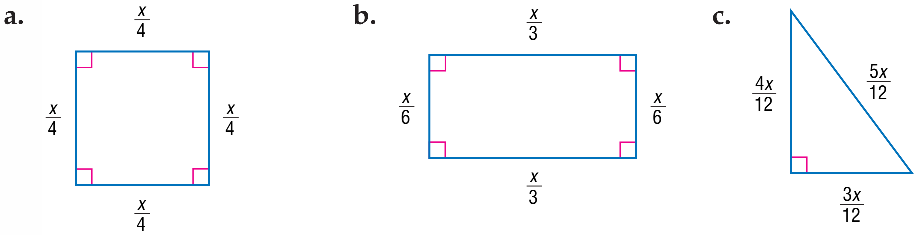
Source: The Wildlife Foundation

HIKING **For Exercises 45 and 46, use the following information.**
A tour guide recommends that hikers carry a gallon of water on hikes to the bottom of the Grand Canyon. Water weighs 62.4 pounds per cubic foot, and one cubic foot of water contains 7.48 gallons.

45. Tanika plans to carry two 1-quart bottles and four 1-pint bottles for her hike. Write a rational expression for this amount of water written as a fraction of a cubic foot.

46. How much does this amount of water weigh?

GEOMETRY **For Exercises 47 and 48, use the following information.**
Each figure has a perimeter of x units.



47. Find the ratio of the area of each figure to its perimeter.

48. Which figure has the greatest ratio?

49. **CRITICAL THINKING** Which of the following rational numbers is not equivalent to the others?

a. $\frac{3}{2-x}$ b. $\frac{-3}{x-2}$ c. $-\frac{3}{2-x}$ d. $-\frac{3}{x-2}$

50. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can you use rational expressions to interpret graphics?

Include the following in your answer:
- an explanation of how the numbers in the graphic relate to rational expressions, and
- a description of how to add two rational expressions whose denominators are $3x - 4y$ and $4y - 3x$.

Standardized Test Practice

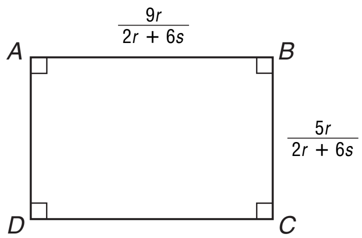
51. Find $\frac{k+2}{k-7} + \frac{-3}{k-7}$.

(A) $\frac{k-1}{k-7}$ (B) $\frac{k-5}{k-7}$ (C) $\frac{k+1}{k-7}$ (D) $\frac{k+5}{k-7}$

52. Which is an expression for the perimeter of rectangle $ABCD$?

(A) $\frac{14r}{2r+6s}$ (B) $\frac{14r}{r+3s}$

(C) $\frac{14r}{r+6s}$ (D) $\frac{28r}{r+3s}$



Maintain Your Skills

Mixed Review

Find each quotient. *(Lessons 12-4 and 12-5)*

53. $\dfrac{x^3 - 7x + 6}{x - 2}$

54. $\dfrac{56x^3 + 32x^2 - 63x - 36}{7x + 4}$

55. $\dfrac{b^2 - 9}{4b} \div (b - 3)$

56. $\dfrac{x}{x + 2} \div \dfrac{x^2}{x^2 + 5x + 6}$

Factor each trinomial. *(Lesson 9-3)*

57. $a^2 + 9a + 14$

58. $p^2 + p - 30$

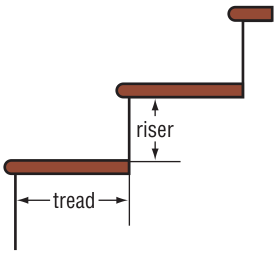
59. $y^2 - 11yz + 28z^2$

Find each sum or difference. *(Lesson 8-5)*

60. $(3x^2 - 4x) - (7 - 9x)$

61. $(5x^2 - 6x + 14) + (2x^2 + 3x + 8)$

62. CARPENTRY When building a stairway, a carpenter considers the ratio of riser to tread. If each stair being built is to have a width of 1 foot and a height of 8 inches, what will be the slope of the stairway?



Getting Ready for the Next Lesson

BASIC SKILL **Find the least common multiple for each set of numbers.**

63. 4, 9, 12

64. 7, 21, 5

65. 6, 12, 24

66. 45, 10, 6

67. 5, 6, 15

68. 8, 9, 12

69. 16, 20, 25

70. 36, 48, 60

71. 9, 16, 24

Practice Quiz 2

Lessons 12-4 through 12-6

Find each quotient. *(Lessons 12-4 and 12-5)*

1. $\dfrac{a}{a + 3} \div \dfrac{a + 11}{a + 3}$

2. $\dfrac{4z + 8}{z + 3} \div (z + 2)$

3. $\dfrac{(2x - 1)(x - 2)}{(x - 2)(x - 3)} \div \dfrac{(2x - 1)(x + 5)}{(x - 3)(x - 1)}$

4. $(9xy^2 - 15xy + 3) \div 3xy$

5. $(2x^2 - 7x - 16) \div (2x + 3)$

6. $\dfrac{y^2 - 19y + 9}{y - 4}$

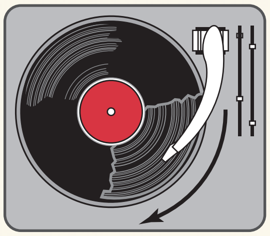
Find each sum or difference. *(Lesson 12-6)*

7. $\dfrac{2}{x + 7} + \dfrac{5}{x + 7}$

8. $\dfrac{2m}{m + 3} - \dfrac{-6}{m + 3}$

9. $\dfrac{5x - 1}{3x + 2} - \dfrac{2x - 1}{3x + 2}$

10. MUSIC Suppose the record shown played for 16.5 minutes on one side and the average of the radii of the grooves on the record was $3\frac{3}{4}$ inches. Write an expression involving units that represents how many inches the needle passed through the grooves while the record was being played. Then evaluate the expression.



$33\frac{1}{3}$ revolutions per minute

12-7 Rational Expressions with Unlike Denominators

What You'll Learn

- Add rational expressions with unlike denominators.
- Subtract rational expressions with unlike denominators.

Vocabulary

- least common multiple (LCM)
- least common denominator (LCD)

How can rational expressions be used to describe elections?

The President of the United States is elected every four years, and senators are elected every six years. A certain senator is elected in 2004, the same year as a presidential election, and is reelected in subsequent elections. In what year is the senator's reelection the same year as a presidential election?

ADD RATIONAL EXPRESSIONS The number of years in which a specific senator's election coincides with a presidential election is related to the common multiples of 4 and 6. The least number of years that will pass until the next election for both a specific senator and the President is the least common multiple of these numbers. The **least common multiple (LCM)** is the least number that is a common multiple of two or more numbers.

Example 1 LCM of Monomials

Find the LCM of $15m^2b^3$ and $18mb^2$.

Find the prime factors of each coefficient and variable expression.

$15m^2b^3 = 3 \cdot 5 \cdot m \cdot m \cdot b \cdot b \cdot b$

$18mb^2 = 2 \cdot 3 \cdot 3 \cdot m \cdot b \cdot b$

Use each prime factor the greatest number of times it appears in any of the factorizations.

$15m^2b^3 = 3 \cdot 5 \cdot m \cdot m \cdot b \cdot b \cdot b$

$18mb^2 = 2 \cdot 3 \cdot 3 \cdot m \cdot b \cdot b$

LCM $= 2 \cdot 3 \cdot 3 \cdot 5 \cdot m \cdot m \cdot b \cdot b \cdot b$ or $90m^2b^3$

Example 2 LCM of Polynomials

Find the LCM of $x^2 + 8x + 15$ and $x^2 + x - 6$.

Express each polynomial in factored form.

$x^2 + 8x + 15 = (x + 3)(x + 5)$

$x^2 + x - 6 = (x - 2)(x + 3)$

Use each factor the greatest number of times it appears.

LCM $= (x - 2)(x + 3)(x + 5)$

Recall that to add fractions with unlike denominators, you need to rename the fractions using the least common multiple (LCM) of the denominators, known as the **least common denominator (LCD)**.

Key Concept *Add Rational Expressions*

Use the following steps to add rational expressions with unlike denominators.

Step 1 Find the LCD.

Step 2 Change each rational expression into an equivalent expression with the LCD as the denominator.

Step 3 Add just as with rational expressions with like denominators.

Step 4 Simplify if necessary.

Example 3 Monomial Denominators

Find $\frac{a+1}{a} + \frac{a-3}{3a}$.

Factor each denominator and find the LCD.

$a = a$

$3a = 3 \cdot a$

$\text{LCD} = 3a$

Since the denominator of $\frac{a-3}{3a}$ is already $3a$, only $\frac{a+1}{a}$ needs to be renamed.

$$\begin{aligned}
\frac{a+1}{a} + \frac{a-3}{3a} &= \frac{3(a+1)}{3(a)} + \frac{a-3}{3a} && \text{Multiply } \frac{a+1}{a} \text{ by } \frac{3}{3}. \\
&= \frac{3a+3}{3a} + \frac{a-3}{3a} && \text{Distributive Property} \\
&= \frac{3a+3+a-3}{3a} && \text{Add the numerators.} \\
&= \frac{4\overset{1}{\cancel{a}}}{3\underset{1}{\cancel{a}}} && \text{Divide out the common factor } a. \\
&= \frac{4}{3} && \text{Simplify.}
\end{aligned}$$

Example 4 Polynomial Denominators

Find $\frac{y-2}{y^2+4y+4} + \frac{y-2}{y+2}$.

$$\begin{aligned}
\frac{y-2}{y^2+4y+4} + \frac{y-2}{y+2} &= \frac{y-2}{(y+2)^2} + \frac{y-2}{y+2} && \text{Factor the denominators.} \\
&= \frac{y-2}{(y+2)^2} + \frac{y-2}{y+2} \cdot \frac{y+2}{y+2} && \text{The LCD is } (y+2)^2. \\
&= \frac{y-2}{(y+2)^2} + \frac{y^2-4}{(y+2)^2} && (y-2)(y+2) = y^2-4 \\
&= \frac{y-2+y^2-4}{(y+2)^2} && \text{Add the numerators.} \\
&= \frac{y^2+y-6}{(y+2)^2} \text{ or } \frac{(y-2)(y+3)}{(y+2)^2} && \text{Simplify.}
\end{aligned}$$

SUBTRACT RATIONAL EXPRESSIONS As with addition, to subtract rational expressions with unlike denominators, you must first rename the expressions using a common denominator.

