Chapter 14 Probability

What You'll Learn

- **Lesson 14-1** Count outcomes using the Fundamental Counting Principle.
- **Lesson 14-2** Determine probabilities using permutations and combinations.
- **Lesson 14-3** Find probabilities of compound events.
- **Lesson 14-4** Use probability distributions.
- **Lesson 14-5** Use probability simulations.

Key Vocabulary

- **permutation** (p. 760)
- **combination** (p. 762)
- **compound event** (p. 769)
- **theoretical probability** (p. 782)
- **experimental probability** (p. 782)

Why It's Important

The United States Senate forms committees to focus on different issues. These committees are made up of senators from various states and political parties. There are many ways these committees could be formed. *You will learn how to find the number of possible committees in Lesson 14-2.*

Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 14.

For Lessons 14-2 through 14-5 — Find Simple Probabilities

Determine the probability of each event if you randomly select a cube from a bag containing 6 red cubes, 3 blue cubes, 4 yellow cubes, and 1 green cube.
(For review, see Lesson 2-6.)

1. $P(\text{red})$ **2.** $P(\text{blue})$ **3.** $P(\text{yellow})$ **4.** $P(\text{not red})$

For Lesson 14-2 — Multiply Fractions

Find each product. *(For review, see pages 800 and 801.)*

5. $\frac{4}{5} \cdot \frac{3}{4}$ **6.** $\frac{5}{12} \cdot \frac{6}{11}$ **7.** $\frac{7}{20} \cdot \frac{4}{19}$

8. $\frac{4}{32} \cdot \frac{7}{32}$ **9.** $\frac{13}{52} \cdot \frac{4}{52}$ **10.** $\frac{56}{100} \cdot \frac{24}{100}$

For Lesson 14-4 — Write Decimals as Percents

Write each decimal as a percent. *(For review, see pages 804 and 805.)*

11. 0.725 **12.** 0.148 **13.** 0.4 **14.** 0.0168

For Lesson 14-5 — Write Fractions as Percents

Write each fraction as a percent. Round to the nearest tenth. *(For review, see pages 804 and 805.)*

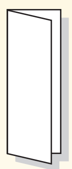
15. $\frac{7}{8}$ **16.** $\frac{33}{80}$ **17.** $\frac{107}{125}$ **18.** $\frac{625}{1024}$

FOLDABLES™ Study Organizer

Probability Make this Foldable to help you organize your notes. Begin with a sheet of plain $8\frac{1}{2}$" by 11" paper.

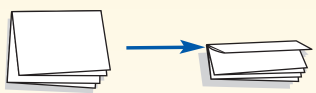
Step 1 **Fold in Half**

Fold in half lengthwise.



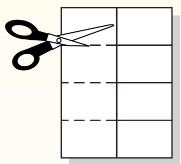
Step 2 **Fold Again in Fourths**

Fold the top to the bottom twice.



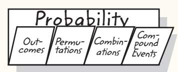
Step 3 **Cut**

Open. Cut along the folds to make four tabs.



Step 4 **Label**

Label as shown.



Reading and Writing As you read and study the chapter, write notes and examples for each concept under the tabs.

14-1 Counting Outcomes

What You'll Learn

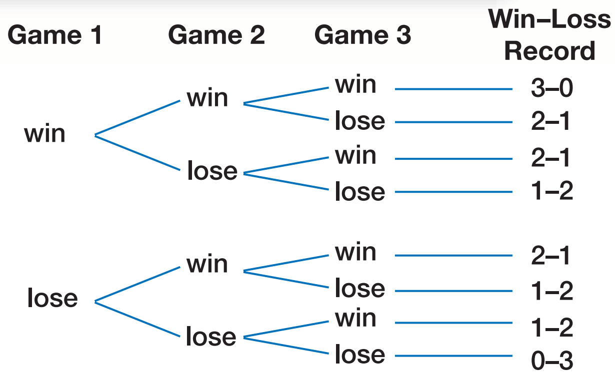
- Count outcomes using a tree diagram.
- Count outcomes using the Fundamental Counting Principle.

Vocabulary

- tree diagram
- sample space
- event
- Fundamental Counting Principle
- factorial

How are possible win–loss records counted in football?

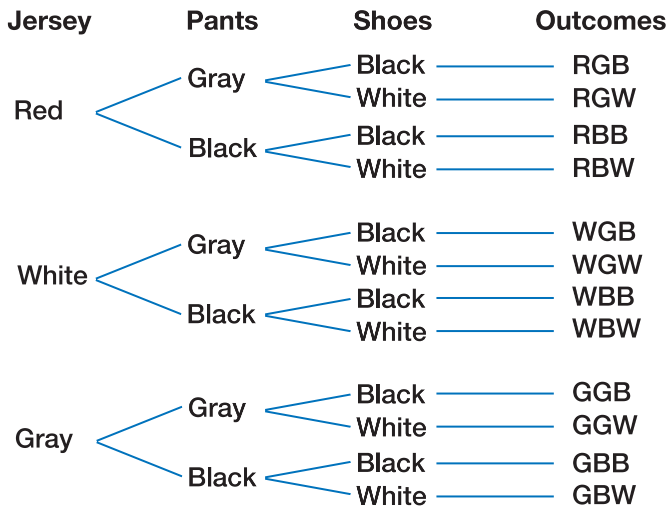
The championship in the Atlantic Coast Conference is decided by the number of conference wins. If there is a tie in conference wins, then the team with more nonconference wins is champion. If Florida State plays 3 nonconference games, the diagram at the right shows the different records they could have for those games.



TREE DIAGRAMS One method used for counting the number of possible outcomes is to draw a **tree diagram**. The last column of a tree diagram shows all of the possible outcomes. The list of all possible outcomes is called the **sample space**, while any collection of one or more outcomes in the sample space is called an **event**.

Example 1 Tree Diagram

A football team uses red jerseys for road games, white jerseys for home games, and gray jerseys for practice games. The team uses gray or black pants, and black or white shoes. Use a tree diagram to determine the number of possible uniforms.



The tree diagram shows that there are 12 possible uniforms.

THE FUNDAMENTAL COUNTING PRINCIPLE The number of possible uniforms in Example 1 can also be found by multiplying the number of choices for each item. If the team can choose from 3 different colored jerseys, 2 different colored pants, and 2 different colored pairs of shoes, there are $3 \cdot 2 \cdot 2$ or 12 possible uniforms. This example illustrates the **Fundamental Counting Principle**.

Key Concept — *Fundamental Counting Principle*

If an event M can occur in m ways and is followed by an event N that can occur in n ways, then the event M followed by event N can occur in $m \cdot n$ ways.

Example 2 Fundamental Counting Principle

The Uptown Deli offers a lunch special in which you can choose a sandwich, a side dish, and a beverage. If there are 10 different sandwiches, 12 different side dishes, and 7 different beverages from which to choose, how many different lunch specials can you order?

Multiply to find the number of lunch specials.

$$\underbrace{10}_{\text{sandwich choices}} \cdot \underbrace{12}_{\text{side dish choices}} \cdot \underbrace{7}_{\text{beverage choices}} = \underbrace{840}_{\text{number of specials}}$$

The number of different lunch specials is 840.

Example 3 Counting Arrangements

Mackenzie is setting up a display of the ten most popular video games from the previous week. If she places the games side-by-side on a shelf, in how many different ways can she arrange them?

The number of ways to arrange the games can be found by multiplying the number of choices for each position.

- Mackenzie has ten games from which to choose for the first position.
- After choosing a game for the first position, there are nine games left from which to choose for the second position.
- There are now eight choices for the third position.
- This process continues until there is only one choice left for the last position.

Let n represent the number of arrangements.

$n = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ or 3,628,800

There are 3,628,800 different ways to arrange the video games.

The expression $n = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ used in Example 3 can be written as 10! using a **factorial**.

Key Concept — *Factorial*

- **Words** The expression $n!$, read n factorial, where n is greater than zero, is the product of all positive integers beginning with n and counting backward to 1.
- **Symbols** $n! = n \cdot (n - 1) \cdot (n - 2) \cdot \ldots \cdot 3 \cdot 2 \cdot 1$
- **Example** $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ or 120

By definition, $0! = 1$.

Example 4 Factorial

Find the value of each expression.

a. 6!

$6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ Definition of factorial

$= 720$ Simplify.

b. 10!

$10! = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ Definition of factorial

$= 3{,}628{,}800$ Simplify.

Example 5 Use Factorials to Solve a Problem

ROLLER COASTERS **Zach and Kurt are going to an amusement park. They cannot decide in which order to ride the 12 roller coasters in the park.**

a. How many different orders can they ride all of the roller coasters if they ride each once?

Use a factorial.

$12! = 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ Definition of factorial

$= 479{,}001{,}600$ Simplify.

There are 479,001,600 ways in which Zach and Kurt can ride all 12 roller coasters.

b. If they only have time to ride 8 of the roller coasters, how many ways can they do this?

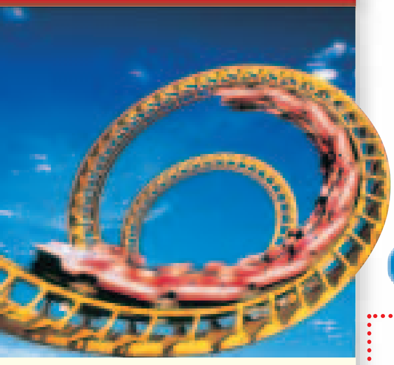
Use the Fundamental Counting Principle to find the sample space.

$s = 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5$ Fundamental Counting Principle

$= 19{,}958{,}400$ Simplify.

There are 19,958,400 ways for Zach and Kurt to ride 8 of the roller coasters.

More About. . .



Roller Coasters

In 2000, there were 646 roller coasters in the United States.

Type	Number
Wood	118
Steel	445
Inverted	35
Stand Up	10
Suspended	11
Wild Mouse	27

Source: Roller Coaster Database

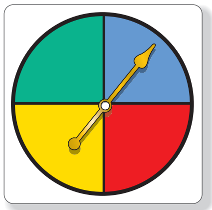
Check for Understanding

Concept Check

1. **OPEN ENDED** Give an example of an event that has $7 \cdot 6$ or 42 outcomes.
2. **Draw** a tree diagram to represent the outcomes of tossing a coin three times.
3. **Explain** what the notation 5! means.

Guided Practice

For Exercises 4–6, suppose the spinner at the right is spun three times.



4. Draw a tree diagram to show the sample space.
5. How many outcomes are possible?
6. How many outcomes involve both green and blue?

7. Find the value of 8!.

Application

8. **SCHOOL** In a science class, each student must choose a lab project from a list of 15, write a paper on one of 6 topics, and give a presentation about one of 8 subjects. How many different ways can students choose to do their assignments?

Practice and Apply

Homework Help

For Exercises	See Examples
9, 10, 19	1
11–14	4
15–18, 20–22	2, 3, 5

Extra Practice
See page 851.

Draw a tree diagram to show the sample space for each event. Determine the number of possible outcomes.

9. earning an A, B, or C in English, Math, and Science classes

10. buying a computer with a choice of a CD-ROM, a CD recorder, or a DVD drive, one of 2 monitors, and either a printer or a scanner

Find the value of each expression.

11. 4! **12.** 7! **13.** 11! **14.** 13!

15. Three dice, one red, one white, and one blue are rolled. How many outcomes are possible?

16. How many outfits are possible if you choose one each of 5 shirts, 3 pairs of pants, 3 pairs of shoes, and 4 jackets?

17. **TRAVEL** Suppose four different airlines fly from Seattle to Denver. Those same four airlines and two others fly from Denver to St. Louis. If there are no direct flights from Seattle to St. Louis, in how many ways can a traveler book a flight from Seattle to St. Louis?

COMMUNICATIONS **For Exercises 18 and 19, use the following information.**
A new 3-digit area code is needed in a certain area to accommodate new telephone numbers.

18. If the first digit must be odd, the second digit must be a 0 or a 1, and the third digit can be anything, how many area codes are possible?

19. Draw a tree diagram to show the different area codes using 4 or 5 for the first digit, 0 or 1 for the second digit, and 7, 8, or 9 for the third digit.

SOCCER **For Exercises 20–22, use the following information.**
The Columbus Crew are playing the D.C. United in a best three-out-of-five championship soccer series.

20. What are the possible outcomes of the series?

21. How many outcomes require exactly four games to determine the champion?

22. How many ways can D.C. United win the championship?

23. **CRITICAL THINKING** To get to and from school, Tucker can walk, ride his bike, or get a ride with a friend. Suppose that one week he walked 60% of the time, rode his bike 20% of the time, and rode with his friend 20% of the time. How many outcomes represent this situation? Assume that he returns home the same way that he went to school.

24. *WRITING IN MATH* Answer the question that was posed at the beginning of the lesson.

How are possible win–loss records counted in football?

Include the following in your answer:

- a few sentences describing how a tree diagram can be used to count the wins and losses of a football team, and
- a demonstration of how to find the number of possible outcomes for a team that plays 4 home games.

Standardized Test Practice

25. Evaluate 9!.

Ⓐ 362,880 Ⓑ 40,320 Ⓒ 36 Ⓓ 8

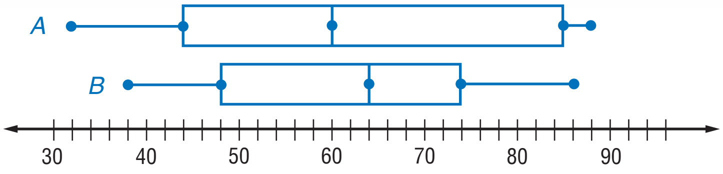
26. A car manufacturer offers a sports car in 4 different models with 6 different option packages. Each model is available in 12 different colors. How many different possibilities are available for this car?

