

UNIT

3

Your study of algebra includes more than just solving equations. Many real-world situations can be modeled by equations and their graphs. In this unit, you will learn about functions and graphs.

Linear Equations, Inequalities, and Functions



Chapter 7
Equations and Inequalities

Chapter 8
Functions and Graphing



WebQuest Internet Project

Just for Fun

What do you like to do in your spare time—shop at the mall, attend a baseball or football game, go to the movies, ride the rides at an amusement park, or hike in the great outdoors?

In this project, you will be exploring how equations, functions, and graphs can help you examine how people spend their leisure time.



Log on to www.pre-alg.com/webquest.
Begin your WebQuest by reading the Task.

Then continue working on your WebQuest as you study Unit 3.

Lesson	7-1	8-8
Page	333	411

USA TODAY Snapshots®

What fans pay after ticket

An average fan at a Major League Baseball game spends \$15.40 on parking, food, drinks and souvenirs in addition to the \$15 ticket price.

MLB NBA NHL NFL

\$15.40 \$18.20 \$18.25 \$19.00

Source: American Demographics, Team Marketing Report
By Anne R. Carey and Marcy E. Mullins, USA TODAY

Equations and Inequalities

What You'll Learn

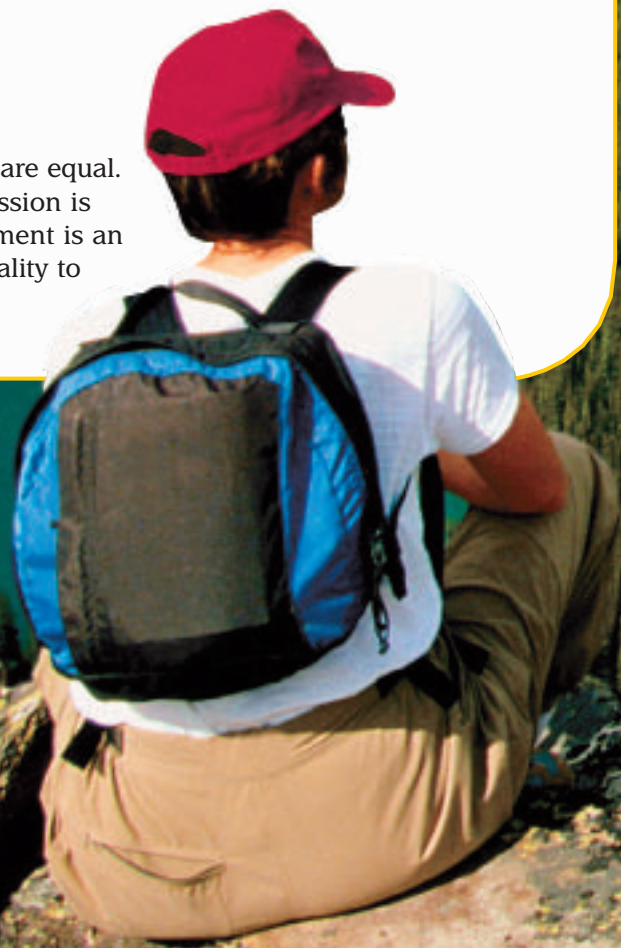
- **Lessons 7-1 and 7-2** Solve equations with variables on each side and with grouping symbols.
- **Lesson 7-3** Write and graph inequalities.
- **Lessons 7-4 and 7-5** Solve inequalities using the Properties of Inequalities.
- **Lesson 7-6** Solve multi-step inequalities.

Why It's Important

An equation is a statement that two expressions are equal. Sometimes, you want to know when one expression is greater or less than another. This kind of statement is an inequality. For example, you can solve an inequality to determine a healthy backpack weight. *You will solve problems involving backpacking in Lesson 7-6.*

Key Vocabulary

- null or empty set (p. 336)
- identity (p. 336)
- inequality (p. 340)



Getting Started

► **Prerequisite Skills** To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 7.

For Lesson 7-1

Solve Two-Step Equations

Solve each equation. Check your solution. (For review, see Lesson 3-5.)