Example 5 Binomials in Denominators

Find $\dfrac{4}{3a-6} - \dfrac{a}{a+2}$.

$$\frac{4}{3a-6} - \frac{a}{a+2} = \frac{4}{3(a-2)} - \frac{a}{a+2} \quad \text{Factor.}$$

$$= \frac{4(a+2)}{3(a-2)(a+2)} - \frac{3a(a-2)}{3(a+2)(a-2)} \quad \text{The LCD is } 3(a+2)(a-2).$$

$$= \frac{4(a+2) - 3a(a-2)}{3(a-2)(a+2)} \quad \text{Subtract the numerators.}$$

$$= \frac{4a + 8 - 3a^2 + 6a}{3(a-2)(a+2)} \quad \text{Multiply.}$$

$$= \frac{-3a^2 + 10a + 8}{3(a-2)(a+2)} \text{ or } -\frac{3a^2 - 10a - 8}{3(a-2)(a+2)} \quad \text{Simplify.}$$

Standardized Test Practice

A B C D

Example 6 Polynomials in Denominators

Multiple-Choice Test Item

Find $\dfrac{h-2}{h^2+4h+4} - \dfrac{h-4}{h^2-4}$.

(A) $\dfrac{2h-12}{(h-2)(h+2)^2}$ (B) $\dfrac{-2h+12}{(h-2)(h+2)^2}$

(C) $\dfrac{2h-12}{(h-2)^2(h+2)}$ (D) $\dfrac{-2h+12}{(h-2)(h+2)}$

Test-Taking Tip

Examine all of the answer choices carefully. Look for differences in operations, positive and negative signs, and exponents.

Read the Test Item

The expression $\dfrac{h-2}{h^2+4h+4} - \dfrac{h-4}{h^2-4}$ represents the difference of two rational expressions with unlike denominators.

Solve the Test Item

Step 1 Factor each denominator and find the LCD.

$h^2 + 4h + 4 = (h+2)^2$

$h^2 - 4 = (h+2)(h-2)$

The LCD is $(h-2)(h+2)^2$.

Step 2 Change each rational expression into an equivalent expression with the LCD. Then subtract.

$$\frac{h-2}{(h+2)^2} - \frac{h-4}{(h+2)(h-2)} = \frac{(h-2)}{(h+2)^2}\cdot\frac{(h-2)}{(h-2)} - \frac{(h-4)}{(h+2)(h-2)}\cdot\frac{(h+2)}{(h+2)}$$

$$= \frac{(h-2)(h-2)}{(h+2)^2(h-2)} - \frac{(h-4)(h+2)}{(h+2)^2(h-2)}$$

$$= \frac{h^2-4h+4}{(h+2)^2(h-2)} - \frac{h^2-2h-8}{(h+2)^2(h-2)}$$

$$= \frac{(h^2-4h+4)-(h^2-2h-8)}{(h+2)^2(h-2)}$$

$$= \frac{h^2-h^2-4h+2h+4+8}{(h+2)^2(h-2)}$$

$$= \frac{-2h+12}{(h-2)(h+2)^2}$$

The correct answer is B.

Check for Understanding

Concept Check

1. **Describe** how to find the LCD of two rational expressions with unlike denominators.
2. **Explain** how to rename rational expressions using their LCD.
3. **OPEN ENDED** Give an example of two rational expressions in which the LCD is equal to twice the denominator of one of the expressions.

Guided Practice

Find the LCM for each pair of expressions.

4. $5a^2, 7a$
5. $2x - 4, 3x - 6$
6. $n^2 + 3n - 4, (n - 1)^2$

Find each sum.

7. $\dfrac{6}{5x} + \dfrac{7}{10x^2}$
8. $\dfrac{a}{a-4} + \dfrac{4}{a+4}$
9. $\dfrac{2y}{y^2-25} + \dfrac{y+5}{y-5}$
10. $\dfrac{a+2}{a^2+4a+3} + \dfrac{6}{a+3}$

Find each difference.

11. $\dfrac{3z}{6w^2} - \dfrac{z}{4w}$
12. $\dfrac{4a}{2a+6} - \dfrac{3}{a+3}$
13. $\dfrac{b+8}{b^2-16} - \dfrac{1}{b-4}$
14. $\dfrac{x}{x-2} - \dfrac{3}{x^2+3x-10}$

Standardized Test Practice

15. Find $\dfrac{2y}{y^2+7y+12} + \dfrac{y+2}{y+4}$.

(A) $\dfrac{y^2+5y+6}{(y+4)(y+3)}$
(B) $\dfrac{y^2+2y+6}{(y+4)(y+3)}$
(C) $\dfrac{y^2+7y+6}{(y+4)(y+3)}$
(D) $\dfrac{y^2-5y+6}{(y+4)(y+3)}$

Practice and Apply

Homework Help

For Exercises	See Examples
16, 17, 54–57	1
18–21	2
22–25	3
26–37	4
38–49	5
50–53	6

Extra Practice
See page 848.

Find the LCM for each pair of expressions.

16. a^2b, ab^3
17. $7xy, 21x^2y$
18. $x - 4, x + 2$
19. $2n - 5, n + 2$
20. $x^2 + 5x - 14, (x - 2)^2$
21. $p^2 - 5p - 6, p + 1$

Find each sum.

22. $\dfrac{3}{x^2} + \dfrac{5}{x}$
23. $\dfrac{2}{a^3} + \dfrac{7}{a^2}$
24. $\dfrac{7}{6a^2} + \dfrac{5}{3a}$
25. $\dfrac{3}{7m} + \dfrac{4}{5m^2}$
26. $\dfrac{3}{x+5} + \dfrac{4}{x-4}$
27. $\dfrac{n}{n+4} + \dfrac{3}{n-3}$
28. $\dfrac{7a}{a+5} + \dfrac{a}{a-2}$
29. $\dfrac{6x}{x-3} + \dfrac{x}{x+1}$
30. $\dfrac{5}{3x-9} + \dfrac{3}{x-3}$
31. $\dfrac{m}{3m+2} + \dfrac{2}{9m+6}$
32. $\dfrac{-3}{5-a} + \dfrac{5}{a^2-25}$
33. $\dfrac{18}{y^2-9} + \dfrac{-7}{3-y}$
34. $\dfrac{x}{x^2+2x+1} + \dfrac{1}{x+1}$
35. $\dfrac{2x+1}{(x-1)^2} + \dfrac{x-2}{x^2+3x-4}$
36. $\dfrac{x^2}{4x^2-9} + \dfrac{x}{(2x+3)^2}$
37. $\dfrac{a^2}{a^2-b^2} + \dfrac{a}{(a-b)^2}$

Find each difference.

38. $\frac{7}{3x} - \frac{3}{6x^2}$

39. $\frac{4}{15x^2} - \frac{5}{3x}$

40. $\frac{11x}{3y^2} - \frac{7x}{6y}$

41. $\frac{5a}{7x} - \frac{3a}{21x^2}$

42. $\frac{x^2 - 1}{x + 1} - \frac{x^2 + 1}{x - 1}$

43. $\frac{k}{k + 5} - \frac{3}{k - 3}$

44. $\frac{k}{2k + 1} - \frac{2}{k + 2}$

45. $\frac{m - 1}{m + 1} - \frac{4}{2m + 5}$

46. $\frac{2x}{x^2 - 5x} - \frac{-3x}{x - 5}$

47. $\frac{-3}{a - 6} - \frac{-6}{a^2 - 6a}$

48. $\frac{n}{5 - n} - \frac{3}{n^2 - 25}$

49. $\frac{3a + 2}{6 - 3a} - \frac{a + 2}{a^2 - 4}$

50. $\frac{3x}{x^2 + 3x + 2} - \frac{3x - 6}{x^2 + 4x + 4}$

51. $\frac{5a}{a^2 + 3a - 4} - \frac{a - 1}{a^2 - 1}$

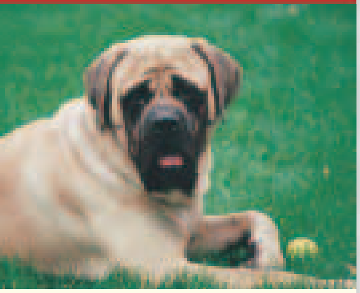
52. $\frac{x^2 + 4x - 5}{x^2 - 2x - 3} - \frac{2}{x + 1}$

53. $\frac{m - 4}{m^2 + 8m + 16} - \frac{m + 4}{m - 4}$

54. **MUSIC** A music director wants to form a group of students to sing and dance at community events. The music they will sing is 2-part, 3-part, or 4-part harmony. The director would like to have the same number of voices on each part. What is the least number of students that would allow for an even distribution on all these parts?

55. **CHARITY** Maya, Makalla, and Monya can walk one mile in 12, 15, and 20 minutes respectively. They plan to participate in a walk-a-thon to raise money for a local charity. Sponsors have agreed to pay $2.50 for each mile that is walked. What is the total number of miles the girls would walk in one hour and how much money would they raise?

More About. . .



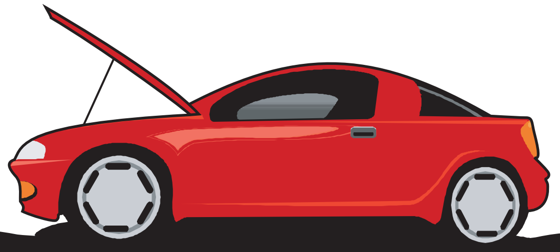
Pet Care

Kell, an English Mastiff owned by Tom Scott of the United Kingdom, is the heaviest dog in the world. Weighing in at 286 pounds, Kell eats a high protein diet of eggs, goat's milk, and beef.

Source: *The Guinness Book of Records*

56. **PET CARE** Kendra takes care of pets while their owners are out of town. One week she has three dogs that all eat the same kind of dog food. The first dog eats a bag of food every 12 days, the second dog eats a bag every 15 days, and the third dog eats a bag every 16 days. How many bags of food should Kendra buy for one week?

57. **AUTOMOBILES** Car owners need to follow a regular maintenance schedule to keep their cars running safely and efficiently. The table shows several items that should be performed on a regular basis. If all of these items are performed when a car's odometer reads 36,000 miles, what would be the car's mileage reading the next time all of the items should be performed?