Ⓐ 96 Ⓑ 144 Ⓒ 288 Ⓓ 384

Maintain Your Skills

Mixed Review

For Exercises 27–30, use box-and-whisker plots A and B. *(Lesson 13-5)*



27. Determine the least value, greatest value, lower quartile, upper quartile, and median for each plot.

28. Which set of data contains the least value?

29. Which plot has the smaller interquartile range?

30. Which plot has the greater range?

For Exercises 31–34, use the stem-and-leaf plot. *(Lesson 13-4)*

31. Find the range of the data.

32. What is the median?

33. Determine the upper quartile, lower quartile, and interquartile range of the data.

34. Identify any outliers.

Stem	Leaf
3	0 1 4 5
4	4 4 8
5	6 9
6	6 8
7	1 6
8	0 1
9	
10	9 $3 \mid 0 = 30$

Find each sum or difference. *(Lesson 12-7)*

35. $\frac{2x + 1}{3x - 1} + \frac{x + 4}{x - 2}$

36. $\frac{4n}{2n + 6} + \frac{3}{n + 3}$

37. $\frac{3z + 2}{3z - 6} - \frac{z + 2}{z^2 - 4}$

38. $\frac{m - n}{m + n} - \frac{1}{m^2 - n^2}$

Solve each equation. *(Lesson 11-3)*

39. $5\sqrt{2n^2 - 28} = 20$

40. $\sqrt{5x^2 - 7} = 2x$

41. $\sqrt{x + 2} = x - 4$

Solve each equation by completing the square. Round to the nearest tenth if necessary. *(Lesson 10-3)*

42. $b^2 - 6b + 4 = 0$

43. $n^2 + 8n - 5 = 0$

44. $x^2 - 11x - 17 = 0$

45. $2p^2 + 10p + 3 = 0$

Study Tip

Deck of Cards

In this text, a *standard deck of cards* always means a deck of 52 playing cards. There are 4 suits—clubs (black), diamonds (red), hearts (red), and spades (black)—with 13 cards in each suit.

Getting Ready for the Next Lesson

PREREQUISITE SKILL **One card is drawn at random from a standard deck of cards. Find each probability.** *(To review **simple probability**, see Lesson 2-6.)*

46. $P(10)$

47. $P(\text{ace})$

48. $P(\text{red } 5)$

49. $P(\text{queen of clubs})$

50. $P(\text{even number})$

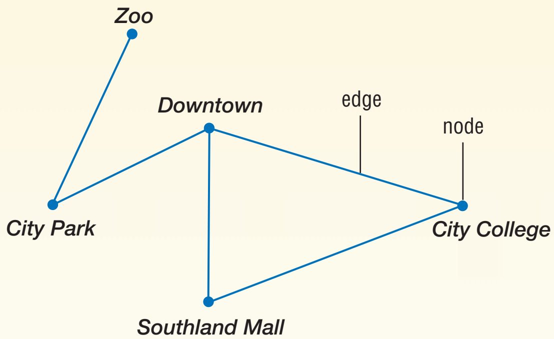
51. $P(3 \text{ or king})$

Algebra Activity

A Follow-Up of Lesson 14-1

Finite Graphs

The City Bus Company provides daily bus service between City College and Southland Mall, City College and downtown, downtown and Southland Mall, downtown and City Park, and City Park and the zoo. The daily routes can be represented using a **finite graph** like the one at the right.

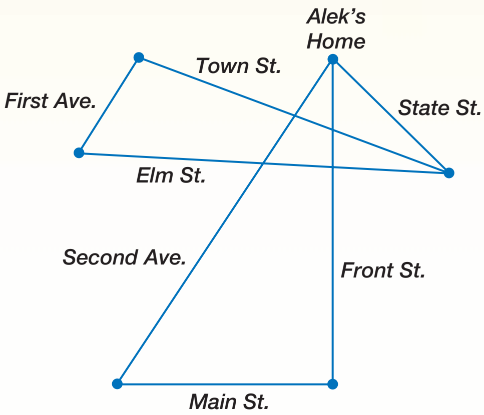


The graph is called a **network**, and each point on the graph is called a **node**. The paths connecting the nodes are called **edges**. A network is said to be **traceable** if all of the nodes can be connected, and each edge can be covered exactly once when the graph is used.

Collect the Data

The graph represents the streets on Alek's newspaper route. To get his route completed as quickly as possible, Alek would like to ride his bike down each street only once.

- Copy the graph onto your paper.
- Beginning at Alek's home, trace over his route without lifting your pencil. Remember to trace each edge only once.
- Compare your graph with those of your classmates.



Analyze the Data

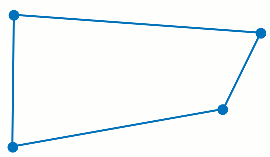
1. Is Alek's route traceable? If so, describe his route.

2. Is there more than one traceable route that begins at Alek's house? If so, how many?

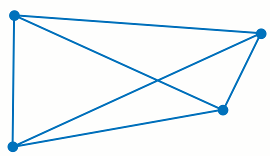
3. Suppose it does not matter where Alek starts his route. How many traceable routes are possible now?

Determine whether each graph is traceable. Explain your reasoning.

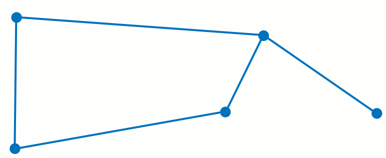
4.



5.



6.



7. The campus for Centerburgh High School has five buildings built around the edge of a circular courtyard. There is a sidewalk between each pair of buildings.

a. Draw a graph of the campus.

b. Is the graph traceable?

c. Suppose that there is not a sidewalk between the pairs of adjacent buildings. Is it possible to reach all five buildings without walking down any sidewalk more than once?

8. **Make a conjecture** for a rule to determine whether a graph is traceable.

14-2 Permutations and Combinations

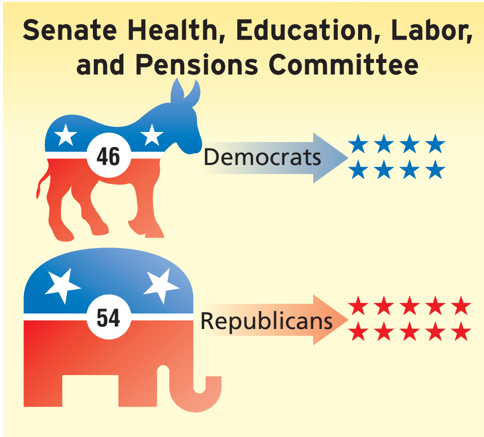
What You'll Learn

- Determine probabilities using permutations.
- Determine probabilities using combinations.

How can combinations be used to form committees?

Vocabulary
- permutation
- combination

The United States Senate forms various committees by selecting senators from both political parties. The Senate Health, Education, Labor, and Pensions Committee of the 106th Congress was made up of 10 Republican senators and 8 Democratic senators. How many different ways could the committee have been selected? The members of the committee were selected in no particular order. This is an example of a combination.



PERMUTATIONS An arrangement or listing in which order or placement is important is called a **permutation**.

Study Tip

Common Misconception
When arranging two objects *A* and *B* using a permutation, the arrangement *AB* is different from the arrangement *BA*.

Example 1 Tree Diagram Permutation

EMPLOYMENT The manager of a coffee shop needs to hire two employees, one to work at the counter and one to work at the drive-through window. Katie, Bob, Alicia, and Jeremiah all applied for a job. How many possible ways are there for the manager to place the applicants?

Use a tree diagram to show the possible arrangements.

Counter	Drive-Through	Outcomes
Katie (K)	Bob	KB
	Alicia	KA
	Jeremiah	KJ
Bob (B)	Katie	BK
	Alicia	BA
	Jeremiah	BJ
Alicia (A)	Jeremiah	AJ
	Katie	AK
	Bob	AB
Jeremiah (J)	Katie	JK
	Bob	JB
	Alicia	JA

There are 12 different ways for the 4 applicants to hold the 2 positions.

In Example 1, the positions are in a specific order, so each arrangement is unique. The symbol ${}_4P_2$ denotes the number of permutations when arranging 4 applicants in 2 positions. You can also use the Fundamental Counting Principle to determine the number of permutations.

$$
\begin{aligned}
{}_4P_2 &= \underbrace{4}_{\text{ways to choose first employee}} \cdot \underbrace{3}_{\text{ways to choose second employee}} \\
&= 4 \cdot 3 \cdot \frac{2 \cdot 1}{2 \cdot 1} && \frac{2 \cdot 1}{2 \cdot 1} = 1 \\
&= \frac{4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} && \text{Multiply.} \\
&= \frac{4!}{2!} && 4 \cdot 3 \cdot 2 \cdot 1 = 4!,\ 2 \cdot 1 = 2!
\end{aligned}
$$

In general, ${}_nP_r$ is used to denote the number of permutations of n objects taken r at a time.

Key Concept — Permutation

- **Words** The number of permutations of n objects taken r at a time is the quotient of $n!$ and $(n - r)!$.
- **Symbols** ${}_nP_r = \frac{n!}{(n-r)!}$

Example 2 Permutation

Find ${}_{10}P_6$.

$${}_nP_r = \frac{n!}{(n-r)!} \quad \text{Definition of } {}_nP_r$$

$${}_{10}P_6 = \frac{10!}{(10-6)!} \quad n = 10,\ r = 6$$

$${}_{10}P_6 = \frac{10!}{4!} \quad \text{Subtract.}$$

$${}_{10}P_6 = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot \overset{1}{\cancel{4 \cdot 3 \cdot 2 \cdot 1}}}{\underset{1}{\cancel{4 \cdot 3 \cdot 2 \cdot 1}}} \quad \text{Definition of factorial}$$

$${}_{10}P_6 = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \text{ or } 151{,}200 \quad \text{Simplify.}$$

There are 151,200 permutations of 10 objects taken 6 at a time.

Permutations are often used to find the probability of events occurring.

Example 3 Permutation and Probability

A word processing program requires a user to enter a 7-digit registration code made up of the digits 1, 2, 4, 5, 6, 7, and 9. Each number has to be used, and no number can be used more than once.

a. How many different registration codes are possible?

Since the order of the numbers in the code is important, this situation is a permutation of 7 digits taken 7 at a time.

$${}_nP_r = \frac{n!}{(n-r)!} \quad \text{Definition of permutation}$$

$${}_7P_7 = \frac{7!}{(7-7)!} \quad n = 7,\ r = 7$$

$${}_7P_7 = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1} \text{ or } 5040 \quad \text{Definition of factorial}$$

There are 5040 possible codes with the digits 1, 2, 4, 5, 6, 7, and 9.

Study Tip

Permutations
The number of permutations of n objects taken n at a time is $n!$.
${}_nP_n = n!$

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b. What is the probability that the first three digits of the code are even numbers?

Use the Fundamental Counting Principle to determine the number of ways for the first three digits to be even.

- There are 3 even digits and 4 odd digits.
- The number of choices for the first three digits, if they are even, is $3 \cdot 2 \cdot 1$.
- The number of choices for the remaining odd digits is $4 \cdot 3 \cdot 2 \cdot 1$.
- The number of favorable outcomes is $3 \cdot 2 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ or 144. There are 144 ways for this event to occur out of the 5040 possible permutations.

$$P(\text{first 3 digits even}) = \frac{144}{5040} \quad \begin{matrix} \leftarrow \text{number of favorable outcomes} \\ \leftarrow \text{number of possible outcomes} \end{matrix}$$

$$= \frac{1}{35} \quad \text{Simplify.}$$

The probability that the first three digits of the code are even is $\frac{1}{35}$ or about 3%.

COMBINATIONS An arrangement or listing in which order is not important is called a **combination**. For example, if you are choosing 2 salad ingredients from a list of 10, the order in which you choose the ingredients does not matter.

Key Concept — Combination

- **Words** The number of combinations of n objects taken r at a time is the quotient of $n!$ and $(n - r)!r!$.
- **Symbols** ${}_nC_r = \frac{n!}{(n - r)!r!}$

Standardized Test Practice

A B C D

Example 4 Combination

Multiple-Choice Test Item

The students of Mr. DeLuca's homeroom had to choose 4 out of the 7 people who were nominated to serve on the Student Council. How many different groups of students could be selected?

(A) 840 (B) 210

(C) 35 (D) 24

Read the Test Item

The order in which the students are chosen does not matter, so this situation represents a combination of 7 people taken 4 at a time.

Solve the Test Item

$${}_nC_r = \frac{n!}{(n - r)!r!} \quad \text{Definition of combination}$$

$${}_7C_4 = \frac{7!}{(7 - 4)!4!} \quad n = 7, r = 4$$

$$= \frac{7 \cdot 6 \cdot 5 \cdot \overset{1}{\cancel{4 \cdot 3 \cdot 2 \cdot 1}}}{3 \cdot 2 \cdot 1 \cdot \underset{1}{\cancel{4 \cdot 3 \cdot 2 \cdot 1}}} \quad \text{Definition of factorial}$$

$$= \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1} \text{ or } 35 \quad \text{Simplify.}$$

There are 35 different groups of students that could be selected. Choice C is correct.

Test-Taking Tip

Read each question carefully to determine whether the situation involves a permutation or a combination. Often, the answer choices include examples of both.

Combinations and the products of combinations can be used to determine probabilities.

Example 5 Use Combinations

SCHOOL A science teacher at Sunnydale High School needs to choose 12 students out of 16 to serve as peer tutors. A group of 7 seniors, 5 juniors, and 4 sophomores have volunteered to be tutors.

a. How many different ways can the teacher choose 12 students?

The order in which the students are chosen does not matter, so we must find the number of combinations of 16 students taken 12 at a time.

$${}_nC_r = \frac{n!}{(n-r)!r!}$$ Definition of combination

$${}_{16}C_{12} = \frac{16!}{(16-12)!12!}$$ $n = 16$, $r = 12$

$$= \frac{16!}{4!12!}$$ $16 - 12 = 4$

$$= \frac{16 \cdot 15 \cdot 14 \cdot 13 \cdot \overset{1}{\cancel{12!}}}{4! \cdot \underset{1}{\cancel{12!}}}$$ Divide by the GCF, 12!.

$$= \frac{43{,}680}{24} \text{ or } 1820$$ Simplify.