1. $2x + 5 = 13$

2. $4n - 3 = 5$

3. $16 = 8 + \frac{d}{3}$

4. $\frac{c}{-4} + 3 = -9$

For Lesson 7-4

Add and Subtract Integers

Find each sum or difference. (For review, see Lessons 2-2 and 2-3.)

5. $-28 + (-16)$

6. $17 + (-25)$

7. $-13 + 24$

8. $36 + (-18)$

9. $31 - 48$

10. $-16 - 7$

11. $4 - (-12)$

12. $-23 - (-29)$

13. $-19 - (-5)$

For Lesson 7-5

Multiply and Divide Integers

Find each product or quotient. (For review, see Lessons 2-4 and 2-5.)

14. $-6(8)$

15. $-3 \cdot 5$

16. $-6(-25)$

17. $2(-4)(-9)$

18. $64 \div (-32)$

19. $-15 \div 3$

20. $-12 \div (-3)$

21. $-6 \div (-6)$

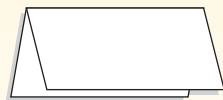
22. $24 \div (-2)$

FOLDABLESTM Study Organizer

Equations and Inequalities Make this Foldable to help you organize your notes. Begin with a plain sheet of $8\frac{1}{2}$ " by 11" paper.

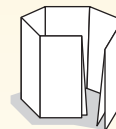
Step 1 Fold in Half

Fold in half lengthwise.



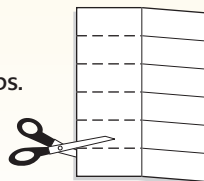
Step 2 Fold in Sixths

Fold in thirds and then fold each third in half.



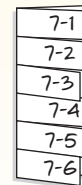
Step 3 Cut

Open. Cut one side along the folds to make tabs.



Step 4 Label

Label each tab with a lesson number as shown.



Reading and Writing As you read and study the chapter, write notes and examples under each tab.



Algebra Activity

A Preview of Lesson 7-1

Equations with Variables on Each Side

In Chapter 3, you used algebra tiles and an equation mat to solve equations in which the variable was on only one side of the equation. You can use algebra tiles and an equation mat to solve equations with variables on each side of the equation.

Activity 1

The following example shows how to solve $x + 3 = 2x + 1$ using algebra tiles.

Model the equation.

$$x + 3 = 2x + 1$$

Remove the same number of x -tiles from each side of the mat until there is an x -tile by itself on one side.

$$x - x + 3 = 2x - x + 1$$

Remove the same number of 1-tiles from each side of the mat until the x -tile is by itself on one side.

$$2 = x$$

There are two 1-tiles on the left side of the mat and one x -tile on the right side. Therefore, $x = 2$. Since $2 + 3 = 2(2) + 1$, the solution is correct.

Model

Use algebra tiles to model and solve each equation.

- | | | |
|---------------------|----------------------|----------------------|
| 1. $2x + 3 = x + 5$ | 2. $3x + 4 = 2x + 8$ | 3. $3x = x + 6$ |
| 4. $6 + x = 4x$ | 5. $2x - 4 = x - 6$ | 6. $5x - 1 = 4x - 5$ |

Analyze

- Which property of equality allows you to remove a 1-tile from each side of the mat?
- Explain why you can remove an x -tile from each side of the mat.

Activity 2

Some equations are solved by using zero pairs. Remember, you may add or subtract a zero pair from either side of an equation mat without changing its value. The following example shows how to solve $2x + 1 = x - 5$.

Model the equation.

$2x + 1 = x - 5$

Remove the same number of x -tiles from each side of the mat until there is an x -tile by itself on one side.

$2x - x + 1 = x - x - 5$

It is not possible to remove the same number of 1-tiles from each side of the mat. Add 1 negative tile to the left side to make a zero pair. Add 1 negative tile to the right side of the mat.

$x + 1 + (-1) = -5 + (-1)$

Remove the zero pair from the left side. There are 6 negative tiles on the right side of the mat.