Inspection or Service	Frequency
engine oil and oil filter change	every 3000 miles (about 3 months)
transmission fluid level check	every oil change
brake system inspection	every oil change
chassis lubrication	every 6000 miles
power steering pump fluid level check	every 6000 miles
tire and wheel rotation and inspection	every 15,000 miles

58. CRITICAL THINKING Janelle says that a shortcut for adding fractions with unlike denominators is to add the cross products for the numerator and write the denominator as the product of the denominators. She gives the following example.

$$\frac{2}{7} + \frac{5}{8} = \frac{2 \cdot 8 + 5 \cdot 7}{7 \cdot 8} = \frac{51}{56}$$

Explain why Janelle's method will always work or provide a counterexample to show that it does not always work.

59. *WRITING IN MATH* Answer the question that was posed at the beginning of the lesson.

How can rational expressions be used to describe elections?

Include the following in your answer:

- an explanation of how to determine the least common multiple of two or more rational expressions, and
- if a certain senator is elected in 2006, when is the next election in which the senator and a President will be elected?

Standardized Test Practice

60. What is the least common denominator of $\frac{6}{a^2 - 2ab + b^2}$ and $\frac{6}{a^2 - b^2}$?

(A) $(a - b)^2$ (B) $(a - b)(a + b)$

(C) $(a + b)^2$ (D) $(a - b)^2(a + b)$

61. Find $\frac{x - 4}{(2 - x)^2} - \frac{x - 5}{x^2 + x - 6}$.

(A) $\frac{8x - 22}{(x + 3)(x - 2)^2}$ (B) $\frac{x^2 - 2x - 17}{(x - 2)(x + 3)}$

(C) $\frac{6x - 22}{(x + 3)(x - 2)^2}$ (D) $\frac{22 - 6x}{(x + 3)(x - 2)}$

Maintain Your Skills

Mixed Review

Find each sum. *(Lesson 12-6)*

62. $\frac{3m}{2m + 1} + \frac{3}{2m + 1}$ **63.** $\frac{4x}{2x + 3} + \frac{5}{2x + 3}$ **64.** $\frac{2y}{y - 3} + \frac{5}{3 - y}$

Find each quotient. *(Lesson 12-5)*

65. $\frac{b^2 + 8b - 20}{b - 2}$ **66.** $\frac{t^3 - 19t + 9}{t - 4}$ **67.** $\frac{4m^2 + 8m - 19}{2m + 7}$

Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write *prime*. *(Lesson 9-4)*

68. $2x^2 + 10x + 8$ **69.** $5r^2 + 7r - 6$ **70.** $16p^2 - 4pq - 30q^2$

71. BUDGETING JoAnne Paulsen's take-home pay is \$1782 per month. She spends \$525 on rent, \$120 on groceries, and \$40 on gas. She allows herself 5% of the remaining amount for entertainment. How much can she spend on entertainment each month? *(Lesson 3-9)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find each quotient.**

*(To review **dividing rational expressions**, see Lesson 12-4.)*

72. $\frac{x}{2} \div \frac{3x}{5}$ **73.** $\frac{a^2}{5b} \div \frac{4a}{10b^2}$ **74.** $\frac{x + 7}{x} \div \frac{x + 7}{x + 3}$

75. $\frac{3n}{2n + 5} \div \frac{12n^2}{2n + 5}$ **76.** $\frac{3x}{x + 2} \div (x - 1)$ **77.** $\frac{x^2 + 7x + 12}{x + 6} \div (x + 3)$

12-8 Mixed Expressions and Complex Fractions

What You'll Learn

- Simplify mixed expressions.
- Simplify complex fractions.

Vocabulary

- mixed expression
- complex fraction

How are rational expressions used in baking?

Katelyn bought $2\frac{1}{2}$ pounds of chocolate chip cookie dough. If the average cookie requires $1\frac{1}{2}$ ounces of dough, the number of cookies that Katelyn can bake can be found by simplifying the expression $\dfrac{2\frac{1}{2}\text{ pounds}}{1\frac{1}{2}\text{ ounces}}$.

SIMPLIFY MIXED EXPRESSIONS Recall that a number like $2\frac{1}{2}$ is a mixed number because it contains the sum of an integer, 2, and a fraction, $\frac{1}{2}$. An expression like $3 + \frac{x+2}{x-3}$ is called a **mixed expression** because it contains the sum of a monomial, 3, and a rational expression, $\frac{x+2}{x-3}$. Changing mixed expressions to rational expressions is similar to changing mixed numbers to improper fractions.

Example 1 Mixed Expression to Rational Expression

Simplify $3 + \frac{6}{x+3}$.

$$3 + \frac{6}{x+3} = \frac{3(x+3)}{x+3} + \frac{6}{x+3} \quad \text{The LCD is } x + 3.$$

$$= \frac{3(x+3)+6}{x+3} \quad \text{Add the numerators.}$$

$$= \frac{3x+9+6}{x+3} \quad \text{Distributive Property}$$

$$= \frac{3x+15}{x+3} \quad \text{Simplify.}$$

SIMPLIFY COMPLEX FRACTIONS If a fraction has one or more fractions in the numerator or denominator, it is called a **complex fraction**. You simplify an algebraic complex fraction in the same way that you simplify a numerical complex fraction.

numerical complex fraction

$$\frac{\frac{8}{3}}{\frac{7}{5}} = \frac{8}{3} \div \frac{7}{5}$$

$$= \frac{8}{3} \cdot \frac{5}{7}$$

$$= \frac{40}{21}$$

algebraic complex fraction

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \div \frac{c}{d}$$

$$= \frac{a}{b} \cdot \frac{d}{c}$$

$$= \frac{ad}{bc}$$

Key Concept — Simplifying a Complex Fraction

Any complex fraction $\frac{\frac{a}{b}}{\frac{c}{d}}$, where $b \neq 0$, $c \neq 0$, and $d \neq 0$, can be expressed as $\frac{ad}{bc}$.

Study Tip

Fraction Bar
Recall that when applying the order of operations, pfy the numerator and denominator of a complex fraction before proceeding with division.

Example 2 Complex Fraction Involving Numbers

BAKING **Refer to the application at the beginning of the lesson. How many cookies can Katelyn make with $2\frac{1}{2}$ pounds of chocolate chip cookie dough?**

To find the total number of cookies, divide the amount of cookie dough by the amount of dough needed for each cookie.

$$\frac{2\frac{1}{2}\text{ pounds}}{1\frac{1}{2}\text{ ounces}} = \frac{2\frac{1}{2}\cancel{\text{ pounds}}}{1\frac{1}{2}\cancel{\text{ ounces}}} \cdot \frac{16\cancel{\text{ ounces}}}{1\cancel{\text{ pound}}}$$ Convert pounds to ounces. Divide by common units.

$$= \frac{16 \cdot 2\frac{1}{2}}{1\frac{1}{2}}$$ Simplify.

$$= \frac{\frac{16}{1} \cdot \frac{5}{2}}{\frac{3}{2}}$$ Express each term as an improper fraction.

$$= \frac{\frac{80}{2}}{\frac{3}{2}}$$ Multiply in the numerator.

$$= \frac{80 \cdot 2}{2 \cdot 3}$$ $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc}$

$$= \frac{160}{6} \text{ or } 26\frac{2}{3}$$ Simplify.

Katelyn can make 27 cookies.

Example 3 Complex Fraction Involving Monomials

Simplify $\frac{\frac{x^2y^2}{a}}{\frac{x^2y}{a^3}}$.

$$\frac{\frac{x^2y^2}{a}}{\frac{x^2y}{a^3}} = \frac{x^2y^2}{a} \div \frac{x^2y}{a^3}$$ Rewrite as a division sentence.

$$= \frac{x^2y^2}{a} \cdot \frac{a^3}{x^2y}$$ Rewrite as multiplication by the reciprocal.

$$= \frac{\overset{1}{\cancel{x^2}}\,\overset{y}{\cancel{y^2}}}{\underset{1}{\cancel{a}}} \cdot \frac{\overset{a^2}{\cancel{a^3}}}{\underset{1}{\cancel{x^2}}\,\underset{1}{\cancel{y}}}$$ Divide by common factors x^2, y, and a.

$$= a^2y$$ Simplify.

Example 4 Complex Fraction Involving Polynomials

Simplify $\dfrac{a - \dfrac{15}{a-2}}{a+3}$.

The numerator contains a mixed expression. Rewrite it as a rational expression first.

$$\frac{a - \frac{15}{a-2}}{a+3} = \frac{\frac{a(a-2)}{a-2} - \frac{15}{a-2}}{a+3}$$ The LCD of the fractions in the numerator is $a - 2$.

$$= \frac{\frac{a^2 - 2a - 15}{a-2}}{a+3}$$ Simplify the numerator.

$$= \frac{\frac{(a+3)(a-5)}{a-2}}{a+3}$$ Factor.

$$= \frac{(a+3)(a-5)}{a-2} \div (a+3)$$ Rewrite as a division sentence.

$$= \frac{(a+3)(a-5)}{a-2} \cdot \frac{1}{a+3}$$ Multiply by the reciprocal of $a + 3$.

$$= \frac{\overset{1}{\cancel{(a+3)}}(a-5)}{a-2} \cdot \frac{1}{\underset{1}{\cancel{a+3}}}$$ Divide by the GCF, $a + 3$.

$$= \frac{a-5}{a-2}$$ Simplify.

Check for Understanding

Concept Check

1. **Describe** the similarities between mixed numbers and mixed rational expressions.

2. **OPEN ENDED** Give an example of a complex fraction and show how to simplify it.

3. **FIND THE ERROR** Bolton and Lian found the LCD of $\frac{4}{2x+1} - \frac{5}{x+1} + \frac{2}{x-1}$.

Bolton

$\frac{4}{2x+1} - \frac{5}{x+1} + \frac{2}{x-1}$

LCD: $(2x+1)(x+1)(x-1)$

Lian

$\frac{4}{2x+1} - \frac{5}{x+1} + \frac{2}{x-1}$

LCD: $2(x+1)(x-1)$

Who is correct? Explain your reasoning.

Guided Practice

Write each mixed expression as a rational expression.

4. $3 + \frac{4}{x}$

5. $7 + \frac{5}{6y}$

6. $\frac{a-1}{3a} + 2a$

Simplify each expression.

7. $\dfrac{3\frac{1}{2}}{4\frac{3}{4}}$

8. $\dfrac{\frac{x^3}{y^2}}{\frac{y^3}{x}}$

9. $\dfrac{\frac{x-y}{a+b}}{\frac{x^2-y^2}{a^2-b^2}}$

Application

10. **ENTERTAINMENT** The student talent committee is arranging the performances for their holiday pageant. The first-act performances and their lengths are shown in the table. What is the average length of the performances?