There are 1820 ways to choose 12 students out of 16.

b. If the students are chosen randomly, what is the probability that 4 seniors, 4 juniors, and 4 sophomores will be selected?

There are three questions to consider.

- How many ways can 4 seniors be chosen from 7?
- How many ways can 4 juniors be chosen from 5?
- How many ways can 4 sophomores be chosen from 4?

Using the Fundamental Counting Principle, the answer can be determined with the product of the three combinations.

$$\underbrace{({}_7C_4)}_{\text{ways to choose 4 seniors out of 7}} \cdot \underbrace{({}_5C_4)}_{\text{ways to choose 4 juniors out of 5}} \cdot \underbrace{({}_4C_4)}_{\text{ways to choose 4 sophomores out of 4}}$$

$$({}_7C_4)({}_5C_4)({}_4C_4) = \frac{7!}{(7-4)!4!} \cdot \frac{5!}{(5-4)!4!} \cdot \frac{4!}{(4-4)!4!}$$ Definition of combination

$$= \frac{7!}{3!4!} \cdot \frac{5!}{1!4!} \cdot \frac{4!}{0!4!}$$ Simplify.

$$= \frac{7 \cdot 6 \cdot 5}{3!} \cdot \frac{5}{1}$$ Divide by the GCF, 4!.

$$= 175$$ Simplify.

There are 175 ways to choose this particular combination out of 1820 possible combinations.

$$P(\text{4 seniors, 4 juniors, 4 sophomores}) = \frac{175}{1820}$$ ← number of favorable outcomes / ← number of possible outcomes

$$= \frac{5}{52}$$ Simplify.

The probability that the science teacher will randomly select 4 seniors, 4 juniors, and 4 sophomores is $\frac{5}{52}$ or about 10%.

Study Tip

Combinations

The number of combinations of n objects taken n at a time is 1. ${}_nC_n = 1$

Check for Understanding

Concept Check

1. **OPEN ENDED** Describe the difference between a permutation and a combination. Then give an example of each.

2. **Demonstrate** and explain why ${}_nC_r = 1$ whenever $n = r$. What does ${}_nP_r$ always equal when $n = r$?

3. **FIND THE ERROR** Eric and Alisa are taking a trip to Washington, D.C. Their tour bus stops at the Lincoln Memorial, the Jefferson Memorial, the Washington Monument, the White House, the Capitol Building, the Supreme Court, and the Pentagon. Both are finding the number of ways they can choose to visit 5 of these 7 sites.

Eric	Alisa
${}_7C_5 = \frac{7!}{2!}$ or 2520	${}_7C_5 = \frac{7!}{2!\,5!}$ or 21

Who is correct? Explain your reasoning.

Guided Practice

Determine whether each situation involves a *permutation* or *combination*. Explain your reasoning.

4. choosing 6 books from a selection of 12 for summer reading
5. choosing digits for a personal identification number

Evaluate each expression.

6. ${}_8P_5$
7. ${}_7C_5$
8. $({}_{10}P_5)({}_3P_2)$
9. $({}_6C_2)({}_4C_3)$

For Exercises 10–12, use the following information.
The digits 0 through 9 are written on index cards. Three of the cards are randomly selected to form a 3-digit code.

10. Does this situation represent a permutation or a combination? Explain.
11. How many different codes are possible?
12. What is the probability that all 3 digits will be odd?

Standardized Test Practice

13. A diner offers a choice of two side items from the list with each entrée. How many ways can two items be selected?

Ⓐ 15 Ⓑ 28
Ⓒ 30 Ⓓ 56

Side Items	
French fries	mixed vegetables
baked potato	rice pilaf
cole slaw	baked beans
small salad	applesauce

Practice and Apply

Determine whether each situation involves a *permutation* or *combination*. Explain your reasoning.

14. team captains for the soccer team
15. three mannequins in a display window
16. a hand of 10 cards from a selection of 52
17. the batting order of the New York Yankees

Homework Help

For Exercises	See Examples
14–21, 34 36, 40	1, 4
22–33, 35, 37–39, 41–49	2, 3, 5

Extra Practice
See page 851.

18. first place and runner-up winners for the table tennis tournament
19. a selection of 5 DVDs from a group of eight
20. selection of 2 candy bars from six equally-sized bars
21. the selection of 2 trombones, 3 clarinets, and 2 trumpets for a jazz combo

Evaluate each expression.

22. ${}_{12}P_3$
23. ${}_{4}P_1$
24. ${}_{6}C_6$
25. ${}_{7}C_3$
26. ${}_{15}C_3$
27. ${}_{20}C_8$
28. ${}_{15}P_3$
29. ${}_{16}P_5$
30. $({}_{7}P_7)({}_{7}P_1)$
31. $({}_{20}P_2)({}_{16}P_4)$
32. $({}_{3}C_2)({}_{7}C_4)$
33. $({}_{8}C_5)({}_{5}P_5)$

SOFTBALL **For Exercises 34 and 35, use the following information.**
The manager of a softball team needs to prepare a batting lineup using her nine starting players.

34. Does this situation involve a permutation or a combination?
35. How many different lineups can she make?

More About. . .

Softball

The game of softball was developed in 1888 as an indoor sport for practicing baseball during the winter months.

Source: www.encyclopedia.com

SCHOOL **For Exercises 36–39, use the following information.**
Mrs. Moyer's class has to choose 4 out of 12 people to serve on an activity committee.

36. Does the selection of the students involve a permutation or a combination? Explain.
37. How many different groups of students could be selected?
38. Suppose the students are selected for the positions of chairperson, activities planner, activity leader, and treasurer. How many different groups of students could be selected?
39. What is the probability that any one of the students is chosen to be the chairperson?

GAMES **For Exercises 40–42, use the following information.**
In your turn of a certain game, you roll five different-colored dice.

40. Do the outcomes of rolling the five dice represent a permutation or a combination? Explain.
41. How many outcomes are possible?
42. What is the probability that all five dice show the same number on one roll?

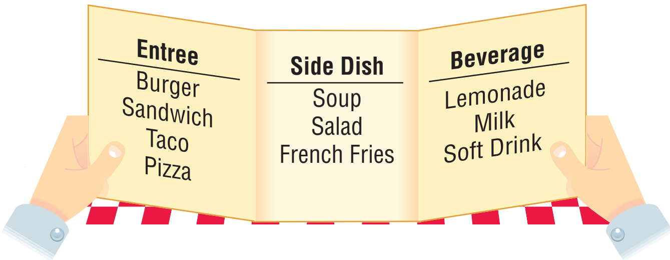
BUSINESS **For Exercises 43 and 44, use the following information.**
There are six positions available in the research department of a software company. Of the applicants, 15 are men and 10 are women.

43. In how many ways could 4 men and 2 women be chosen if each were equally qualified?
44. What is the probability that five women are selected if the positions are randomly filled?

TRACK **For Exercises 45 and 46, use the following information.**
Central High School is competing against West High School at a track meet. Each team entered 4 girls to run the 1600-meter event. The top three finishers are awarded medals.

45. How many different ways can the runners place first, second, and third?
46. If all eight runners have an equal chance of placing, what is the probability that the first and second place finishers are from West and the third place finisher is from Central?

DINING **For Exercises 47–49, use the following information.**
For lunch in the school cafeteria, you can select one item from each category to get the daily combo.



47. Find the number of possible meal combinations.

48. If a side dish is chosen at random, what is the probability that a student will choose soup?

49. What is the probability that a student will randomly choose a sandwich and soup?

CRITICAL THINKING **For Exercises 50 and 51, use the following information.**
Larisa is trying to solve a word puzzle. She needs to arrange the letters H, P, S, T, A, E, and O into a two-word arrangement.

50. How many different arrangements of the letters can she make?

51. Assuming that each arrangement has an equal chance of occurring, what is the probability that she will form the words *tap shoe* on her first try?

WebQuest

You can use permutations and combinations to analyze data on U.S. schools. Visit www.algebra1.com/webquest to continue work on your WebQuest project.

SWIMMING **For Exercises 52–54, use the following information.**
A swimming coach plans to pick four swimmers out of a group of 6 to form the 400-meter freestyle relay team.

52. How many different teams can he form?

53. The coach must decide in which order the four swimmers should swim. He timed the swimmers in each possible order and chose the best time. How many relays did the four swimmers have to swim so that the coach could collect all of the data necessary?

54. If Tomás is chosen to be on the team, what is the probability that he will swim in the third leg?

55. *WRITING IN MATH* Answer the question that was posed at the beginning of the lesson.

How can combinations be used to form committees?

Include the following in your answer:

- a few sentences explaining why forming a Senate committee is a combination, and
- an explanation of how to find the number of ways to select the committee if committee positions are based upon seniority.

Standardized Test Practice

56. There are 12 songs on a CD. If 10 songs are played randomly and each song is played once, how many arrangements are there?

(A) 479,001,600 (B) 239,500,800 (C) 66 (D) 1

57. Julie remembered that the 4 digits of her locker combination were 4, 9, 15, and 22, but not their order. What is the maximum number of attempts Julie has to make to find the correct combination?

(A) 4 (B) 16 (C) 24 (D) 256

Maintain Your Skills

Mixed Review

58. The Sanchez family acts as a host family for a foreign exchange student during each school year. It is equally likely that they will host a girl or a boy. How many different ways can they host boys and girls over the next four years? *(Lesson 14-1)*

STATISTICS **For Exercises 59–62, use the table at the right.** *(Lesson 13-5)*

59. Make a box-and-whisker plot of the data.

60. What is the range of the data?

61. Identify the lower and upper quartiles.

62. Name any outliers.

Highest Paying Occupations in America

Occupation	Median Salary
Physician	\$148,000
Dentist	\$93,000
Lobbyist	\$91,300
Management Consultant	\$61,900
Lawyer	\$60,500
Electrical Engineer	\$59,100
School Principal	\$57,300
Aeronautical Engineer	\$56,700
Airline Pilot	\$56,500
Civil Engineer	\$55,800

Source: U.S. Bureau of Labor Statistics

Online Research **Data Update** For current data on the highest-paying occupations, visit: www.algebra1.com/data_update

Simplify each expression. *(Lesson 12-2)*

63. $\dfrac{x + 3}{x^2 + 6x + 9}$ **64.** $\dfrac{x^2 - 49}{x^2 - 2x - 35}$ **65.** $\dfrac{n^2 - n - 20}{n^2 + 9n + 20}$

Find the distance between each pair of points whose coordinates are given. Express answers in simplest radical form and as decimal approximations rounded to the nearest hundredth if necessary. *(Lesson 11-5)*

66. $(12, 20), (16, 34)$ **67.** $(-18, 7), (2, 15)$ **68.** $(-2, 5), \left(-\frac{1}{2}, 3\right)$

Solve each equation by using the Quadratic Formula. Approximate irrational roots to the nearest hundredth. *(Lesson 10-4)*

69. $m^2 + 4m + 2 = 0$ **70.** $2s^2 + s - 15 = 0$ **71.** $2n^2 - n = 4$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find each sum or difference.**
(To review ***fractions****, see pages 798 and 799.)*

72. $\frac{8}{52} + \frac{4}{52}$ **73.** $\frac{7}{32} + \frac{5}{8}$ **74.** $\frac{5}{15} + \frac{6}{15} - \frac{2}{15}$

75. $\frac{15}{24} + \frac{11}{24} - \frac{3}{4}$ **76.** $\frac{2}{3} + \frac{15}{36} - \frac{1}{4}$ **77.** $\frac{16}{25} + \frac{3}{10} - \frac{1}{4}$

Practice Quiz 1 — *Lessons 14-1 and 14-2*

Find the number of outcomes for each event. *(Lesson 14-1)*

1. A die is rolled and two coins are tossed.

2. A certain model of mountain bike comes in 5 sizes, 4 colors, with regular or off-road tires, and with a choice of 1 of 5 accessories.

Find each value. *(Lesson 14-2)*

3. ${}_{13}C_8$ **4.** ${}_{9}P_6$

5. A flower bouquet has 5 carnations, 6 roses, and 3 lilies. If four flowers are selected at random, what is the probability of selecting two roses and two lilies? *(Lesson 14-2)*

Reading Mathematics

Mathematical Words and Related Words

You may have noticed that many words used in mathematics contain roots of other words and are closely related to other English words. You can use the more familiar meanings of these related words to better understand mathematical meanings.

The table shows two mathematical terms along with related words and their meanings as well as additional notes.

Mathematical Term and Meaning	Related Words and Meanings	Notes
combination A combination is a selection of distinct objects from a group of objects, where the order in which they were selected does not matter.	*combine* (n): a harvesting machine that performs many functions *binary*: a base-two numerical system	*Combine* originally meant to put just two things together; it now means to put any number of things together.
permutation A permutation is an arrangement of distinct objects from a group of objects, where the arrangement is in a certain order.	*mutation*: a change in genes or other characteristics *commute*: to change places; for example, $2 + 5 = 5 + 2$	

Notice how the meanings of the related words can give an insight to the meanings of the mathematical terms.

Reading to Learn

1. Do the related words of combination and permutation help you to remember their mathematical meanings? Explain.

2. What is a similarity and a difference between the mathematical meanings of combination and permutation?

3. **RESEARCH** Use the Internet or other reference to find the mathematical meaning of the word *factorial* and meanings of at least two related words. How are these meanings connected?

4. **RESEARCH** Use the Internet or other reference to find the meanings of the word *probability* and its Latin origins *probus* and *probare*. Compare the three.

14-3 Probability of Compound Events

What You'll Learn

- Find the probability of two independent events or dependent events.
- Find the probability of two mutually exclusive or inclusive events.