$x = -6$

Therefore, $x = -6$. Since $2(-6) + 1 = -6 - 5$, the solution is correct.

Model

Use algebra tiles to model and solve each equation.

- | | | |
|----------------------|-----------------------|-----------------------|
| 9. $2x + 3 = x - 5$ | 10. $3x - 2 = x + 6$ | 11. $x - 1 = 3x + 7$ |
| 12. $x + 6 = 2x - 3$ | 13. $2x + 4 = 3x - 2$ | 14. $4x - 1 = 2x + 5$ |

Analyze

15. Does it matter whether you remove x -tiles or 1-tiles first? Explain.
16. Explain how you could use models to solve $-2x + 5 = -x - 2$.

Solving Equations with Variables on Each Side

What You'll Learn

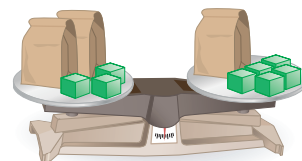
- Solve equations with variables on each side.

How

is solving equations with variables on each side like solving equations with variables on one side?

On the balance at the right, each bag contains the same number of blocks. (Assume that the paper bag weighs nothing.)

- The two sides balance. Without looking in a bag, how can you determine the number of blocks in each bag?
- Explain why your method works.
- Suppose x represents the number of blocks in the bag. Write an equation that is modeled by the balance.
- Explain how you could solve the equation.



EQUATIONS WITH VARIABLES ON EACH SIDE To solve equations with variables on each side, use the Addition or Subtraction Property of Equality to write an equivalent equation with the variables on one side. Then solve the equation.

Study Tip

Look Back

To review **Addition and Subtraction Properties of Equality**, see Lesson 3-3.

Example 1 Equations with Variables on Each Side

Solve $2x + 3 = 3x$. Check your solution.

$$2x + 3 = 3x \quad \text{Write the equation.}$$

$$2x - 2x + 3 = 3x - 2x \quad \text{Subtract } 2x \text{ from each side.}$$

$$3 = x \quad \text{Simplify.}$$

Subtract $2x$ from the left side of the equation to isolate the variable.

Subtract $2x$ from the right side of the equation to keep it balanced.

To check your solution, replace x with 3 in the original equation.

CHECK $2x + 3 = 3x$ Write the equation.

$$2(3) + 3 \stackrel{?}{=} 3(3) \quad \text{Replace } x \text{ with } 3.$$

$$6 + 3 \stackrel{?}{=} 9 \quad \text{Check to see whether this statement is true.}$$

$$9 = 9 \quad \checkmark \quad \text{The statement is true.}$$

The solution is 3.



Concept Check

What property allows you to add the same quantity to each side of an equation?

Example 2 Equations with Variables on Each Side

- a. Solve $5x + 4 = 3x - 2$. Check your solution.

$$5x + 4 = 3x - 2 \quad \text{Write the equation.}$$

$$5x - 3x + 4 = 3x - 3x - 2 \quad \text{Subtract } 3x \text{ from each side.}$$

$$2x + 4 = -2 \quad \text{Simplify.}$$

$$2x + 4 - 4 = -2 - 4 \quad \text{Subtract 4 from each side.}$$

$$2x = -6 \quad \text{Simplify.}$$

$$x = -3 \quad \text{Mentally divide each side by 2.}$$

CHECK $5x + 4 = 3x - 2$ Write the equation.

$$5(-3) + 4 \stackrel{?}{=} 3(-3) - 2 \quad \text{Is this statement true?}$$

$$-11 = -11 \quad \checkmark \quad \text{The solution checks.}$$

The solution is -3 .

- b. Solve $2.4 + a = 2.5a - 4.5$.

$$2.4 + a = 2.5a - 4.5 \quad \text{Write the equation.}$$

$$2.4 + a - a = 2.5a - a - 4.5 \quad \text{Subtract } a \text{ from each side.}$$

$$2.4 = 1.5a - 4.5 \quad \text{Simplify.}$$

$$2.4 + 4.5 = 1.5a - 4.5 + 4.5 \quad \text{Add 4.5 to each side.}$$

$$6.9 = 1.5a \quad \text{Simplify.}$$

$$\frac{6.9}{1.5} = \frac{1.5a}{1.5} \quad \text{Divide each side by 1.5.}$$

$$4.6 = a \quad \text{Check your solution.}$$

The solution is 4.6 .