Holiday Pageant Line-Up

Performance	Length (min)
A	7
B	$4\frac{1}{2}$
C	$6\frac{1}{2}$
D	$8\frac{1}{4}$
E	$10\frac{1}{5}$

Practice and Apply

Homework Help

For Exercises	See Examples
11–22, 35	1
23–26, 37–40	2
27–32, 36	3
33, 34	4

Extra Practice
See page 848.

Write each mixed expression as a rational expression.

11. $8 + \frac{3}{n}$
12. $4 + \frac{5}{a}$
13. $2x + \frac{x}{y}$
14. $6z + \frac{2z}{w}$
15. $2m - \frac{4 + m}{m}$
16. $3a - \frac{a + 1}{2a}$
17. $b^2 + \frac{a - b}{a + b}$
18. $r^2 + \frac{r - 4}{r + 3}$
19. $5n^2 - \frac{n + 3}{n^2 - 9}$
20. $3s^2 - \frac{s + 1}{s^2 - 1}$
21. $(x - 5) + \frac{x + 2}{x - 3}$
22. $(p + 4) + \frac{p + 1}{p - 4}$

Simplify each expression.

23. $\dfrac{5\frac{3}{4}}{7\frac{2}{3}}$
24. $\dfrac{8\frac{2}{7}}{4\frac{4}{5}}$
25. $\dfrac{\frac{a}{b^3}}{\frac{a^2}{b}}$
26. $\dfrac{\frac{n^3}{m^2}}{\frac{n^2}{m^2}}$
27. $\dfrac{\frac{x + 4}{y - 2}}{\frac{x^2}{y^2}}$
28. $\dfrac{\frac{s^3}{t^2}}{\frac{s + t}{s - t}}$
29. $\dfrac{\frac{y^2 - 1}{y^2 + 3y - 4}}{y + 1}$
30. $\dfrac{\frac{a^2 - 2a - 3}{a^2 - 1}}{a - 3}$
31. $\dfrac{\frac{n^2 + 2n}{n^2 + 9n + 18}}{\frac{n^2 - 5n}{n^2 + n - 30}}$
32. $\dfrac{\frac{x^2 + 4x - 21}{x^2 - 9x + 18}}{\frac{x^2 + 3x - 28}{x^2 - 10x + 24}}$
33. $\dfrac{x - \frac{15}{x - 2}}{x - \frac{20}{x - 1}}$
34. $\dfrac{n + \frac{35}{n + 12}}{n - \frac{63}{n - 2}}$

35. What is the quotient of $b + \frac{1}{b}$ and $a + \frac{1}{a}$?

36. What is the product of $\frac{2b^2}{5c}$ and the quotient of $\frac{4b^3}{2c}$ and $\frac{7b^3}{8c^2}$?

37. **PARTIES** The student council is planning a party for the school volunteers. There are five 66-ounce bottles of soda left from a recent dance. When poured over ice, $5\frac{1}{2}$ ounces of soda fills a cup. How many servings of soda can they get from the bottles they have?

ACOUSTICS **For Exercises 38 and 39, use the following information.**
If a vehicle is moving toward you at v miles per hour and blowing its horn at a frequency of f, then you hear the horn as if it were blowing at a frequency of h. This can be defined by the equation $h = \frac{f}{1 - \frac{v}{s}}$, where s is the speed of sound, approximately 760 miles per hour.

38. Simplify the complex fraction in the formula.

39. Suppose a truck horn blows at 370 cycles per second and is moving toward you at 65 miles per hour. Find the frequency of the horn as you hear it.

40. **POPULATION** According to the 2000 Census, New Jersey was the most densely populated state, and Alaska was the least densely populated state. The population of New Jersey was 8,414,350, and the population of Alaska was 626,932. The land area of New Jersey is about 7419 square miles, and the land area of Alaska is about 570,374 square miles. How many more people were there per square mile in New Jersey than in Alaska?

41. **BICYCLES** When air is pumped into a bicycle tire, the pressure P required varies inversely as the volume of the air V and is given by the equation $P = \frac{k}{V}$. If the pressure is 30 lb/in^2 when the volume is $1\frac{2}{3}$ cubic feet, find the pressure when the volume is $\frac{3}{4}$ cubic feet.

42. **CRITICAL THINKING** Which expression is equivalent to 0?

a. $\dfrac{a}{1 - \frac{3}{a}} + \dfrac{a}{\frac{3}{a} - 1}$ **b.** $\dfrac{a - \frac{1}{3}}{b} - \dfrac{a + \frac{1}{3}}{b}$ **c.** $\dfrac{\frac{1}{2} + 2a}{b - 1} - \dfrac{2a + \frac{1}{2}}{1 - b}$

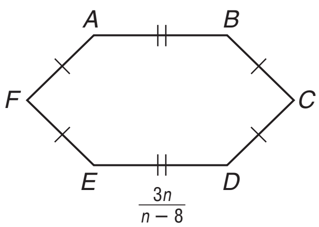
43. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are rational expressions used in baking?

Include the following in your answer:
- an example of a situation in which you would divide a measurement by a fraction when cooking, and
- an explanation of the process used to simplify a complex fraction.

Standardized Test Practice

44. The perimeter of hexagon $ABCDEF$ is 12. Which expression can be used to represent the measure of $\overline{BC}$?



Ⓐ $\dfrac{6n - 96}{n - 8}$ Ⓑ $\dfrac{9n - 96}{n - 8}$

Ⓒ $\dfrac{6n - 96}{4n - 32}$ Ⓓ $\dfrac{9n - 96}{4n - 32}$

45. Express $\dfrac{\frac{6mn}{5p}}{\frac{24n^2}{20mp}}$ in simplest form.

Ⓐ $\dfrac{n}{m^2}$ Ⓑ $\dfrac{1}{n}$ Ⓒ $\dfrac{m^2}{n}$ Ⓓ $\dfrac{36n^3}{25p^2}$

Maintain Your Skills

Mixed Review

Find each sum. *(Lesson 12-7)*

46. $\dfrac{12x}{4y^2} + \dfrac{8}{6y}$

47. $\dfrac{a}{a - b} + \dfrac{b}{2b + 3a}$

48. $\dfrac{a + 3}{3a^2 - 10a - 8} + \dfrac{2a}{a^2 - 8a + 16}$

49. $\dfrac{n - 4}{(n - 2)^2} + \dfrac{n - 5}{n^2 + n - 6}$

Find each difference. *(Lesson 12-6)*

50. $\dfrac{7}{x^2} - \dfrac{3}{x^2}$

51. $\dfrac{x}{(x - 3)^2} - \dfrac{3}{(x - 3)^2}$

52. $\dfrac{2}{t^2 - t - 2} - \dfrac{t}{t^2 - t - 2}$

53. $\dfrac{2n}{n^2 + 2n - 24} - \dfrac{8}{n^2 + 2n - 24}$

54. **BIOLOGY** Ana is working on a biology project for her school's science fair. For her experiment, she needs to have a certain type of bacteria that doubles its population every hour. Right now Ana has 1000 bacteria. If Ana does not interfere with the bacteria, predict how many there will be in ten hours. *(Lesson 10-6)*

Solve each equation by factoring. Check your solutions. *(Lesson 9-5)*

55. $s^2 = 16$

56. $9p^2 = 64$

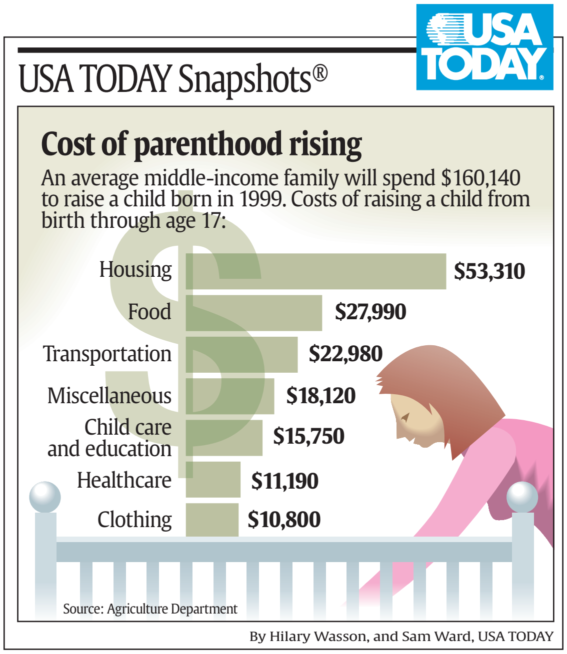
57. $z^3 - 9z = 45 - 5z^2$

FAMILIES **For Exercises 58–60, refer to the graph.** *(Lesson 8-3)*

58. Write each number in the graph using scientific notation.

59. How many times as great is the amount spent on food as the amount spent on clothing? Express your answer in scientific notation.

60. What percent of the total amount is spent on housing?



TELEPHONE RATES **For Exercises 61 and 62, use the following information.** *(Lesson 5-4)*

A 15-minute call to Mexico costs $3.39. A 24-minute call costs $4.83.

61. Write a linear equation to find the total cost C of an m-minute call.

62. Find the cost of a 9-minute call.

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Solve each equation.**
*(To review **solving equations**, see Lessons 3-2 through 3-4.)*

63. $-12 = \dfrac{x}{4}$

64. $1.8 = g - 0.6$

65. $\dfrac{3}{4}n - 3 = 9$

66. $7x^2 = 28$

67. $3.2 = \dfrac{-8 + n}{-7}$

68. $\dfrac{-3n - (-4)}{-6} = -9$

12-9 Solving Rational Equations

What You'll Learn

- Solve rational equations.
- Eliminate extraneous solutions.

Vocabulary

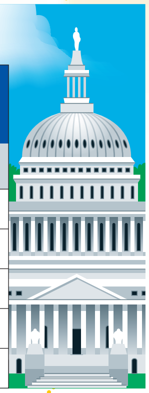
- rational equations
- work problems
- rate problems
- extraneous solutions

How are rational equations important in the operation of a subway system?

The Washington, D.C., Metrorail is one of the safest subway systems in the world, serving a population of more than 3.5 million. It is vital that a rail system of this size maintain a consistent schedule. Rational equations can be used to determine the exact positions of trains at any given time.

Washington Metropolitan Area Transit Authority

Train	Distance
Red Line	19.4 mi
Orange Line	24.14 mi
Blue Line	19.95 mi
Green Line	20.59 mi
Yellow Line	9.46 mi



SOLVE RATIONAL EQUATIONS **Rational equations** are equations that contain rational expressions. You can use cross products to solve rational equations, but only when both sides of the equation are single fractions.