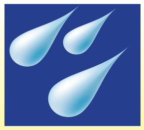
Vocabulary

- simple event
- compound event
- independent events
- dependent events
- complements
- mutually exclusive
- inclusive

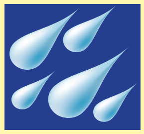
How are probabilities used by meteorologists?

The weather forecast for the weekend calls for rain. By using the probabilities for both days, we can find other probabilities for the weekend. What is the probability that it will rain on both days? only on Saturday? Saturday or Sunday?

Weekend Forecast: Rain Likely



Saturday 40%

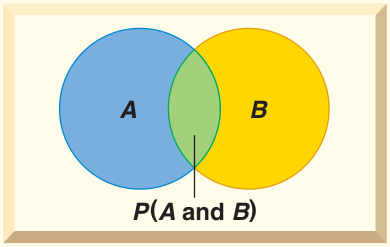


Sunday 80%

INDEPENDENT AND DEPENDENT EVENTS A single event, like rain on Saturday, is called a **simple event**. Suppose you wanted to determine the probability that it will rain both Saturday and Sunday. This is an example of a **compound event**, which is made up of two or more simple events. The weather on Saturday does not affect the weather on Sunday. These two events are called **independent events** because the outcome of one event does not affect the outcome of the other.

Key Concept — *Probability of Independent Events*

- **Words** If two events, A and B, are independent, then the probability of both events occurring is the product of the probability of A and the probability of B.
- **Symbols** $P(A \text{ and } B) = P(A) \cdot P(B)$
- **Model**



Example 1 *Independent Events*

Refer to the application above. Find the probability that it will rain on Saturday and Sunday.

$$P(A \text{ and } B) = P(A) \cdot P(B) \quad \text{Definition of independent events}$$

$$P(\text{Saturday and Sunday}) = \underbrace{P(\text{Saturday})}_{0.4} \cdot \underbrace{P(\text{Sunday})}_{0.8}$$

$$= 0.4 \cdot 0.8 \quad 40\% = 0.4 \text{ and } 80\% = 0.8$$

$$= 0.32 \quad \text{Multiply.}$$

The probability that it will rain on Saturday and Sunday is 32%.

CONTENTS

When the outcome of one event affects the outcome of another event, the events are **dependent events**. For example, drawing a card from a deck, not returning it, then drawing a second card are dependent events because the drawing of the second card is dependent on the drawing of the first card.

Key Concept — *Probability of Dependent Events*

- **Words** If two events, A and B, are dependent, then the probability of both events occurring is the product of the probability of A and the probability of B after A occurs.
- **Symbols** $P(A \text{ and } B) = P(A) \cdot P(B \text{ following } A)$

Example 2 Dependent Events

A bag contains 8 red marbles, 12 blue marbles, 9 yellow marbles, and 11 green marbles. Three marbles are randomly drawn from the bag and not replaced. Find each probability if the marbles are drawn in the order indicated.

a. *P*(red, blue, green)

The selection of the first marble affects the selection of the next marble since there is one less marble from which to choose. So, the events are dependent.

First marble: $P(\text{red}) = \frac{8}{40}$ or $\frac{1}{5}$ ← $\frac{\text{number of red marbles}}{\text{total number of marbles}}$

Second marble: $P(\text{blue}) = \frac{12}{39}$ or $\frac{4}{13}$ ← $\frac{\text{number of blue marbles}}{\text{number of marbles remaining}}$

Third marble: $P(\text{green}) = \frac{11}{38}$ ← $\frac{\text{number of green marbles}}{\text{number of marbles remaining}}$

$$P(\text{red, blue, green}) = P(\text{red}) \cdot P(\text{blue}) \cdot P(\text{green})$$

$$= \frac{1}{5} \cdot \frac{4}{13} \cdot \frac{11}{38} \quad \text{Substitution}$$

$$= \frac{44}{2470} \text{ or } \frac{22}{1235} \quad \text{Multiply.}$$

The probability of drawing red, blue, and green marbles is $\frac{22}{1235}$.

b. *P*(blue, yellow, yellow)

Notice that after selecting a yellow marble, not only is there one fewer marble from which to choose, there is also one fewer yellow marble.

$$P(\text{blue, yellow, yellow}) = P(\text{blue}) \cdot P(\text{yellow}) \cdot P(\text{yellow})$$

$$= \frac{12}{40} \cdot \frac{9}{39} \cdot \frac{8}{38} \quad \text{Substitution}$$

$$= \frac{864}{59{,}280} \text{ or } \frac{18}{1235} \quad \text{Multiply.}$$

The probability of drawing a blue and then two yellow marbles is $\frac{18}{1235}$.

c. *P*(red, yellow, *not* green)

Since the marble that is not green is selected after the first two marbles, there are 29 − 2 or 27 marbles that are not green.

$$P(\text{red, yellow, } not \text{ green}) = P(\text{red}) \cdot P(\text{yellow}) \cdot P(\text{not green})$$

$$= \frac{8}{40} \cdot \frac{9}{39} \cdot \frac{27}{38}$$

$$= \frac{1944}{59{,}280} \text{ or } \frac{81}{2470}$$

The probability of drawing a red, a yellow, and *not* a green marble is $\frac{81}{2470}$.

Study Tip

More Than Two Dependent Events
Notice that the formula for the probability of dependent events can be applied to more than two events.

In part **c** of Example 2, the events for drawing a marble that is green and for drawing a marble that is *not* green are called **complements**. Consider the probabilities for drawing the third marble.

Study Tip

Reading Math
A complement is one of two parts that make up a whole.

P(green) | P(*not* green) | sum of probabilities

$$\frac{11}{38} + \frac{27}{38} = 1$$

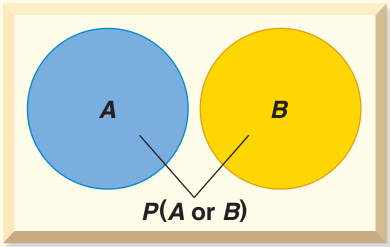
This is always true for any two complementary events.

MUTUALLY EXCLUSIVE AND INCLUSIVE EVENTS

Events that cannot occur at the same time are called **mutually exclusive**. Suppose you want to find the probability of rolling a 2 *or* a 4 on a die. Since a die cannot show both a 2 and a 4 at the same time, the events are mutually exclusive.

Key Concept — *Mutually Exclusive Events*

- **Words** If two events, A and B, are mutually exclusive, then the probability that either A or B occurs is the sum of their probabilities.
- **Symbols** $P(A \text{ or } B) = P(A) + P(B)$
- **Model**



Example 3 Mutually Exclusive Events

During a magic trick, a magician randomly draws one card from a standard deck of cards. What is the probability that the card drawn is a heart or a diamond?

Since a card cannot be both a heart and a diamond, the events are mutually exclusive.

$P(\text{heart}) = \frac{13}{52}$ or $\frac{1}{4}$ ← number of hearts / total number of cards

$P(\text{diamond}) = \frac{13}{52}$ or $\frac{1}{4}$ ← number of diamonds / total number of cards

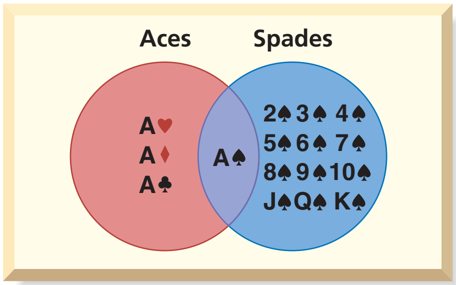
$P(\text{heart or diamond}) = P(\text{heart}) + P(\text{diamond})$ Definition of mutually exclusive events

$= \frac{1}{4} + \frac{1}{4}$ Substitution

$= \frac{2}{4}$ or $\frac{1}{2}$ Add.

The probability of drawing a heart or a diamond is $\frac{1}{2}$.

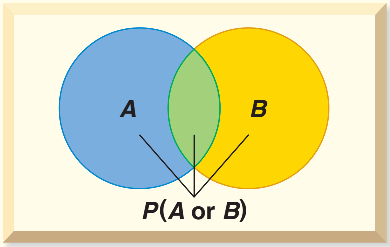
Suppose you wanted to find the probability of randomly selecting an ace or a spade from a standard deck of cards. Since it is possible to draw a card that is both an ace and a spade, these events are not mutually exclusive. They are called **inclusive** events.



If the formula for the probability of mutually exclusive events is used, the probability of drawing an ace of spades is counted twice, once for an ace and once for a spade. To correct this, you must subtract the probability of drawing the ace of spades from the sum of the individual probabilities.

Key Concept — *Probability of Inclusive Events*

- **Words** If two events, A and B, are inclusive, then the probability that either A or B occurs is the sum of their probabilities decreased by the probability of both occurring.
- **Model**



- **Symbols** $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Example 4 Inclusive Events

GAMES **In the game of bingo, balls or tiles are numbered 1 through 75. These numbers correspond to columns on a bingo card. The numbers 1 through 15 can appear in the B column, 16 through 30 in the I column, 31 through 45 in the N column, 46 through 60 in the G column, and 61 through 75 in the O column. A number is selected at random. What is the probability that it is a multiple of 4 or is in the O column?**

Since the numbers 64, 68, and 72 are multiples of 4 and they can be in the O column, these events are inclusive.

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ Definition of inclusive events

$P(\text{multiple of 4 or O column})$

$$= \underbrace{P(\text{multiple of 4})}_{} + \underbrace{P(\text{O column})}_{} - \underbrace{P(\text{multiple of 4 and O column})}_{}$$

$$= \frac{18}{75} + \frac{15}{75} - \frac{3}{75} \quad \text{Substitution}$$

$$= \frac{18 + 15 - 3}{75} \quad \text{LCD is 75.}$$

$$= \frac{30}{75} \text{ or } \frac{2}{5} \quad \text{Simplify.}$$

The probability of a number being a multiple of 4 or in the O column is $\frac{2}{5}$ or 40%.

Check for Understanding

Concept Check

1. **Explain** the difference between a simple event and a compound event.
2. **Find a counterexample** for the following statement.
   If two events are independent, then the probability of both events occurring is less than 1.
3. **OPEN ENDED** Explain how dependent events are different than independent events. Give specific examples in your explanation.

4. **FIND THE ERROR** On the school debate team, 6 of the 14 girls are seniors, and 9 of the 20 boys are seniors. Chloe and Amber are both seniors on the team. Each girl calculated the probability that either a girl or a senior would randomly be selected to argue a position at a state debate.

Chloe

P(girl or senior)

$= \frac{14}{34} + \frac{15}{34} - \frac{6}{34}$

$= \frac{23}{34}$

Amber

P(girl or senior)

$= \frac{6}{34} + \frac{15}{34} - \frac{14}{34}$

$= \frac{7}{34}$

Who is correct? Explain your reasoning.

Guided Practice

A bin contains 8 blue chips, 5 red chips, 6 green chips, and 2 yellow chips. Find each probability.

5. drawing a red chip, replacing it, then drawing a green chip
6. selecting two yellow chips without replacement
7. choosing green, then blue, then red, replacing each chip after it is drawn
8. choosing green, then blue, then red without replacing each chip

A student is selected at random from a group of 12 male and 12 female students. There are 3 male students and 3 female students from each of the 9th, 10th, 11th, and 12th grades. Find each probability.

9. P(9th or 12th grader)
10. P(10th grader or female)
11. P(male or female)
12. P(male or not 11th grader)

Application

BUSINESS For Exercises 13–15, use the following information.
Mr. Salyer is a buyer for an electronics store. He received a shipment of 5 DVD players in which one is defective. He randomly chose 3 of the DVD players to test.

13. Determine whether choosing one DVD player after another indicates independent or dependent events.
14. What is the probability that he selected the defective player?
15. Suppose the defective player is one of the three that Mr. Salyer tested. What is the probability that the last one tested was the defective one?

Practice and Apply

Homework Help

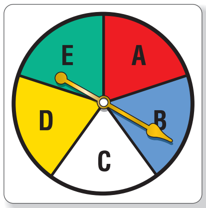
For Exercises	See Examples
16–19, 24, 25, 28–31	2
20–23, 32–34	1
26, 27, 41, 44, 45	4
36–40, 42, 43, 46, 47	3

Extra Practice
See page 851.

A bag contains 2 red, 6 blue, 7 yellow, and 3 orange marbles. Once a marble is selected, it is not replaced. Find each probability.

16. P(2 orange)
17. P(blue, then red)
18. P(2 yellows in a row then orange)
19. P(blue, then yellow, then red)

A die is rolled and a spinner like the one at the right is spun. Find each probability.



20. P(3 and D)
21. P(an odd number and a vowel)
22. P(a prime number and A)
23. P(2 and A, B, or C)

Raffle tickets numbered 1 through 30 are placed in a box. Tickets for a second raffle numbered 21 to 48 are placed in another box. One ticket is randomly drawn from each box. Find each probability.

24. Both tickets are even.

25. Both tickets are greater than 20 and less than 30.

26. The first ticket is greater than 10, and the second ticket is less than 40 or odd.

27. The first ticket is greater than 12 or prime, and the second ticket is a multiple of 6 or a multiple of 4.

More About. . .

Safety

In the U.S., 60% of carbon monoxide emissions come from transportation sources. The largest contributor is highway motor vehicles. In urban areas, motor vehicles can contribute more than 90%.

Source: U.S. Environmental Protection Agency

SAFETY **For Exercises 28–31, use the following information.**
A carbon monoxide detector system uses two sensors, *A* and *B*. If carbon monoxide is present, there is a 96% chance that sensor *A* will detect it, a 92% chance that sensor *B* will detect it, and a 90% chance that both sensors will detect it.

28. Draw a Venn diagram that illustrates this situation.

29. If carbon monoxide is present, what is the probability that it will be detected?

30. What is the probability that carbon monoxide would go undetected?

31. Do sensors *A* and *B* operate independently of each other? Explain.

BIOLOGY **For Exercises 32–34, use the table and following information.**
Each person carries two types of genes for eye color. The gene for brown eyes (B) is dominant over the gene for blue eyes (b). That is, if a person has one gene for brown eyes and the other for blue, that person will have brown eyes. The Punnett square at the right shows the genes for two parents.