Example 1 Use Cross Products

Solve $\frac{12}{x+5} = \frac{4}{(x+2)}$.

$$\frac{12}{x+5} = \frac{4}{(x+2)} \quad \text{Original equation}$$

$$12(x+2) = 4(x+5) \quad \text{Cross multiply.}$$

$$12x + 24 = 4x + 20 \quad \text{Distributive Property}$$

$$8x = -4 \quad \text{Add } -4x \text{ and } -24 \text{ to each side.}$$

$$x = -\frac{4}{8} \text{ or } -\frac{1}{2} \quad \text{Divide each side by 8.}$$

Another method you can use to solve rational equations is to multiply each side of the equation by the LCD to eliminate fractions.

Example 2 Use the LCD

Solve $\frac{n-2}{n} - \frac{n-3}{n-6} = \frac{1}{n}$.

$$\frac{n-2}{n} - \frac{n-3}{n-6} = \frac{1}{n} \quad \text{Original equation}$$

$$n(n-6)\left(\frac{n-2}{n} - \frac{n-3}{n-6}\right) = n(n-6)\left(\frac{1}{n}\right) \quad \text{The LCD is } n(n-6).$$

$$\left(\frac{\overset{1}{\cancel{n}}(n-6)}{1} \cdot \frac{n-2}{\underset{1}{\cancel{n}}}\right) - \left(\frac{n\overset{1}{\cancel{(n-6)}}}{1} \cdot \frac{n-3}{\underset{1}{\cancel{n-6}}}\right) = \frac{\overset{1}{\cancel{n}}(n-6)}{1} \cdot \frac{1}{\underset{1}{\cancel{n}}} \quad \text{Distributive Property}$$

$$(n-6)(n-2) - n(n-3) = n - 6 \quad \text{Simplify.}$$

$$(n^2 - 8n + 12) - (n^2 - 3n) = n - 6 \quad \text{Multiply.}$$

$$n^2 - 8n + 12 - n^2 + 3n = n - 6 \quad \text{Subtract.}$$

$$-5n + 12 = n - 6 \quad \text{Simplify.}$$

$$-6n = -18 \quad \text{Subtract 12 and } n \text{ from each side.}$$

$$n = 3 \quad \text{Divide each side by } -6.$$

A rational equation may have more than one solution.

Example 3 Multiple Solutions

Solve $\frac{-4}{a+1} + \frac{3}{a} = 1$.

$$\frac{-4}{a+1} + \frac{3}{a} = 1 \quad \text{Original equation}$$

$$a(a+1)\left(\frac{-4}{a+1} + \frac{3}{a}\right) = a(a+1)(1) \quad \text{The LCD is } a(a+1).$$

$$\left(\frac{a\cancel{(a+1)}^{1}}{1} \cdot \frac{-4}{\cancel{a+1}_{1}}\right) + \left(\frac{\cancel{a}^{1}(a+1)}{1} \cdot \frac{3}{\cancel{a}_{1}}\right) = a(a+1) \quad \text{Distributive Property}$$

$$-4a + 3a + 3 = a^2 + a \quad \text{Simplify.}$$

$$-a + 3 = a^2 + a \quad \text{Add like terms.}$$

$$0 = a^2 + 2a - 3 \quad \text{Set equal to 0.}$$

$$0 = (a+3)(a-1) \quad \text{Factor.}$$

$$a + 3 = 0 \quad \text{or} \quad a - 1 = 0$$

$$a = -3 \qquad\qquad a = 1$$

CHECK Check by substituting each value in the original equation.

$$\frac{-4}{a+1} + \frac{3}{a} = 1 \qquad\qquad \frac{-4}{a+1} + \frac{3}{a} = 1$$

$$\frac{-4}{-3+1} + \frac{3}{-3} \stackrel{?}{=} 1 \quad a = -3 \qquad\qquad \frac{-4}{1+1} + \frac{3}{1} \stackrel{?}{=} 1 \quad a = 1$$

$$2 + (-1) \stackrel{?}{=} 1 \qquad\qquad -2 + 3 \stackrel{?}{=} 1$$

$$1 = 1 \qquad\qquad 1 = 1$$

The solutions are 1 or -3.

Study Tip

Look Back
To review **solving quadratic equations by factoring**, see Lessons 9-3 through 9-6.

Rational equations can be used to solve **work problems**.

Example 4 Work Problem

LAWN CARE **Abbey has a lawn care service. One day she asked her friend Jamal to work with her. Normally, it takes Abbey two hours to mow and trim Mrs. Harris' lawn. When Jamal worked with her, the job took only 1 hour and 20 minutes. How long would it have taken Jamal to do the job himself?**

Explore Since it takes Abbey two hours to do the yard, she can finish $\frac{1}{2}$ the job in one hour. The amount of work Jamal can do in one hour can be represented by $\frac{1}{t}$. To determine how long it takes Jamal to do the job, use the formula Abbey's work + Jamal's work = 1 completed yard.

Plan The time that both of them worked was $1\frac{1}{3}$ hours. Each rate multiplied by this time results in the amount of work done by each person.

Solve

$$\underbrace{\frac{1}{2}\left(\frac{4}{3}\right)}_{\text{Abbey's work}} \quad \underbrace{+}_{\text{plus}} \quad \underbrace{\frac{1}{t}\left(\frac{4}{3}\right)}_{\text{Jamal's work}} \quad \underbrace{=}_{\text{equals}} \quad \underbrace{1}_{\text{total work.}}$$

$$\frac{4}{6} + \frac{4}{3t} = 1 \quad \text{Multiply.}$$

(continued on the next page)

Study Tip

Work Problems
When solving work problems, remember that each term should represent the portion of a job completed in one unit of time.

$$6t\left(\frac{4}{6} + \frac{4}{3t}\right) = 6t \cdot 1$$ The LCD is $6t$.

$$\overset{1}{\cancel{6}}t\left(\frac{4}{\underset{1}{\cancel{6}}}\right) + \overset{2}{\cancel{6t}}\left(\frac{4}{\underset{1}{\cancel{3t}}}\right) = 6t$$ Distributive Property

$$4t + 8 = 6t$$ Simplify.

$$8 = 2t$$ Add $-4t$ to each side.

$$4 = t$$ Divide each side by 2.

Examine The time that it would take Jamal to do the yard by himself is four hours. This seems reasonable because the combined efforts of the two took longer than half of Abbey's usual time.

Rational equations can also be used to solve **rate problems**.

Example 5 Rate Problem

TRANSPORTATION **Refer to the application at the beginning of the lesson. The Yellow Line runs between Huntington and Mt. Vernon Square. Suppose one train leaves Mt. Vernon Square at noon and arrives at Huntington 24 minutes later, and a second train leaves Huntington at noon and arrives at Mt. Vernon Square 28 minutes later. At what time do the two trains pass each other?**

Study Tip

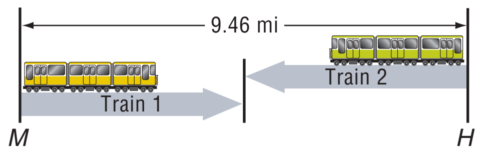
Rate Problems

You can solve rate problems, also called *uniform motion problems*, more easily if you first make a drawing.

Determine the rates of both trains. The total distance is 9.46 miles.

Train 1 $\frac{9.46 \text{ mi}}{24 \text{ min}}$ **Train 2** $\frac{9.46 \text{ mi}}{28 \text{ min}}$

Next, since both trains left at the same time, the time both have traveled when they pass will be the same. And since they started at opposite ends of the route, the sum of their distances is equal to the total route, 9.46 miles.



	r	t	d
Train 1	$\frac{9.46 \text{ mi}}{24 \text{ min}}$	t min	$\frac{9.46t}{24}$ mi
Train 2	$\frac{9.46 \text{ mi}}{28 \text{ min}}$	t min	$\frac{9.46t}{28}$ mi

$$\frac{9.46t}{24} + \frac{9.46t}{28} = 9.46$$ The sum of the distances is 9.46.

$$168\left(\frac{9.46t}{24} + \frac{9.46t}{28}\right) = 168 \cdot 9.46$$ The LCD is 168.

$$\frac{\overset{7}{\cancel{168}}}{1} \cdot \frac{9.46t}{\underset{1}{\cancel{24}}} + \frac{\overset{6}{\cancel{168}}}{1} \cdot \frac{9.46t}{\underset{1}{\cancel{28}}} = 1589.28$$ Distributive Property

$$66.22t + 56.76t = 1589.28$$ Simplify.

$$122.98t = 1589.28$$ Add.

$$t = 12.92$$ Divide each side by 122.98.

The trains passed at about 12.92 or about 13 minutes after leaving their stations, which is 12:13 P.M.

EXTRANEOUS SOLUTIONS Multiplying each side of an equation by the LCD of two rational expressions can yield results that are not solutions to the original equation. Recall that such solutions are called **extraneous solutions**.

Example 6 No Solution

Solve $\frac{3x}{x-1} + \frac{6x-9}{x-1} = 6$.

$$\frac{3x}{x-1} + \frac{6x-9}{x-1} = 6 \quad \text{Original equation}$$

$$(x-1)\left(\frac{3x}{x-1} + \frac{6x-9}{x-1}\right) = (x-1)6 \quad \text{The LCD is } x-1.$$

$$\cancel{(x-1)}^{1}\left(\frac{3x}{\cancel{x-1}_{1}}\right) + \cancel{(x-1)}^{1}\left(\frac{6x-9}{\cancel{x-1}_{1}}\right) = (x-1)6 \quad \text{Distributive Property}$$

$$3x + 6x - 9 = 6x - 6 \quad \text{Simplify.}$$

$$9x - 9 = 6x - 6 \quad \text{Add like terms.}$$

$$3x = 3 \quad \text{Add 9 to each side.}$$

$$x = 1 \quad \text{Divide each side by 3.}$$

Since 1 is an excluded value for x, the number 1 is an extraneous solution. Thus, the equation has no solution.

Rational equations can have both valid solutions and extraneous solutions.

Example 7 Extraneous Solution

Solve $\frac{2n}{1-n} + \frac{n+3}{n^2-1} = 1$.

$$\frac{2n}{1-n} + \frac{n+3}{n^2-1} = 1$$

$$\frac{2n}{1-n} + \frac{n+3}{(n-1)(n+1)} = 1$$

$$-\frac{2n}{n-1} + \frac{n+3}{(n-1)(n+1)} = 1$$

$$(n-1)(n+1)\left(-\frac{2n}{n-1} + \frac{n+3}{(n-1)(n+1)}\right) = (n-1)(n+1)1$$

$$\cancel{(n-1)}^{1}(n+1)\left(-\frac{2n}{\cancel{n-1}_{1}}\right) + \cancel{(n-1)}^{1}\cancel{(n+1)}^{1}\left(\frac{n+3}{\cancel{(n-1)}_{1}\cancel{(n+1)}_{1}}\right) = (n-1)(n+1)$$

$$-2n(n+1) + (n+3) = n^2 - 1$$

$$-2n^2 - 2n + n + 3 = n^2 - 1$$

$$-3n^2 - n + 4 = 0$$

$$3n^2 + n - 4 = 0$$

$$(3n+4)(n-1) = 0$$

$3n + 4 = 0$ or $n - 1 = 0$

$n = -\frac{4}{3}$ $\qquad n = 1$

The number 1 is an extraneous solution, since 1 is an excluded value for n. Thus, $-\frac{4}{3}$ is the solution of the equation.

Check for Understanding

Concept Check

1. **OPEN ENDED** Explain why the equation $n + \frac{1}{n-1} = \frac{1}{n-1} + 1$ has no solution.

2. **Write** an expression to represent the amount of work Aminta can do in h hours if it normally takes her 3 hours to change the oil and tune up her car.

3. **Find a counterexample** for the following statement.
The solution of a rational equation can never be zero.

Guided Practice

Solve each equation. State any extraneous solutions.

4. $\frac{2}{x} = \frac{3}{x+1}$
5. $\frac{7}{a-1} = \frac{5}{a+3}$
6. $\frac{3x}{5} + \frac{3}{2} = \frac{7x}{10}$
7. $\frac{x+1}{x} + \frac{x+4}{x} = 6$
8. $\frac{5}{k+1} - \frac{7}{k} = \frac{1}{k+1}$
9. $\frac{x+2}{x-2} - \frac{2}{x+2} = \frac{-7}{3}$

Application

10. **BASEBALL** Omar has 32 hits in 128 times at bat. He wants his batting average to be .300. His current average is $\frac{32}{128}$ or .250. How many at bats does Omar need to reach his goal if he gets a hit in each of his next b at bats?

Practice and Apply

Homework Help

For Exercises	See Examples
11–14	1
15–19, 21, 23, 26, 27	2
22, 24, 25	3
29–34	4, 5
20, 28	6, 7

Extra Practice
See page 849.

Solve each equation. State any extraneous solutions.

11. $\frac{4}{a} = \frac{3}{a-2}$
12. $\frac{3}{x} = \frac{1}{x-2}$
13. $\frac{x-3}{x} = \frac{x-3}{x-6}$
14. $\frac{x}{x+1} = \frac{x-6}{x-1}$
15. $\frac{2n}{3} + \frac{1}{2} = \frac{2n-3}{6}$
16. $\frac{5}{4} + \frac{3y}{2} = \frac{7y}{6}$
17. $\frac{a-1}{a+1} - \frac{2a}{a-1} = -1$
18. $\frac{7}{x^2-5x} + \frac{3}{5-x} = \frac{4}{x}$
19. $\frac{4x}{2x+3} - \frac{2x}{2x-3} = 1$
20. $\frac{5}{5-p} - \frac{p^2}{p-5} = -8$
21. $\frac{a}{3a+6} - \frac{a}{5a+10} = \frac{2}{5}$
22. $\frac{c}{c-4} - \frac{6}{4-c} = c$
23. $\frac{2b-5}{b-2} - 2 = \frac{3}{b+2}$
24. $\frac{7}{k-3} - \frac{1}{2} = \frac{3}{k-4}$
25. $\frac{x^2-4}{x-2} + x^2 = 4$
26. $\frac{2n}{n-1} + \frac{n-5}{n^2-1} = 1$
27. $\frac{3z}{z^2-5z+4} = \frac{2}{z-4} + \frac{3}{z-1}$
28. $\frac{4}{m^2-8m+12} = \frac{m}{m-2} + \frac{1}{m-6}$

29. **QUIZZES** Each week, Mandy's algebra teacher gives a 10-point quiz. After 5 weeks, Mandy has earned a total of 36 points for an average of 7.2 points per quiz. She would like to raise her average to 9 points. On how many quizzes must she score 10 points in order to reach her goal?

BOATING **For Exercises 30 and 31, use the following information.**
Jim and Mateo live across a lake from each other at a distance of about 3 miles. Jim can row his boat to Mateo's house in 1 hour and 20 minutes. Mateo can drive his motorboat the same distance in a half hour.

30. If they leave their houses at the same time and head toward each other, how long will it be before they meet?

31. How far from the nearest shore will they be when they meet?

32. **CAR WASH** Ian and Nadya can each wash a car and clean its interior in about 2 hours, but Chris needs 3 hours to do the work. If the three work together, how long will it take to clean seven cars?

SWIMMING POOLS **For Exercises 33 and 34, use the following information.** The pool in Kara's backyard is cleaned and ready to be filled for the summer. It measures 15 feet long and 10 feet wide with an average depth of 4 feet.

33. What is the volume of the pool?

34. How many gallons of water will it take to fill the pool? ($1 \text{ ft}^3 = 7.5$ gal)

35. **CRITICAL THINKING** Solve $\dfrac{\frac{x+3}{x-2} \cdot \frac{x^2+x-2}{x+5}}{x-1} + 2 = 0$.

36. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are rational equations important in the operation of a subway system?

Include the following in your answer:

- an explanation of how rational equations can be used to approximate the time that trains will pass each other if they leave distant stations and head toward each other.

Standardized Test Practice

37. What is the value of a in the equation $\dfrac{a-2}{a} - \dfrac{a-3}{a-6} = \dfrac{1}{a}$?

(A) 3 (B) 2 (C) 6 (D) 0

38. Which value is an extraneous solution of $\dfrac{-1}{n+2} = \dfrac{n^2-7n-8}{3n^2+2n-8}$?

(A) 6 (B) 2 (C) -1 (D) -2

Maintain Your Skills

Mixed Review

Simplify each expression. *(Lesson 12-8)*

39. $\dfrac{\frac{x^2+8x+15}{x^2+x-6}}{\frac{x^2+2x-15}{x^2-2x-3}}$

40. $\dfrac{\frac{a^2-6a+5}{a^2+13a+42}}{\frac{a^2-4a+3}{a^2+3a-18}}$

41. $\dfrac{x+2+\frac{2}{x+5}}{x+6+\frac{6}{x+1}}$

Find each difference. *(Lesson 12-7)*

42. $\dfrac{3}{2m-3} - \dfrac{m}{6-4m}$

43. $\dfrac{y}{y^2-2y+1} - \dfrac{1}{y-1}$

44. $\dfrac{a+2}{a^2-9} - \dfrac{2a}{6a^2-17a-3}$

Factor each polynomial. *(Lesson 9-2)*

45. $20x - 8y$

46. $14a^2b + 21ab^2$

47. $10p^2 - 12p + 25p - 30$

48. **CHEMISTRY** One solution is 50% glycol, and another is 30% glycol. How much of each solution should be mixed to make a 100-gallon solution that is 45% glycol? *(Lesson 7-2)*

WebQuest Internet Project

Building the Best Roller Coaster

It is time to complete your project. Use the information and data you have gathered about the building and financing of a roller coaster to prepare a portfolio or Web page. Be sure to include graphs and/or tables in the presentation.

www.algebra1.com/webquest

Chapter 12

Study Guide and Review

Vocabulary and Concept Check

complex fraction (p. 684)
excluded values (p. 648)
extraneous solutions (p. 693)
inverse variation (p. 642)
least common multiple (p. 678)
least common denominator (p. 679)
mixed expression (p. 684)
product rule (p. 643)
rate problem (p. 692)
rational equation (p. 690)
rational expression (p. 648)
work problem (p. 691)

State whether each sentence is *true* or *false*. If false, replace the underlined expression to make a true sentence.

1. A <u>mixed</u> expression is a fraction whose numerator and denominator are polynomials.
2. The complex fraction $\dfrac{\frac{4}{5}}{\frac{2}{3}}$ can be simplified as $\underline{\frac{6}{5}}$.
3. The equation $\dfrac{x}{x-1} + \dfrac{2x-3}{x-1} = 2$ has an extraneous solution of $\underline{1}$.
4. The mixed expression $6 - \dfrac{a-2}{a+3}$ can be rewritten as $\underline{\dfrac{5a+16}{a+3}}$.
5. The least common multiple for $(x^2 - 144)$ and $(x + 12)$ is $\underline{x + 12}$.
6. The excluded values for $\dfrac{4x}{x^2 - x - 12}$ are $\underline{-3 \text{ and } 4}$.

Lesson-by-Lesson Review

12-1 Inverse Variation

See pages 642–647.

Concept Summary

- The product rule for inverse variations states that if (x_1, y_1) and (x_2, y_2) are solutions of an inverse variation, then $x_1y_1 = k$ and $x_2y_2 = k$.
- You can use $\dfrac{x_1}{x_2} = \dfrac{y_2}{y_1}$ to solve problems involving inverse variation.

Example **If y varies inversely as x and $y = 24$ when $x = 30$, find x when $y = 10$.**

$\dfrac{x_1}{x_2} = \dfrac{y_2}{y_1}$ Proportion for inverse variations

$\dfrac{30}{x_2} = \dfrac{10}{24}$ $x_1 = 30$, $y_1 = 24$, and $y_2 = 10$

$720 = 10x_2$ Cross multiply.

$72 = x_2$ Thus, $x = 72$ when $y = 10$.

Exercises **Write an inverse variation equation that relates x and y. Assume that y varies inversely as x. Then solve.** *See Examples 3 and 4 on page 644.*

7. If $y = 28$ when $x = 42$, find y when $x = 56$.
8. If $y = 15$ when $x = 5$, find y when $x = 3$.
9. If $y = 18$ when $x = 8$, find x when $y = 3$.
10. If $y = 35$ when $x = 175$, find y when $x = 75$.

12-2 Rational Expressions

See pages 648–653.

Concept Summary

- Excluded values are values of a variable that result in a denominator of zero.

Example **Simplify $\frac{x+4}{x^2+12x+32}$. State the excluded values of x.**

$$\frac{x+4}{x^2+12x+32} = \frac{\cancel{x+4}^{\,1}}{\cancel{(x+4)}_{1}(x+8)} \quad \text{Factor.}$$

$$= \frac{1}{x+8} \quad \text{Simplify.}$$

The expression is undefined when $x = -4$ and $x = -8$.