	B	b
B	BB	Bb
b	Bb	bb

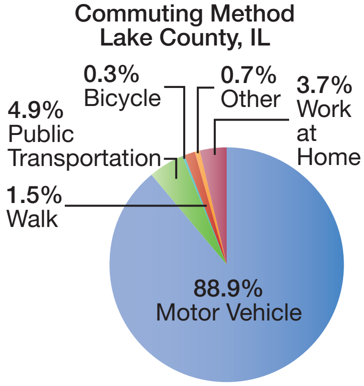
32. What is the probability that any child will have blue eyes?

33. What is the probability that the couple's two children both have brown eyes?

34. Find the probability that the first or the second child has blue eyes.

35. **RESEARCH** Use the Internet or other reference to investigate various blood types. Use this information to determine the probability of a child having blood type O if the father has blood type A(Ai) and the mother has blood type B(Bi).

TRANSPORTATION **For Exercises 36 and 37, use the graph and the following information.**
The U.S. Census Bureau conducted an American Community Survey in Lake County, Illinois. The circle graph at the right shows the survey results of how people commute to work.



Source: U.S. Census Bureau

36. If a person from Lake County was chosen at random, what is the probability that he or she uses public transportation or walks to work?

37. If offices are being built in Lake County to accommodate 400 employees, what is the minimum number of parking spaces an architect should plan for the parking lot?

ECONOMICS **For Exercises 38–40, use the table below that compares the total number of hourly workers who earned the minimum wage of $5.15 with those making less than minimum wage.**

Number of Hourly Workers (thousands)			
Age (years)	Total	At $5.15	Below $5.15
16–24	15,793	1145	2080
25+	55,287	970	2043

Source: U.S. Bureau of Labor Statistics

38. If an hourly worker was chosen at random, what is the probability that he or she earned minimum wage? less than minimum wage?

39. What is the probability that a randomly-chosen hourly worker earned less than or equal to minimum wage?

40. If you randomly chose an hourly worker from each age group, which would you expect to have earned no more than minimum wage? Explain.

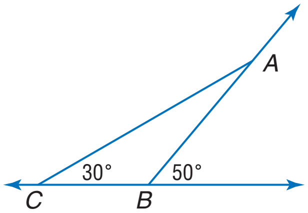
More About. . .

Economics

The first federal minimum wage was set in 1938 at $0.25 per hour. That was the equivalent of $3.05 in 2000.

Source: U.S. Department of Labor

GEOMETRY **For Exercises 41–43, use the figure and the following information.**
Two of the six non-straight angles in the figure are chosen at random.

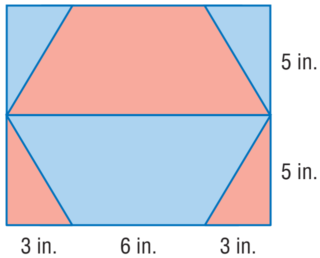


41. What is the probability of choosing an angle inside $\triangle ABC$ or an obtuse angle?

42. What is the probability of selecting a straight angle or a right angle inside $\triangle ABC$?

43. Find the probability of picking a 20° angle or a 130° angle.

A dart is thrown at a dartboard like the one at the right. If the dart can land anywhere on the board, find the probability that it lands in each of the following.



44. a triangle or a red region

45. a trapezoid or a blue region

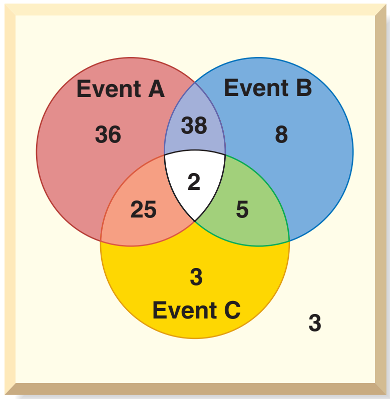
46. a blue triangle or a red triangle

47. a square or a hexagon

CRITICAL THINKING **For Exercises 48–51, use the following information.**
A sample of high school students were asked if they:

A) drive a car to school,
B) are involved in after-school activities, or
C) have a part-time job.

The results of the survey are shown in the Venn diagram.



48. How many students were surveyed?

49. How many students said that they drive a car to school?

50. If a high school student is chosen at random, what is the probability that he or she does all three?

51. What is the probability that a randomly-chosen student drives a car to school or is involved in after-school activities or has a part-time job?

52. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are probabilities used by meteorologists?

Include the following in your answer:

- a few sentences about how compound probabilities can be used to predict the weather, and
- assuming that the events are independent, the probability that it will rain either Saturday or Sunday if there is a 30% chance of rain on Saturday and a 50% chance of rain Sunday.

Standardized Test Practice

53. A bag contains 8 red marbles, 5 blue marbles, 4 green marbles, and 7 yellow marbles. Five marbles are randomly drawn from the bag and not replaced. What is the probability that the first three marbles drawn are red?

Ⓐ $\frac{1}{27}$ Ⓑ $\frac{28}{1771}$ Ⓒ $\frac{7}{253}$ Ⓓ $\frac{7}{288}$

54. Yolanda usually makes 80% of her free throws. What is the probability that she will make at least one free throw in her next three attempts?

Ⓐ 99.2% Ⓑ 51.2% Ⓒ 38.4% Ⓓ 9.6%

Maintain Your Skills

Mixed Review

CIVICS **For Exercises 55 and 56, use the following information.**
The Stratford town council wants to form a 3-person parks committee. Five people have applied to be on the committee. *(Lesson 14-2)*

55. How many committees are possible?

56. What is the probability of any one person being selected if each has an equal chance?

57. **BUSINESS** A real estate developer built a strip mall with seven different-sized stores. Ten small businesses have shown interest in renting space in the mall. The developer must decide which business would be best suited for each store. How many different arrangements are possible? *(Lesson 14-1)*

Find each sum or difference. *(Lesson 13-2)*

58. $\begin{bmatrix} 3 & -6 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 4 \\ 1 & 5 \end{bmatrix}$

59. $\begin{bmatrix} -4 & -5 \\ 8 & 8 \end{bmatrix} - \begin{bmatrix} -9 & -7 \\ 4 & 9 \end{bmatrix}$

60. Find the quotient of $\frac{2m^2 + 7m - 15}{m + 5}$ and $\frac{9m^2 - 4}{3m + 2}$. *(Lesson 12-4)*

Simplify. *(Lesson 11-1)*

61. $\sqrt{45}$

62. $\sqrt{128}$

63. $\sqrt{40b^4}$

64. $\sqrt{120a^3b}$

65. $3\sqrt{7} \cdot 6\sqrt{2}$

66. $\sqrt{3}(\sqrt{3} + \sqrt{6})$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Express each fraction as a decimal. Round to the nearest thousandth.** *(To review **expressing fractions as decimals**, see pages 804 and 805.)*

67. $\frac{9}{24}$

68. $\frac{2}{15}$

69. $\frac{63}{128}$

70. $\frac{5}{52}$

71. $\frac{8}{36}$

72. $\frac{11}{38}$

73. $\frac{81}{2470}$

74. $\frac{18}{1235}$

75. $\frac{128}{3570}$

14-4 Probability Distributions

What You'll Learn

- Use random variables to compute probability.
- Use probability distributions to solve real-world problems.

Vocabulary

- random variable
- probability distribution
- probability histogram

How can a pet store owner use a probability distribution?

The owner of a pet store asked customers how many pets they owned. The results of this survey are shown in the table.

Number of Pets	Number of Customers
0	3
1	37
2	33
3	18
4	9

RANDOM VARIABLES AND PROBABILITY A **random variable** is a variable whose value is the numerical outcome of a random event. In the situation above, we can let the random variable X represent the number of pets owned. Thus, X can equal 0, 1, 2, 3, or 4.

Example 1 Random Variable

Refer to the application above.

a. Find the probability that a randomly-chosen customer has 2 pets.

There is only one outcome in which there are 2 pets owned, and there are 100 survey results.

$$P(X = 2) = \frac{\text{2 pets owned}}{\text{customers surveyed}}$$

$$= \frac{33}{100}$$

The probability that a randomly-chosen customer has 2 pets is $\frac{33}{100}$ or 33%.

b. Find the probability that a randomly-chosen customer has at least 3 pets.

There are 18 + 9 or 27 outcomes in which a customer owns at least 3 pets.

$$P(X \geq 3) = \frac{27}{100}$$

The probability that a randomly-chosen customer owns at least 3 pets is $\frac{27}{100}$ or 27%.

Study Tip

Reading Math

The notation $P(X = 2)$ means the same as P(2 pets), the probability of a customer having 2 pets.

PROBABILITY DISTRIBUTIONS The probability of every possible value of the random variable X is called a **probability distribution**.

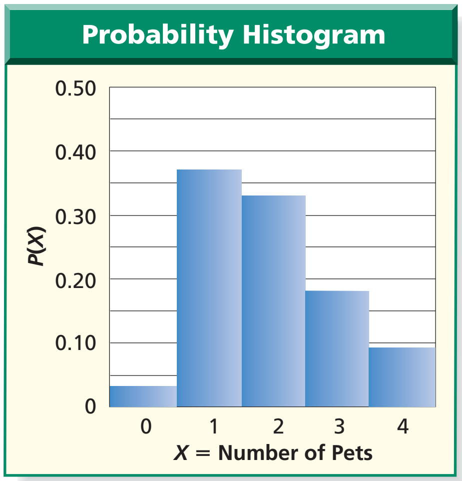
Key Concept — Properties of Probability Distributions

1. The probability of each value of X is greater than or equal to 0 and less than or equal to 1.
2. The probabilities of all of the values of X add up to 1.

www.algebra1.com/extra_examples

The probability distribution for a random variable can be given in a table or in a **probability histogram**. The probability distribution and a probability histogram for the application at the beginning of the lesson are shown below.

Probability Distribution Table	
X = Number of Pets	$P(X)$
0	0.03
1	0.37
2	0.33
3	0.18
4	0.09



Example 2 Probability Distribution

CARS **The table shows the probability distribution of the number of vehicles per household for the Columbus, Ohio, area.**

Vehicles per Household Columbus, OH	
X = Number of Vehicles	Probability
0	0.10
1	0.42
2	0.36
3+	0.12

Source: U.S. Census Bureau

a. Show that the distribution is valid.

Check to see that each property holds.

1. For each value of X, the probability is greater than or equal to 0 and less than or equal to 1.
2. $0.10 + 0.42 + 0.36 + 0.12 = 1$, so the probabilities add up to 1.

b. What is the probability that a household has fewer than 2 vehicles?

Recall that the probability of a compound event is the sum of the probabilities of each individual event.

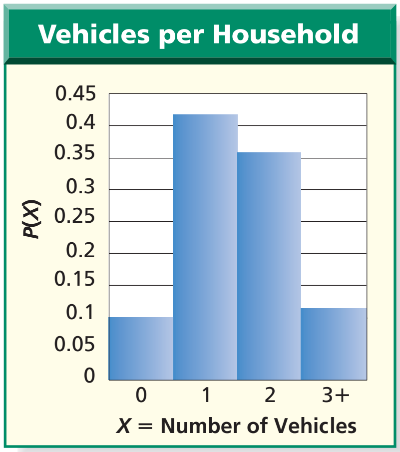
The probability of a household having fewer than 2 vehicles is the sum of the probability of 0 vehicles and the probability of 1 vehicle.

$P(X < 2) = P(X = 0) + P(X = 1)$ Sum of individual probabilities

$= 0.10 + 0.42$ or 0.52 $P(X = 0) = 0.10$, $P(X = 1) = 0.42$

c. Make a probability histogram of the data.

Draw and label the vertical and horizontal axes. Remember to use equal intervals on each axis. Include a title.



More About...

Cars

In 1900, there were 8000 registered cars in the United States. By 2000, there were over 133 million registered cars. This is an increase of more than 1,662,400%.

Source: *The World Almanac*

Check for Understanding

Concept Check

1. **List** the conditions that must be satisfied to have a valid probability distribution.
2. **Explain** why the probability of tossing a coin three times and getting 1 head and 2 tails is the same as the probability of getting 1 tail and 2 heads.
3. **OPEN ENDED** Describe a situation that could be displayed in a probability histogram.

Guided Practice

For Exercises 4–6, use the table that shows the possible sums when rolling two dice and the number of ways each sum can be found.

Sum of Two Dice	2	3	4	5	6	7	8	9	10	11	12
Ways to Achieve Sum	1	2	3	4	5	6	5	4	3	2	1

4. Draw a table to show the sample space of all possible outcomes.
5. Find the probabilities for $X = 4$, $X = 5$, and $X = 6$.
6. What is the probability that the sum of two dice is greater than 6 on each of three separate rolls?

Application

GRADES **For Exercises 7–9, use the table that shows a class's grade distribution, where A = 4.0, B = 3.0, C = 2.0, D = 1.0, and F = 0.**

X = Grade	0	1.0	2.0	3.0	4.0
Probability	0.05	0.10	0.40	0.40	0.05

7. Show that the probability distribution is valid.
8. What is the probability that a student passes the course?
9. What is the probability that a student chosen at random from the class receives a grade of B or better?

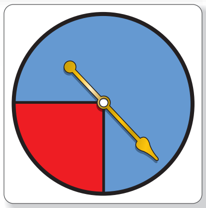
Practice and Apply

Homework Help

For Exercises	See Examples
10, 11, 14, 18	1
12, 13, 15–17, 19–22	2

Extra Practice
See page 852.

For Exercises 10–13, the spinner shown is spun three times.



10. Write the sample space with all possible outcomes.
11. Find the probability distribution X, where X represents the number of times the spinner lands on blue for $X = 0$, $X = 1$, $X = 2$, and $X = 3$.
12. Make a probability histogram.
13. Do all possible outcomes have an equal chance of occurring? Explain.

SALES **For Exercises 14–17, use the following information.**
A music store manager takes an inventory of the top 10 CDs sold each week. After several weeks, the manager has enough information to estimate sales and make a probability distribution table.

Number of Top 10 CDs Sold Each Week	0–100	101–200	201–300	301–400	401–500
Probability	0.10	0.15	0.40	0.25	0.10

14. Define a random variable and list its values.
15. Show that this is a valid probability distribution.
16. In a given week, what is the probability that no more than 400 CDs sell?
17. In a given week, what is the probability that more than 200 CDs sell?

www.algebra1.com/self_check_quiz

EDUCATION **For Exercises 18–20, use the table that shows the education level of persons aged 25 and older in the United States.**

X = Level of Education	Probability
Some High School	0.167
High School Graduate	0.333
Some College	0.173
Associate's Degree	0.075
Bachelor's Degree	0.170
Advanced Degree	0.082

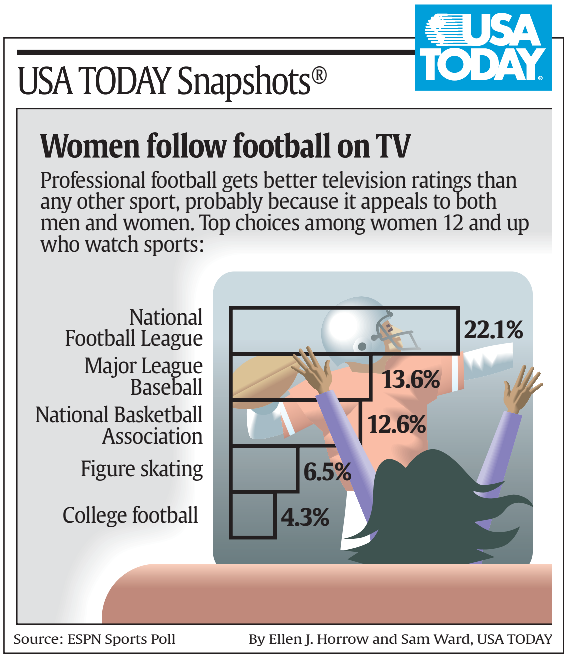
Source: U.S. Census Bureau

18. If a person was randomly selected, what is the probability that he or she completed at most some college?

19. Make a probability histogram of the data.

20. Explain how you can find the probability that a randomly selected person has earned at least a bachelor's degree.

SPORTS **For Exercises 21 and 22, use the graph that shows the sports most watched by women on TV.**



21. Determine whether this is a valid probability distribution. Justify your answer.

22. Based on the graph, in a group of 35 women how many would you expect to say they watch figure skating?

23. **CRITICAL THINKING** Suppose a married couple has children until they have a girl. Let the random variable X represent the number of children in their family.

a. Calculate the probabilities for $X = 1, 2, 3,$ and 4.

b. Find the probability that the couple will have more than 4 children.

24. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can a pet store owner use a probability distribution?

Include the following in your answer:

- a sentence or two describing how to create a probability distribution, and
- an explanation of how the store owner could use a probability distribution to establish a frequent buyer program.

Standardized Test Practice

25. The table shows the probability distribution for the number of heads when four coins are tossed. What is the probability that there are no more than two heads showing on a random toss?

X = Number of Heads	0	1	2	3	4
Probability $P(X)$	0.0625	0.25	0.375	0.25	0.0625

(A) 0.6875 (B) 0.375 (C) 0.875 (D) 0.3125

26. On a random roll of two dice, what is the probability that the sum of the numbers showing is less than 5?

(A) 0.08 (B) 0.17 (C) 0.11 (D) 0.28

Maintain Your Skills

Mixed Review

A card is drawn from a standard deck of 52 cards. Find each probability. *(Lesson 14-3)*

27. P(ace or 10) **28.** P(3 or diamond) **29.** P(odd number or spade)

Evaluate. *(Lesson 14-2)*

30. ${}_{10}C_7$ **31.** ${}_{12}C_5$ **32.** $({}_6P_3)({}_5P_3)$

Let $A = \begin{bmatrix} 1 & 4 \\ 5 & 7 \end{bmatrix}$ **and** $B = \begin{bmatrix} -3 & 0 \\ -2 & 5 \end{bmatrix}$. *(Lesson 13-2)*

33. Find $A + B$. **34.** Find $B - A$.

Write an inverse variation equation that relates x and y. Assume that y varies inversely as x. Then solve. *(Lesson 12-1)*

35. If $y = -2.4$ when $x = -0.6$, find y when $x = 1.8$.

36. If $y = 4$ when $x = -1$, find x when $y = -3$.

Simplify each expression. *(Lesson 11-2)*

37. $3\sqrt{8} + 7\sqrt{2}$ **38.** $2\sqrt{3} + \sqrt{12}$ **39.** $3\sqrt{7} - 2\sqrt{28}$

SAVINGS **For Exercises 40–42, use the following information.**
Selena is investing her \$900 tax refund in a certificate of deposit that matures in 4 years. The interest rate is 8.25% compounded quarterly. *(Lesson 10-6)*

40. Determine the balance in the account after 4 years.

41. Her friend Monique invests the same amount of money at the same interest rate, but her bank compounds interest monthly. Determine how much she will have after 4 years.

42. Which type of compounding appears more profitable? Explain.

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Write each fraction as a percent rounded to the nearest whole number.** *(To review* ***writing fractions as percents****, see pages 804 and 805.)*

43. $\frac{16}{80}$ **44.** $\frac{20}{52}$ **45.** $\frac{30}{114}$

46. $\frac{57}{120}$ **47.** $\frac{72}{340}$ **48.** $\frac{54}{162}$

Practice Quiz 2 — Lessons 14-3 and 14-4

For Exercises 1–3, use the probability distribution for the number of people in a household. *(Lesson 14-4)*

1. Show that the probability distribution is valid.

2. If a household is chosen at random, what is the probability that 4 or more people live in it?

3. Make a histogram of the data.

American Households

X = Number of People	Probability
1	0.25
2	0.32
3	0.18
4	0.15
5	0.07
6	0.02
7+	0.01

Source: U.S. Census Bureau

A ten-sided die, numbered 1 through 10, is rolled. Find each probability.

4. P(odd or greater than 4)

5. P(less than 3 or greater than 7)

14-5 Probability Simulations

What You'll Learn

- Use theoretical and experimental probability to represent and solve problems involving uncertainty.
- Perform probability simulations to model real-world situations involving uncertainty.

Vocabulary

- theoretical probability
- experimental probability
- relative frequency
- empirical study
- simulation

How can probability simulations be used in health care?

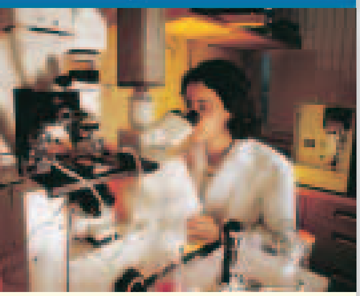
A pharmaceutical company is developing a new medication to treat a certain heart condition. Based on similar drugs, researchers at the company expect the new drug to work successfully in 70% of patients.

To test the drug's effectiveness, the company performs three clinical studies. Each study involves 100 volunteers who use the drug for six months. The results of the studies are shown in the table.

Study Of New Medication

Result	Study 1	Study 2	Study 3
Expected Success Rate	70%	70%	70%
Condition Improved	61%	74%	67%
No Improvement	39%	25%	33%
Condition Worsened	0%	1%	0%

Career Choices



Medical Researcher

Many medical researchers conduct research to advance knowledge of living organisms, including viruses and bacteria.

Online Research For information about a career as a medical researcher, visit: www.algebra1.com/careers

THEORETICAL AND EXPERIMENTAL PROBABILITY The probability we have used to describe events in previous lessons is theoretical probability. **Theoretical probabilities** are determined mathematically and describe what should happen. In the situation above, the expected success rate of 70% is a theoretical probability.

A second type of probability we can use is **experimental probability**, which is determined using data from tests or experiments. Experimental probability is the ratio of the number of times an outcome occurred to the total number of events or trials. This ratio is also known as the **relative frequency**.

$$\text{experimental probability} = \frac{\text{frequency of an outcome}}{\text{total number of trials}}$$

Example 1 Experimental Probability

MEDICAL RESEARCH **Refer to the application at the beginning of the lesson. What is the experimental probability that the drug was successful for a patient in Study 1?**

In Study 1, the drug worked successfully in 61 of the 100 patients.

$$\text{experimental probability} = \frac{61}{100} \quad \begin{matrix} \leftarrow \text{frequency of successes} \\ \leftarrow \text{total number of patients} \end{matrix}$$

The experimental probability of Study 1 is $\frac{61}{100}$ or 61%.

It is often useful to perform an experiment repeatedly, collect and combine the data, and analyze the results. This is known as an **empirical study**.

Example 2 Empirical Study

Refer to the application at the beginning of the lesson. What is the experimental probability that the drug was successful for all three studies?

The number of successful outcomes of the three studies was 61 + 74 + 67 or 202 out of the 300 total patients.

experimental probability $= \frac{202}{300}$ or $\frac{101}{150}$

The experimental probability of the three studies was $\frac{101}{150}$ or about 67%.

PERFORMING SIMULATIONS A method that is often used to find experimental probability is a simulation. A **simulation** allows you to use objects to act out an event that would be difficult or impractical to perform.

Algebra Activity

Simulations

Collect the Data

- Roll a die 20 times. Record the value on the die after each roll.
- Determine the experimental probability distribution for *X*, the value on the die.
- Combine your results with the rest of the class to find the experimental probability distribution for *X* given the new number of trials.
  (20 · the number of students in your class)

Analyze the Data

1. Find the theoretical probability of rolling a 2.
2. Find the theoretical probability of rolling a 1 or a 6.
3. Find the theoretical probability of rolling a value less than 4.
4. Compare the experimental and theoretical probabilities. Which pair of probabilities was closer to each other: your individual probabilities or your class's probabilities?
5. Suppose each person rolls the die 50 times. Explain how this would affect the experimental probabilities for the class.

Make a Conjecture

6. What can you conclude about the relationship between the number of experiments in a simulation and the experimental probability?

Study Tip

Reading Math
The *Law of Large Numbers* states that as the number of trials increases, the experimental probability gets closer to the theoretical probability.

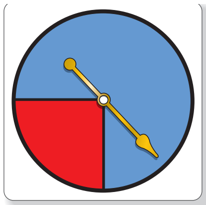
You can conduct simulations of the outcomes for many problems by using one or more objects such as dice, coins, marbles, or spinners. The objects you choose should have the same number of outcomes as the number of possible outcomes of the problem, and all outcomes should be equally likely.

Example 3 Simulation

In one season, Malcolm made 75% of the field goals he attempted.

a. What could be used to simulate his kicking a field goal? Explain.

You could use a spinner like the one at the right, where 75% of the spinner represents making a field goal.



b. Describe a way to simulate his next 8 attempts.

Spin the spinner once to simulate a kick. Record the result, then repeat this 7 more times.

Example 4 Theoretical and Experimental Probability

DOGS **Ali raises purebred dogs. One of her dogs is expecting a litter of four puppies, and Ali would like to figure out the most likely mix of male and female puppies. Assume that $P(\text{male}) = P(\text{female}) = \frac{1}{2}$.**

a. What objects can be used to model the possible outcomes of the puppies?

Each puppy can be male or female, so there are $2 \cdot 2 \cdot 2 \cdot 2$ or 16 possible outcomes for the litter. Use a simulation that also has 2 outcomes for each of 4 events. One possible simulation would be to toss four coins, one for each puppy, with heads representing female and tails representing male.

Study Tip

Alternative Simulation

You could also create a spinner with two even parts and spin it 4 times to simulate the outcomes of the puppies.

b. Find the theoretical probability that there will be two female and two male puppies.

There are 16 possible outcomes, and the number of combinations that have two female and two male puppies is ${}_4C_2$ or 6. So the theoretical probability is $\frac{6}{16}$ or $\frac{3}{8}$.

c. The results of a simulation Ali performed are shown in the table below. What is the experimental probability that there will be three male puppies?

Outcomes	Frequency
4 female, 0 male	3
3 female, 1 male	13
2 female, 2 male	18
1 female, 3 male	12
0 female, 4 male	4

Ali performed 50 trials and 12 of those resulted in three males. So, the experimental probability is $\frac{12}{50}$ or 24%.

d. How does the experimental probability compare to the theoretical probability of a litter with three males?

Theoretical probability

$$P(3 \text{ males}) = \frac{{}_4C_3}{16} \quad \begin{array}{l}\leftarrow \text{combinations with 3 male puppies} \\ \leftarrow \text{possible outcomes}\end{array}$$

$$= \frac{4}{16} \text{ or } 25\% \quad \text{Simplify.}$$

The experimental probability, 24%, is very close to the theoretical probability.

Check for Understanding

Concept Check

1. **Explain** why it is useful to carry out an empirical study when calculating experimental probabilities.

2. **Analyze** the relationship between the theoretical and experimental probability of an event as the number of trials in a simulation increases.

3. **OPEN ENDED** Describe a situation that could be represented by a simulation. What objects would you use for this experiment?

4. **Tell** whether the theoretical probability and the experimental probability of an event are *sometimes*, *always*, or *never* the same.

Guided Practice

5. So far this season, Rita has made 60% of her free throws. Describe a simulation that could be used to predict the outcome of her next 25 free throws.

For Exercises 6–8, roll a die 25 times and record your results.