Exercises **Simplify each expression.** *See Example 5 on page 650.*

11. $\frac{3x^2y}{12xy^3z}$ **12.** $\frac{n^2-3n}{n-3}$ **13.** $\frac{a^2-25}{a^2+3a-10}$ **14.** $\frac{x^2+10x+21}{x^3+x^2-42x}$

12-3 Multiplying Rational Expressions

See pages 655–659.

Concept Summary

- Multiplying rational expressions is similar to multiplying rational numbers.

Example **Find $\frac{1}{x^2+x-12} \cdot \frac{x-3}{x+5}$.**

$$\frac{1}{x^2+x-12} \cdot \frac{x-3}{x+5} = \frac{1}{(x+4)\cancel{(x-3)}_{1}} \cdot \frac{\cancel{x-3}^{\,1}}{x+5} \quad \text{Factor.}$$

$$= \frac{1}{(x+4)(x+5)} \quad \text{Simplify.}$$

Exercises **Find each product.** *See Examples 1–3 on pages 655 and 656.*

15. $\frac{7b^2}{9} \cdot \frac{6a^2}{b}$ **16.** $\frac{5x^2y}{8ab} \cdot \frac{12a^2b}{25x}$ **17.** $(3x+30) \cdot \frac{10}{x^2-100}$

18. $\frac{3a-6}{a^2-9} \cdot \frac{a+3}{a^2-2a}$ **19.** $\frac{x^2+x-12}{x+2} \cdot \frac{x+4}{x^2-x-6}$ **20.** $\frac{b^2+19b+84}{b-3} \cdot \frac{b^2-9}{b^2+15b+36}$

12-4 Dividing Rational Expressions

See pages 660–664.

Concept Summary

- Divide rational expressions by multiplying by the reciprocal of the divisor.

Example **Find $\frac{y^2-16}{y^2-64} \div \frac{y+4}{y-8}$.**

$$\frac{y^2-16}{y^2-64} \div \frac{y+4}{y-8} = \frac{y^2-16}{y^2-64} \cdot \frac{y-8}{y+4} \quad \text{Multiply by the reciprocal of } \frac{y+4}{y-8}.$$

$$= \frac{(y-4)\cancel{(y+4)}^{\,1}}{\cancel{(y-8)}_{1}(y+8)} \cdot \frac{\cancel{y-8}^{\,1}}{\cancel{y+4}_{1}} \text{ or } \frac{y-4}{y+8} \quad \text{Simplify.}$$

Exercises Find each quotient. *See Examples 1–4 on pages 660 and 661.*

21. $\frac{p^3}{2q} \div \frac{p^2}{4q}$

22. $\frac{y^2}{y+4} \div \frac{3y}{y^2-16}$

23. $\frac{3y-12}{y+4} \div (y^2 - 6y + 8)$

24. $\frac{2m^2 + 7m - 15}{m+5} \div \frac{9m^2-4}{3m+2}$

12-5 Dividing Polynomials

See pages 666–671.

Concept Summary

- To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.
- To divide a polynomial by a binomial, use long division.

Example **Find $(x^3 - 2x^2 - 22x + 21) \div (x - 3)$.**

$$
\begin{array}{r l}
x^2 \quad + x - 19 & \\
x - 3\overline{)x^3 - 2x^2 - 22x + 21} & \\
\underline{(-)x^3 - 3x^2} \qquad\qquad & \text{Multiply } x^2 \text{ and } x - 3. \\
x^2 - 22x \qquad & \text{Subtract.} \\
\underline{(-)x^2 - \quad 3x} \qquad & \text{Multiply } x \text{ and } x - 3. \\
-19x + 21 & \text{Subtract.} \\
\underline{(-) -19x + 57} & \text{Multiply } -19 \text{ and } x - 3. \\
-36 & \text{Subtract.}
\end{array}
$$

The quotient is $x^2 + x - 19 - \frac{36}{x-3}$.

Exercises Find each quotient. *See Examples 1–5 on pages 666–668.*

25. $(4a^2b^2c^2 - 8a^3b^2c + 6abc^2) \div 2ab^2$

26. $(x^3 + 7x^2 + 10x - 6) \div (x + 3)$

27. $\frac{x^3 - 7x + 6}{x - 2}$

28. $(48b^2 + 8b + 7) \div (12b - 1)$

12-6 Rational Expressions with Like Denominators

See pages 672–677.

Concept Summary

- Add (or subtract) rational expressions with like denominators by adding (or subtracting) the numerators and writing the sum (or difference) over the denominator.

Example **Find $\frac{m^2}{m+4} - \frac{16}{m+4}$.**

$\frac{m^2}{m+4} - \frac{16}{m+4} = \frac{m^2 - 16}{m+4}$ Subtract the numerators.

$= \frac{(m-4)\overset{1}{\cancel{(m+4)}}}{\underset{1}{\cancel{m+4}}}$ or $m - 4$ Factor.

Exercises Find each sum or difference. *See Examples 1–4 on pages 672 and 673.*

29. $\frac{m+4}{5} + \frac{m-1}{5}$

30. $\frac{-5}{2n-5} + \frac{2n}{2n-5}$

31. $\frac{a^2}{a-b} + \frac{-b^2}{a-b}$

32. $\frac{7a}{b^2} - \frac{5a}{b^2}$

33. $\frac{2x}{x-3} - \frac{6}{x-3}$

34. $\frac{m^2}{m-n} - \frac{2mn - n^2}{m-n}$

12-7 Rational Expressions with Unlike Denominators

See pages 678–683.

Concept Summary

- Rewrite rational expressions with unlike denominators using the least common denominator (LCD). Then add or subtract.

Example Find $\frac{x}{x+3} + \frac{5}{x-2}$.

$$\frac{x}{x+3} + \frac{5}{x-2} = \frac{x-2}{x-2} \cdot \frac{x}{x+3} + \frac{x+3}{x+3} \cdot \frac{5}{x-2}$$ The LCD is $(x + 3)(x - 2)$.

$$= \frac{x^2 - 2x}{(x+3)(x-2)} + \frac{5x + 15}{(x+3)(x-2)}$$ Multiply.

$$= \frac{x^2 + 3x + 15}{(x+3)(x-2)}$$ Add.

Exercises **Find each sum or difference.** *See Examples 3–5 on pages 679 and 680.*

35. $\frac{2c}{3d^2} + \frac{3}{2cd}$ **36.** $\frac{r^2 + 21r}{r^2 - 9} + \frac{3r}{r+3}$ **37.** $\frac{3a}{a-2} + \frac{5a}{a+1}$

38. $\frac{7n}{3} - \frac{9n}{7}$ **39.** $\frac{7}{3a} - \frac{3}{6a^2}$ **40.** $\frac{2x}{2x+8} - \frac{4}{5x+20}$

12-8 Mixed Expressions and Complex Fractions

See pages 684–689.

Concept Summary

- Write mixed expressions as rational expressions in the same way as mixed numbers are changed to improper fractions.
- Simplify complex fractions by writing them as division problems.

Example Simplify $\frac{y - \frac{40}{y-3}}{y+5}$.

$$\frac{y - \frac{40}{y-3}}{y+5} = \frac{\frac{y(y-3)}{(y-3)} - \frac{40}{y-3}}{y+5}$$ The LCD in the numerator is $y - 3$.

$$= \frac{\frac{y^2 - 3y - 40}{y-3}}{y+5}$$ Add in the numerator.

$$= \frac{y^2 - 3y - 40}{y-3} \div (y+5)$$ Rewrite as a division sentence.

$$= \frac{y^2 - 3y - 40}{y-3} \cdot \frac{1}{y+5}$$ Multiply by the reciprocal of $y + 5$.

$$= \frac{(y-8)\overset{1}{\cancel{(y+5)}}}{y-3} \cdot \frac{1}{\underset{1}{\cancel{y+5}}} \text{ or } \frac{y-8}{y-3}$$ Factor.

Exercises **Write each mixed expression as a rational expression.**
See Example 1 on page 684.

41. $4 + \frac{x}{x-2}$ **42.** $2 - \frac{x+2}{x^2-4}$ **43.** $3 + \frac{x^2 + y^2}{x^2 - y^2}$

- Extra Practice, see pages 846–849.
- Mixed Problem Solving, see page 864.

Simplify each expression. *See Examples 3 and 4 on pages 685 and 686.*

44. $\dfrac{\frac{x^2}{y^3}}{\frac{3x}{9y^2}}$ **45.** $\dfrac{5+\frac{4}{a}}{\frac{a}{2}-\frac{3}{4}}$ **46.** $\dfrac{y+9-\frac{6}{y+4}}{y+4+\frac{2}{y+1}}$

12-9 Solving Rational Equations

See pages 690–695.

Concept Summary

- Use cross products to solve rational equations with a single fraction on each side of the equal sign.
- Multiply every term of a more complicated rational equation by the LCD to eliminate fractions.

Example **Solve $\frac{5n}{6} + \frac{1}{n-2} = \frac{n+1}{3(n-2)}$.**

$$\frac{5n}{6} + \frac{1}{n-2} = \frac{n+1}{3(n-2)} \quad \text{Original equation}$$

$$6(n-2)\left(\frac{5n}{6} + \frac{1}{n-2}\right) = 6(n-2)\frac{n+1}{3(n-2)} \quad \text{The LCD is } 6(n-2)$$

$$\frac{\overset{1}{\cancel{6}}(n-2)(5n)}{\underset{1}{\cancel{6}}} + \frac{6\overset{1}{\cancel{(n-2)}}}{\underset{1}{\cancel{n-2}}} = \frac{\overset{2}{\cancel{6}}\overset{1}{\cancel{(n-2)}}(n+1)}{\underset{1}{\cancel{3}}\underset{1}{\cancel{(n-2)}}} \quad \text{Distributive Property}$$

$$(n-2)(5n) + 6 = 2(n+1) \quad \text{Simplify.}$$

$$5n^2 - 10n + 6 = 2n + 2 \quad \text{Multiply.}$$

$$5n^2 - 12n + 4 = 0 \quad \text{Subtract.}$$

$$(5n-2)(n-2) = 0 \quad \text{Factor.}$$

$$n = \frac{2}{5} \text{ or } n = 2$$

CHECK Let $n = \frac{2}{5}$.

$$\frac{\frac{2}{5}+1}{3\left(\frac{2}{5}-2\right)} \stackrel{?}{=} \frac{5\left(\frac{2}{5}\right)}{6} + \frac{1}{\frac{2}{5}-2}$$

$$-\frac{7}{24} = -\frac{7}{24} \checkmark$$

Let $n = 2$.

$$\frac{2+1}{3(2-2)} \stackrel{?}{=} \frac{5(2)}{6} + \frac{1}{2-2}$$

$$\frac{3}{3(0)} \stackrel{?}{=} \frac{10}{6} + \frac{1}{0}$$

When you check the value 2, you get a zero in the denominator. So, 2 is an extraneous solution.