6. Based on your results, what is the probability of rolling a 3?

7. Based on your results, what is the probability of rolling a 5 or an odd number?

8. Compare your results to the theoretical probabilities.

Application

ASTRONOMY **For Exercises 9–12, use the following information.**
Enrique is writing a report about meteorites and wants to determine the probability that a meteor reaching Earth's surface hits land. He knows that 70% of Earth's surface is covered by water. He places 7 blue marbles and 3 brown marbles in a bag to represent hitting water $\left(\frac{7}{10}\right)$ and hitting land $\left(\frac{3}{10}\right)$. He draws a marble from the bag, records the color, and then replaces the marble. The table shows the results of his experiment.

Blue	Brown
56	19

9. Did Enrique choose an appropriate simulation for his research? Explain.

10. What is the theoretical probability that a meteorite reaching Earth's surface hits land?

11. Based on his results, what is the probability that a meteorite hits land?

12. Using the experimental probability, how many of the next 500 meteorites that strike Earth would you expect to hit land?

Practice and Apply

Homework Help

For Exercises	See Examples
13–16	3
17–21, 25–31	4
22–24	1, 2

Extra Practice
See page 852.

13. What could you use to simulate the outcome of guessing on 15 true-false questions?

14. There are 12 cans of cola, 8 cans of diet cola, and 4 cans of root beer in a cooler. What could be used for a simulation determining the probability of randomly picking any one type of soft drink?

For Exercises 15 and 16, use the following information.
Central City Mall is randomly giving each shopper one of 12 different gifts during the holidays.

15. What could be used to perform a simulation of this situation? Explain your choice.

16. How could you use this simulation to model the next 100 gifts handed out?

For Exercises 17 and 18, toss 3 coins, one at a time, 25 times and record your results.

17. Based on your results, what is the probability that any two coins will show heads?

18. Based on your results, what is the probability that the first and third coins show tails?

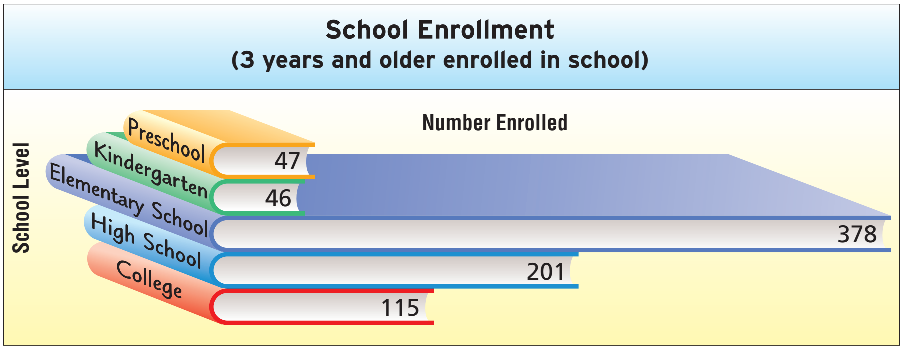
For Exercises 19–21, roll two dice 50 times and record the sums.

19. Based on your results, what is the probability that the sum is 8?

20. Based on your results, what is the probability that the sum is 7, or the sum is greater than 5?

21. If you roll the dice 25 more times, which sum would you expect to see about 10% of the time?

CITY PLANNING **For Exercises 22–24, use the following information.**
The Lewiston City Council sent surveys to randomly selected households to determine current and future enrollment for the local school district. The results of the survey are shown in the table.



22. Find the experimental probability distribution for the number of people enrolled at each level.

23. Based on the survey, what is the probability that a student chosen at random is in elementary school or high school?

24. Suppose the school district is expecting school enrollment to increase by 1800 over the next 5 years due to new buildings in the area. Of the new enrollment, how many will most likely be in kindergarten?

RESTAURANTS **For Exercises 25–27, use the following information.**
A family restaurant gives children a free toy with each children's meal. There are eight different toys that are randomly given. There is an equally likely chance of getting each toy each time.

25. What objects could be used to perform a simulation of this situation?

26. Conduct a simulation until you have one of each toy. Record your results.

27. Based on your results, how many meals must be purchased so that you get all 8 toys?

More About. . .

Animals

Labrador retrievers are the most popular breed of dog in the United States.

Source: American Kennel Club

ANIMALS **For Exercises 28–31, use the following information.**
Refer to Example 4 on page 784. Suppose Ali's dog is expecting a litter of 5 puppies.

28. List the possible outcomes of the genders of the puppies.

29. Perform a simulation and list your results in a table.

30. Based on your results, what is the probability that there will be 3 females and two males in the litter?

31. What is the experimental probability of the litter having at least three male puppies?

32. CRITICAL THINKING The captain of a football team believes that the coin the referee uses for the opening coin toss gives an advantage to one team. The referee has players toss the coin 50 times each and record their results. Based on the results, do you think the coin is fair? Explain your reasoning.

Player	1	2	3	4	5	6
Heads	38	31	29	27	26	30
Tails	12	19	21	23	24	20

33. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How can probability simulations be used in health care?

Include the following in your answer:

- a few sentences explaining experimental probability, and
- an explanation of why an experimental probability of 75% found in 400 trials is more reliable than an experimental probability of 75% found in 50 trials.

Standardized Test Practice

34. Ramón tossed two coins and rolled a die. What is the probability that he tossed two tails and rolled a 3?

(A) $\frac{1}{4}$ (B) $\frac{1}{6}$ (C) $\frac{5}{12}$ (D) $\frac{1}{24}$

35. If a coin is tossed three times, what is the probability that the results will be heads exactly one time?

(A) $\frac{2}{3}$ (B) $\frac{3}{8}$ (C) $\frac{1}{5}$ (D) $\frac{1}{8}$

Graphing Calculator

SIMULATION **For Exercises 36–38, use the following information.**

When you are performing an experiment that involves a large number of trials that cannot be simulated using an object like a coin or a spinner, you can use the random number generator function on a graphing calculator. The TI-83 Plus program at the right will perform T trials by generating random numbers between 1 and P, the number of possible outcomes.

```
PROGRAM: SIMULATE
:Disp "ENTER THE NUMBER"
:Disp "OF POSSIBLE"
:Disp "OUTCOMES"
:Input P
:Disp "ENTER THE NUMBER"
:Disp "OF TRIALS"
:Input T
:For(N, 1, T)
:randInt(1, P)→S
:Disp S
:Pause
:End
```

36. Run the program to simulate 50 trials of an event that has 15 outcomes. Record your results.

37. What is the experimental probability of displaying the number 10?

38. Repeat the experiment several times. Find the experimental probability of displaying the number 10. Has the probability changed from the probability found in Exercise 37? Explain why or why not.

ENTERTAINMENT **For Exercises 39–41, use the following information and the graphing calculator program above.**

A CD changer contains 5 CDs with 14 songs each. When "Random" is selected, each CD is equally likely to be chosen as each song.

39. Use the program SIMULATE to perform a simulation of randomly playing 40 songs from the 5 CDs. (*Hint*: Number the songs sequentially from 1, CD 1 track 1, to 70, CD 5 track 14.)

40. Do the experimental probabilities for your simulation support the statement that each CD is equally likely to be chosen? Explain.

41. Based on your results, what is the probability that the first three songs played are on the third disc?

Maintain Your Skills

Mixed Review

For Exercises 42–44, use the probability distribution for the random variable X, the number of computers per household. *(Lesson 14-4)*

Computers per Household	
X = Number of Computers	$P(X)$
0	0.579
1	0.276
2	0.107
3+	0.038

Source: U.S. Dept. of Commerce

42. Show that the probability distribution is valid.

43. If a household is chosen at random, what is the probability that it has at least 2 computers?

44. Determine the probability of randomly selecting a household with no more than one computer.

For Exercises 45–47, use the following information.
A jar contains 18 nickels, 25 dimes, and 12 quarters. Three coins are randomly selected. Find each probability. *(Lesson 14-3)*

45. picking three dimes, replacing each after it is drawn

46. a nickel, then a quarter, then a dime without replacing the coins

47. 2 dimes and a quarter, without replacing the coins, if order does not matter

Solve each equation. *(Lesson 12-9)*

48. $\frac{2a-3}{a-3} - 2 = \frac{12}{a+3}$ **49.** $\frac{r^2}{r-7} + \frac{50}{7-r} = 14$ **50.** $\frac{x-2}{x} - \frac{x-3}{x-6} = \frac{1}{x}$

51. $\frac{2x-3}{7} - \frac{x}{2} = \frac{x+3}{14}$ **52.** $\frac{5n}{n+1} + \frac{1}{n} = 5$ **53.** $\frac{a+2}{a-2} - \frac{2}{a+2} = \frac{-7}{3}$

54. CONSTRUCTION To paint his house, Lonnie needs to purchase an extension ladder that reaches at least 24 feet off the ground. Ladder manufacturers recommend the angle formed by the ladder and the ground be no more than 75°. What is the shortest ladder he could buy to reach 24 feet safely? *(Lesson 11-7)*

Determine whether the following side measures would form a right triangle. *(Lesson 11-4)*

55. 5, 7, 9 **56.** $3\sqrt{34}$, 9, 15 **57.** 36, 86.4, 93.6

Solve each equation. Check your solutions. *(Lesson 9-6)*

58. $(x-6)^2 = 4$ **59.** $x^2 + 121 = 22x$ **60.** $4x^2 + 12x + 9 = 0$

61. $25x^2 + 20x = -4$ **62.** $49x^2 - 84x + 36 = 0$ **63.** $180x - 100 = 81x^2$

WebQuest Internet Project

America Counts!

It is time to complete your project. Use the information and data you have gathered about populations to prepare a brochure or Web page. Be sure to identify the state you have chosen for this project. Include graphs, tables, and/or calculations in the presentation.

www.algebra1.com/webquest

Chapter 14

Study Guide and Review

Vocabulary and Concept Check

combination (p. 762)
complements (p. 771)
compound event (p. 769)
dependent events (p. 770)
edge (p. 759)
empirical study (p. 783)
event (p. 754)
experimental probability (p. 782)
factorial (p. 755)
finite graph (p. 759)
Fundamental Counting Principle (p. 755)
inclusive (p. 771)
independent events (p. 769)
mutually exclusive (p. 771)
network (p. 759)
node (p. 759)
permutation (p. 760)
probability distribution (p. 777)
probability histogram (p. 778)
random variable (p. 777)
relative frequency (p. 782)
sample space (p. 754)
simple event (p. 769)
simulation (p. 783)
theoretical probability (p. 782)
traceable (p. 759)
tree diagram (p. 754)

Choose the word or term that best completes each sentence.

1. The arrangement or listing in which order is important is called a (*combination*, *permutation*).
2. The notation 10! refers to a (*prime factor*, *factorial*).
3. Rolling one die and then another die are (*dependent*, *independent*) events.
4. The sum of probabilities of complements equals (*0*, *1*).
5. Randomly drawing a coin from a bag and then drawing another coin are dependent events if the coins (*are*, *are not*) replaced.
6. Events that cannot occur at the same time are (*inclusive*, *mutually exclusive*).
7. The sum of the probabilities in a probability distribution equals (*0*, *1*).
8. (*Experimental*, *Theoretical*) probabilities are precise and predictable.

Lesson-by-Lesson Review

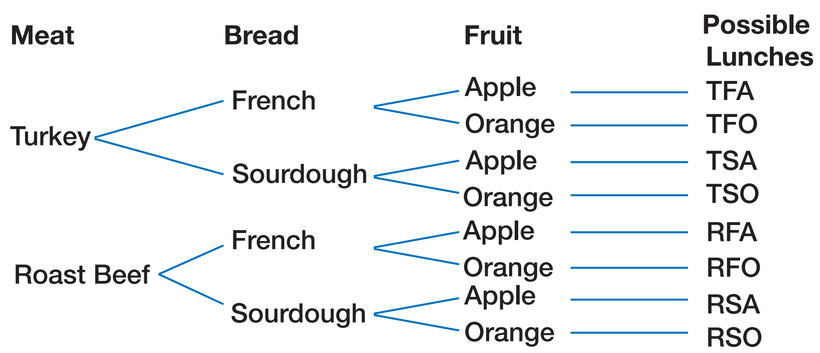
14-1 Counting Outcomes

See pages 754–758.

Concept Summary

- Use a tree diagram to make a list of possible outcomes.
- If an event M can occur m ways and is followed by an event N that can occur n ways, the event M followed by event N can occur $m \cdot n$ ways.

Example When Jerri packs her lunch, she can choose to make a turkey or roast beef sandwich on French or sourdough bread. She also can pack an apple or an orange. Draw a tree diagram to show the number of different ways Jerri can select these items.



There are 8 different ways for Jerri to select these items.

Exercises **Determine the number of outcomes for each event.**
See Examples 1–3 on pages 754 and 755.

9. Samantha wants to watch 3 videos one rainy afternoon. She has a choice of 3 comedies, 4 dramas, and 3 musicals.

10. Marquis buys 4 books, one from each category. He can choose from 12 mystery, 8 science fiction, 10 classics, and 5 biographies.

11. The Jackson Jackals and the Westfield Tigers are going to play a best three-out-of-five games baseball tournament.

14-2 Permutations and Combinations

See pages 760–767.

Concept Summary

- In a permutation, the order of objects is important. $_nP_r = \frac{n!}{(n-r)!}$
- In a combination, the order of objects is not important. $_nC_r = \frac{n!}{(n-r)!r!}$

Examples

1 **Find $_{12}C_8$.**

$$\begin{aligned} {}_{12}C_8 &= \frac{12!}{(12-8)!8!} \\ &= \frac{12!}{4!8!} \\ &= \frac{12 \cdot 11 \cdot 10 \cdot 9}{4!} \\ &= 495 \end{aligned}$$

2 **Find $_9P_4$.**

$$\begin{aligned} {}_9P_4 &= \frac{9!}{(9-4)!} \\ &= \frac{9!}{5!} \\ &= \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} \\ &= 3024 \end{aligned}$$

Exercises **Evaluate each expression.** *See Examples 1, 2, and 4 on pages 760–762.*

12. $_4P_2$ **13.** $_8C_3$ **14.** $_4C_4$

15. $(_7C_1)(_6C_3)$ **16.** $(_7P_3)(_7P_2)$ **17.** $(_3C_2)(_4P_1)$

14-3 Probability of Compound Events

See pages 769–776.