Exercises **Solve each equation. State any extraneous solutions.**
See Examples 6 and 7 on page 693.

47. $\frac{4x}{3} + \frac{7}{2} = \frac{7x}{12} - \frac{1}{4}$ **48.** $\frac{11}{2x} - \frac{2}{3x} = \frac{1}{6}$

49. $\frac{2}{3r} - \frac{3r}{r-2} = -3$ **50.** $\frac{x-2}{x} - \frac{x-3}{x-6} = \frac{1}{x}$

51. $\frac{3}{x^2+3x} + \frac{x+2}{x+3} = \frac{1}{x}$ **52.** $\frac{1}{n+4} - \frac{1}{n-1} = \frac{2}{n^2+3n-4}$

Chapter

12 Practice Test

Vocabulary and Concepts

Choose the letter that best matches each algebraic expression.

1. $\dfrac{\frac{a}{b}}{\frac{x}{y}}$ 2. $3 - \dfrac{a+1}{a-1}$ 3. $\dfrac{2}{x^2 + 2x - 4}$

a. complex fraction
b. rational expression
c. mixed expression

Skills and Applications

Write an inverse variation equation that relates x and y. Assume that y varies inversely as x. Then solve.

4. If $y = 21$ when $x = 40$, find y when $x = 84$.
5. If $y = 22$ when $x = 4$, find x when $y = 16$.

Simplify each expression. State the excluded values of the variables.

6. $\dfrac{5 - 2m}{6m - 15}$
7. $\dfrac{3 + x}{2x^2 + 5x - 3}$
8. $\dfrac{4c^2 + 12c + 9}{2c^2 - 11c - 21}$
9. $\dfrac{1 - \frac{9}{t}}{1 - \frac{81}{t^2}}$
10. $\dfrac{\frac{5}{6} + \frac{u}{t}}{\frac{2u}{t} - 3}$
11. $\dfrac{x + 4 + \frac{5}{x-2}}{x + 6 + \frac{15}{x-2}}$

Perform the indicated operations.

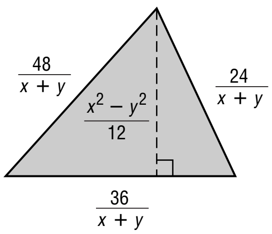
12. $\dfrac{2x}{x-7} - \dfrac{14}{x-7}$
13. $\dfrac{n+3}{2n-8} \cdot \dfrac{6n-24}{2n+1}$
14. $(10m^2 + 9m - 36) \div (2m - 3)$
15. $\dfrac{x^2 + 4x - 32}{x + 5} \cdot \dfrac{x - 3}{x^2 - 7x + 12}$
16. $\dfrac{z^2 + 2z - 15}{z^2 + 9z + 20} \div (z - 3)$
17. $\dfrac{4x^2 + 11x + 6}{x^2 - x - 6} \div \dfrac{x^2 + 8x + 16}{x^2 + x - 12}$
18. $(10z^4 + 5z^3 - z^2) \div 5z^3$
19. $\dfrac{y}{7y + 14} + \dfrac{6}{6 - 3y}$
20. $\dfrac{x + 5}{x + 2} + 6$
21. $\dfrac{x^2 - 1}{x + 1} - \dfrac{x^2 + 1}{x - 1}$

Solve each equation. State any extraneous solutions.

22. $\dfrac{2n}{n-4} - 2 = \dfrac{4}{n+5}$
23. $\dfrac{3}{x^2 + 5x + 6} - \dfrac{7}{x + 3} = -\dfrac{x - 1}{x + 2}$

24. **FINANCE** Barrington High School is raising money for Habitat for Humanity by doing lawn work for friends and neighbors. Scott can rake a lawn and bag the leaves in 5 hours, while Kalyn can do it in 3 hours. If Scott and Kalyn work together, how long will it take them to rake a lawn and bag the leaves?

25. **STANDARDIZED TEST PRACTICE** Which expression can be used to represent the area of the triangle?



(A) $\frac{1}{2}(x - y)$ (B) $\frac{3}{2}(x - y)$

(C) $\frac{1}{4}(x - y)$ (D) $\dfrac{108}{x + y}$

Chapter 12 Standardized Test Practice

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. A cylindrical container is 8 inches in height and has a radius of 2.5 inches. What is the volume of the container to the nearest cubic inch? (*Hint*: $V = \pi r^2 h$) (Lesson 3-8)

Ⓐ 63 Ⓑ 126
Ⓒ 150 Ⓓ 157

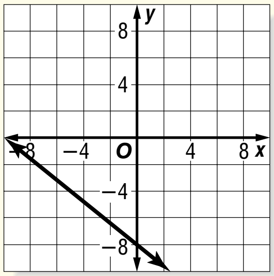
2. Which function includes all of the ordered pairs in the table? (Lesson 4-8)

x	−3	−1	1	3	5
y	10	4	−2	−8	−14

Ⓐ $y = -2x$ Ⓑ $y = -3x + 1$
Ⓒ $y = 2x - 4$ Ⓓ $y = 3x + 1$

3. Which equation describes the graph below? (Lesson 5-4)

Ⓐ $4x - 5y = 40$
Ⓑ $4x + 5y = -40$
Ⓒ $4x + 5y = -8$
Ⓓ $rx - 5y = 10$



4. Which equation represents the line that passes through $(-12, 5)$ and has a slope of $-\frac{1}{4}$? (Lesson 5-5)

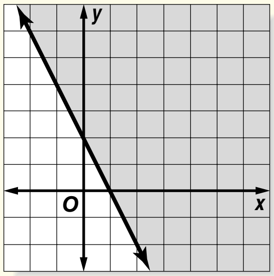
Ⓐ $x + 4y = 8$ Ⓑ $-x + 4y = 20$
Ⓒ $-4x + y = 65$ Ⓓ $x + 4y = 5$

Test-Taking Tip

Questions 2, 4, 8
Sometimes sketching the graph of a function can help you to see the relationship between *x* and *y* and answer the question.

5. Which inequality represents the shaded region? (Lesson 6-6)

Ⓐ $y \leq -\frac{1}{2}x - 2$
Ⓑ $y \geq -\frac{1}{2}x + 2$
Ⓒ $y \leq -2x + 2$
Ⓓ $y \geq -2x + 2$



6. Which ordered pair is the solution of the following system of equations? (Lesson 7-4)

$$3x + y = -2$$
$$-2x + y = 8$$

Ⓐ $(-6, 16)$ Ⓑ $(-2, 4)$
Ⓒ $(-3, 2)$ Ⓓ $(2, -8)$

7. The length of a rectangular door is 2.5 times its width. If the area of the door is 9750 square inches, which equation will determine the width *w* of the door? (Lesson 8-1)

Ⓐ $w^2 + 2.5w = 9750$
Ⓑ $2.5w^2 = 9750$
Ⓒ $2.5w^2 + 9750 = 0$
Ⓓ $7w = 9750$

8. A scientist monitored a 144-gram sample of a radioactive substance, which decays into a nonradioactive substance. The table shows the amount, in grams, of the radioactive substance remaining at intervals of 20 hours. How many grams of the radioactive substance are likely to remain after 100 hours? (Lessons 10-6 and 10-7)

Time (h)	0	20	40	60	80	100
Mass (g)	144	72	36			

Ⓐ 1 g Ⓑ 2.25 g
Ⓒ 4.5 g Ⓓ 9 g

Preparing for Standardized Tests
For test-taking strategies and more practice, see pages 867–884.

Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

9. A family drove an average of 350 miles per day during three days of their trip. They drove 360 miles on the first day and 270 miles on the second day. How many miles did they drive on the third day? (Lesson 3-4)

10. The area of the rectangular playground at Hillcrest School is 750 square meters. The length of the playground is 5 meters greater than its width. What are the length and width of the playground in meters? (Lesson 9-5)

11. How many roots does the graph of $y = x^2 + 2x + 3$ have? (Lesson 10-2)

12. Use the Quadratic Formula or factoring to determine whether the graph of $y = 16x^2 + 24x + 9$ intersects the x-axis in zero, one, or two points. (Lesson 10-4)

13. Find the length of the hypotenuse of a right triangle if one leg is 6 inches and the other leg is $\sqrt{10}$ inches. Round to the nearest tenth of an inch. (Lesson 11-4)

14. Simplify $\frac{x^2 - 7x + 6}{x^2 + 6x - 7}$. State the excluded values of x. (Lesson 12-2).

15. An airplane travels approximately 1030 kilometers per hour. Approximately what is its speed in meters per second? (Lesson 12-3)

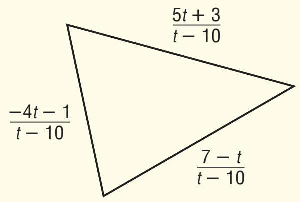
16. Express $\frac{x^2 - 9}{x^3 + x} \cdot \frac{3x}{x - 3}$ as a quotient of two polynomials written in simplest form. (Lesson 12-3)

17. Find $\frac{a + 3}{2a + 6} \div \frac{6a - 24}{4a + 12}$. (Lesson 12-4)

18. Express the following quotient in simplest form. (Lesson 12-5)

$$\frac{x}{x + 4} \div \frac{4x}{x^2 - 16}$$

19. Find the expression for the perimeter of the triangle. (Lesson 12-6)

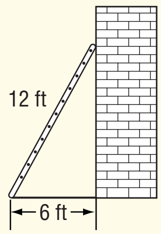


Part 3 Extended Response

Record your answers on a sheet of paper. Show your work.

20. A 12-foot ladder is placed against the side of a building so that the bottom of the ladder is 6 feet from the base of the building. (Lesson 12-1)

a. Suppose the bottom of the ladder is moved closer to the base of the building. Does the height that the ladder reaches increase or decrease?



b. What conclusion can you make about the height the ladder reaches and the distance between the bottom of the ladder and the base of the building?

c. Does this relationship form an inverse variation? Explain your reasoning.

21. The maximum speed of a barge in still water is 8 miles per hour. At this rate, a 30-mile trip downstream (with the current) takes as much time as an 18-mile trip upstream (against the current). (Lesson 12-9)

a. Find the rational expressions that measure the amount of time it takes for the barge to go downstream and upstream if c represents the speed of the current.

b. What is the speed of the current?