Concept Summary

- For independent events, use $P(A \text{ and } B) = P(A) \cdot P(B)$.
- For dependent events, use $P(A \text{ and } B) = P(A) \cdot P(B \text{ following } A)$.
- For mutually exclusive events, use $P(A \text{ or } B) = P(A) + P(B)$.
- For inclusive events, use $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

Example

A box contains 8 red chips, 6 blue chips, and 12 white chips. Three chips are randomly drawn from the box and not replaced. Find P(red, white, blue).

First chip: $P(\text{red}) = \frac{8}{26}$ ← number of red chips / ← total number of chips

Second chip: $P(\text{white}) = \frac{12}{25}$ ← number of white chips / ← number of chips remaining

Third chip: $P(\text{blue}) = \frac{6}{24}$ ← number of blue chips / ← number of chips remaining

$$P(\text{red, white, blue}) = \underbrace{P(\text{red})} \cdot \underbrace{P(\text{white})} \cdot \underbrace{P(\text{blue})}$$
$$= \frac{8}{26} \cdot \frac{12}{25} \cdot \frac{6}{24}$$
$$= \frac{576}{15{,}600} \text{ or } \frac{12}{325}$$

Exercises **A bag of colored paper clips contains 30 red clips, 22 blue clips, and 22 green clips. Find each probability if three clips are drawn randomly from the bag and are not replaced.** *See Example 2 on page 770.*

18. P(blue, red, green) **19.** P(red, red, blue) **20.** P(red, green, not blue)

One card is randomly drawn from a standard deck of 52 cards. Find each probability. *See Examples 3 and 4 on pages 771 and 772.*

21. P(diamond or club) **22.** P(heart or red) **23.** P(10 or spade)

14-4 Probability Distributions

See pages 777–781.

Concept Summary

Probability distributions have the following properties.

- For each value of X, $0 \leq P(X) \leq 1$.
- The sum of the probabilities of each value of X is 1.

Example **A local cable provider asked its subscribers how many televisions they had in their homes. The results of their survey are shown in the probability distribution.**

Televisions per Household	
X = Number of Televisions	Probability
1	0.18
2	0.36
3	0.34
4	0.08
5+	0.04

a. Show that the probability distribution is valid.

For each value of X, the probability is greater than or equal to 0 and less than or equal to 1.

$0.18 + 0.36 + 0.34 + 0.08 + 0.04 = 1$, so the probabilities add up to 1.

b. If a household is selected at random, what is the probability that it has fewer than 4 televisions?

$$P(X < 4) = P(X = 1) + P(X = 2) + P(X = 3)$$
$$= 0.18 + 0.36 + 0.34$$
$$= 0.88$$

Exercises **The table shows the probability distribution for the number of extracurricular activities in which students at Boardwalk High School participate.** *See Example 2 on page 778.*

Extracurricular Activities	
X = Number of Activities	Probability
0	0.04
1	0.12
2	0.37
3	0.30
4+	0.17

24. Show that the probability distribution is valid.

25. If a student is chosen at random, what is the probability that the student participates in 1 to 3 activities?

26. Make a probability histogram of the data.

• Extra Practice, see pages 851–852.
• Mixed Problem Solving, see page 866.

14-5 Probability Simulations

See pages 782–788.

Concept Summary

- Theoretical probability describes expected outcomes, while experimental probabilities describe tested outcomes.
- Simulations are used to perform experiments that would be difficult or impossible to perform in real life.

Example **A group of 3 coins are tossed.**

a. Find the theoretical probability that there will be 2 heads and 1 tail.

Each coin toss can be heads or tails, so there are $2 \cdot 2 \cdot 2$ or 8 possible outcomes. There are 3 possible combinations of 2 heads and one tail, HHT, HTH, or THH. So, the theoretical probability is $\frac{3}{8}$.

b. The results of a simulation in which three coins are tossed ten times are shown in the table. What is the experimental probability that there will be 1 head and 2 tails?

Of the 10 trials, 3 resulted in 1 head and 2 tails, so the experimental probability is $\frac{3}{10}$ or 30%.

Outcomes	Frequency
3 heads, 0 tails	1
2 heads, 1 tail	4
1 head, 2 tails	3
0 heads, 3 tails	2

c. Compare the theoretical probability of 2 heads and 1 tail and the experimental probability of 2 heads and 1 tail.

The theoretical probability is $\frac{3}{8}$ or 37.5%, while the experimental probability is $\frac{3}{10}$ or 30%. The probabilities are close.

Exercises **While studying flower colors in biology class, students are given the Punnett square at the right. The Punnett square shows that red parent plant flowers (Rr) produce red flowers (RR and Rr) and pink flowers (rr).**

	R	r
R	RR	Rr
r	Rr	rr

See Examples 1, 3, and 4 on pages 782 and 784.

27. If 5 flowers are produced, find the theoretical probability that there will be 4 red flowers and 1 pink flower.
28. Describe items that the students could use to simulate the colors of 5 flowers.
29. The results of a simulation of flowers are shown in the table. What is the experimental probability that there will be 3 red flowers and 2 pink flowers?

Outcomes	Frequency
5 red, 0 pink	15
4 red, 1 pink	30
3 red, 2 pink	23
2 red, 3 pink	7
1 red, 4 pink	4
0 red, 5 pink	1

Chapter 14 Practice Test

Vocabulary and Concepts

1. Seven students lining up to buy tickets for a school play is an example of a (*permutation*, *combination*).
2. Rolling a die and recording the result 25 times would be used to find (*theoretical*, *experimental*) probability.
3. A (*random variable*, *probability distribution*) is the numerical outcome of an event.

Skills and Applications

There are two roads from Ashville to Bakersville, four roads from Bakersville to Clifton, and two roads from Clifton to Derry.

4. Draw a tree diagram showing the possible routes from Ashville to Derry.
5. How many different routes are there from Ashville to Derry?

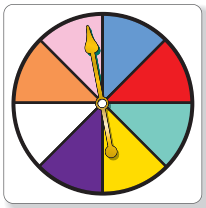
Determine whether each situation involves a *permutation* or a *combination*. Then determine the number of possible arrangements.

6. Six students in a class meet in a room that has nine chairs.
7. The top four finishers in a race with ten participants.
8. A class has 15 girls and 19 boys. A committee is formed with two girls and two boys, each with a separate responsibility.

A bag contains 4 red, 6 blue, 4 yellow, and 2 green marbles. Once a marble is selected, it is not replaced. Find each probability.

9. P(blue, green)
10. P(yellow, yellow)
11. P(red, blue, yellow)
12. P(blue, red, not green)

The spinner is spun, and a die is rolled. Find each probability.



13. P(yellow, 4)
14. P(red, even)
15. P(purple or white, not prime)
16. P(green, even or less than 5)

During a magic trick, a magician randomly selects a card from a standard deck of 52 cards. Without replacing it, the magician has a member of the audience randomly select a card. Find each probability.

17. P(club, heart)
18. P(black 7, diamond)
19. P(queen or red, jack of spades)
20. P(black 10, ace or heart)

The table shows the number of ways four coins can land heads up when they are tossed at the same time.

Four Coins Tossed

Number of Heads	Possible Outcomes
0	1
1	4
2	6
3	4
4	1

21. Set up a probability distribution of the possible outcomes.
22. Find the probability that there will be no heads.
23. Find the probability that there will be at least two heads.
24. Find the probability that there will be two tails.

25. **STANDARDIZED TEST PRACTICE** Two numbers a and b can be arranged in two different orders, a, b and b, a. In how many ways can three numbers be arranged?

(A) 3 (B) 4 (C) 5 (D) 6

www.algebra1.com/chapter_test

Chapter 14 Standardized Test Practice

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. If the average of a and b is 20, and the average of a, b, and c is 25, then what is the value of c? (Prerequisite Skill)

Ⓐ 10 Ⓑ 15
Ⓒ 25 Ⓓ 35

2. The volume of a cube is 27 cubic inches. Its total surface area, in square inches, is (Lesson 3-8)

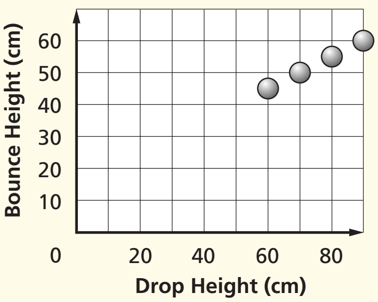
Ⓐ 9. Ⓑ $6\sqrt{3}$.
Ⓒ $18\sqrt{3}$. Ⓓ 54.

3. A truck travels 50 miles from Oakton to Newton in exactly 1 hour. When the truck is halfway between Oakton and Newton, a car leaves Oakton and travels at 60 miles per hour. How many miles has the car traveled when the truck reaches Newton? (Lesson 3-8)

Ⓐ 25 Ⓑ 30
Ⓒ 50 Ⓓ 60

4. Which equation would best represent the graphed data? (Lesson 5-7)

Table-Tennis Ball Bounce



Ⓐ $y = \frac{1}{2}x + 15$ Ⓑ $y = 2x + 15$
Ⓒ $y = 2x$ Ⓓ $y = \frac{1}{2}x$

5. If a child is equally likely to be born a boy or a girl, what is the probability that a family of 3 children will contain exactly one boy? (Lesson 7-5)

Ⓐ $\frac{1}{8}$ Ⓑ $\frac{1}{4}$
Ⓒ $\frac{3}{8}$ Ⓓ $\frac{1}{2}$

6. What is the value of 5^{-2}? (Lesson 8-2)

Ⓐ -25 Ⓑ $-\frac{1}{25}$
Ⓒ $\frac{1}{25}$ Ⓓ $-\sqrt{5}$

7. What are the solutions of $x^2 + x = 20$? (Lesson 9-4)

Ⓐ $-4, 5$ Ⓑ $-2, 10$
Ⓒ $2, 10$ Ⓓ $4, -5$

8. Two airplanes are flying at the same altitude. One plane is two miles west and two miles north of an airport. The other plane is seven miles west and eight miles north of the same airport. How many miles apart are the airplanes? (Lesson 11-4)

Ⓐ 2.8 Ⓑ 7.8
Ⓒ 10.6 Ⓓ 11.0

9. A certain password consists of three characters, and each character is a letter of the alphabet. Each letter can be used more than once. How many different passwords are possible? (Lesson 14-1)

Ⓐ 78 Ⓑ 2600
Ⓒ 15,600 Ⓓ 17,576

Test-Taking Tip

If you are allowed to write in your test booklet, underline key words, do calculations, sketch diagrams, cross out answer choices as you eliminate them, and mark any questions that you skip. But do not make any marks on the *answer sheet* except your answers.

Preparing for Standardized Tests
For test-taking strategies and more practice, see pages 867– 884.

Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

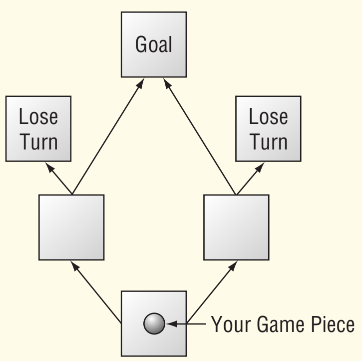
10. What are the coordinates of the point of intersection of the lines represented by the equations $x + 4y = 0$ and $2x - 3y = 11$? (Lesson 7-2)

11. Is $4\left(x - \frac{1}{2}\right)^2 - 1 = 4x^2 - 4x$ true for *all* values of x, *some* values of x, or *no* values of x? (Lesson 8-8)

12. Triangle ABC has sides of length $a = 5$, $b = 7$, and $c = \sqrt{74}$. What is the measure, in degrees, of the angle opposite side c? (Lesson 11-4)

13. All seven-digit telephone numbers in a town begin with the same three digits. Of the last four digits in any given phone number, neither the first nor the last digit can be 0. How many telephone numbers are available in this town? (Lesson 14-2)

14. In the board game shown below, you move your game piece along the arrows from square to square. To determine which direction to move your game piece, you roll a number cube with sides numbered 1, 2, 3, 4, 5, and 6. If you roll 1 or 2, you move your game piece one space to the left. If you roll 3, 4, 5, or 6, you move your game piece one square to the right. What is the probability that you will reach the goal within two turns? (Lesson 14-3)



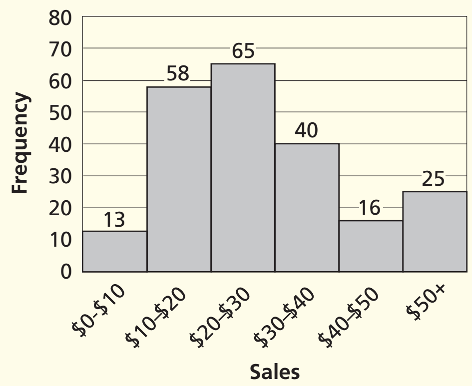
15. An eight-sided die numbered 1–8 is rolled 50 times during the span of a board game. If a 7 is rolled twelve times, what is the theoretical probability of rolling a number other than 7? (Lesson 14-5)

Part 3 Extended Response

Record your answers on a sheet of paper. Show your work.

16. The histogram shows the number of sales DVD World has made during one weekend of business. (Lesson 13-3)

Sales at DVD World



a. What was the total number of sales during the weekend?

b. In what measurement class does the median occur?

c. Describe the distribution of the data.

17. At WackyWorld Pizza, the Random Special is a random selection of two different toppings on a large cheese pizza. The available toppings are pepperoni, sausage, onion, mushrooms, and green peppers. (Lessons 14-2 and 14-3)

a. How many different Random Specials are possible? Show how you found your answer.

b. If you order the Random Special, what is the probability that it will have mushrooms?

c. If you order the Random Special, what is the probabililty that it will have neither onion nor green peppers?