You can use equations with variables on each side to solve problems.

Example 3 Use an Equation to Solve a Problem

- **VIDEOS** A video store has two membership plans. Under plan A, a yearly membership costs \$30 plus \$1.50 for each rental. Under plan B, the yearly membership costs \$12 plus \$3 for each rental. What number of rentals results in the same yearly cost?

Let v represent the number of videos rented.

Words $\underbrace{\$30 \text{ plus } \$1.50 \text{ for each video}}_{30 + 1.50v}$ $\underbrace{\$12 \text{ plus } \$3 \text{ for each video}}_{12 + 3v}$

Variables $30 + 1.50v$ $12 + 3v$

Equation $30 + 1.50v = 12 + 3v$ Write an equation.

$$30 + 1.5v - 1.5v = 12 + 3v - 1.5v \quad \text{Subtract } 1.5v \text{ from each side.}$$

$$30 = 12 + 1.5v \quad \text{Simplify.}$$

$$30 - 12 = 12 - 12 + 1.5v \quad \text{Subtract 12 from each side.}$$

$$18 = 1.5v \quad \text{Simplify.}$$

$$\frac{18}{1.5} = \frac{1.5v}{1.5} \quad \text{Divide each side by 1.5.}$$

$$12 = v \quad \text{Simplify.}$$

The yearly cost is the same for 12 rentals.

More About...



Videos

In 1980, only 1% of American households owned a VCR. Today, more than 80% do.

Source: Statistical Abstracts



Check for Understanding

Concept Check

1. **Name** the property of equality that allows you to subtract the same quantity from each side of an equation.
2. **OPEN ENDED** Write an example of an equation with variables on each side. State the steps you would use to isolate the variable.

Guided Practice

Solve each equation. Check your solution.

3. $4x - 8 = 5x$
4. $12x = 2x + 40$
5. $4x - 1 = 3x + 2$
6. $4k + 24 = 6k - 10$
7. $n + 0.4 = -n + 1$
8. $3.1w + 5 = 0.8 + w$

Application

9. **CAR RENTAL** Suppose you can rent a car from ABC Auto for either \$25 a day plus \$0.45 a mile or for \$40 a day plus \$0.25 a mile. What number of miles results in the same cost for one day?

Practice and Apply

Homework Help

For Exercises

10–27, 30–33;
28, 29, 34–36:

See Examples
1, 2
3

Extra Practice
See page 739.

Solve each equation. Check your solution.

10. $4x + 9 = 7x$
11. $6a = 26 + 4a$
12. $3y + 16 = 5y$
13. $n - 14 = 3n$
14. $8 - 3c = 2c - 2$
15. $3 - 4b = 10b + 10$
16. $7d - 13 = 3d + 7$
17. $2f - 6 = 7f + 24$
18. $-s + 4 = 7s - 3$
19. $4a - 2 = 7a - 6$
20. $12n - 24 = -14n + 28$
21. $13y - 18 = -5y + 36$
22. $12 + 1.5a = 3a$
23. $12.6 - x = 2x$
24. $2b + 6.2 = 13.2 - 8b$
25. $3c + 4.5 = 7.2 - 6c$
26. $12.4y + 14 = 6y - 2$
27. $4.3n - 1.6 = 2.3n + 5.2$

Define a variable and write an equation to find each number. Then solve.

28. Twice a number is 220 less than six times the number. What is the number?
29. Fourteen less than three times a number equals the number. What is the number?

Solve each equation. Check your solution.

30. $\frac{4}{5}y - 8 = \frac{2}{5}y + 16$
31. $\frac{3}{4}k + 16 = 2 - \frac{1}{8}k$
32. $\frac{x}{0.4} = 2x + 1.2$
33. $\frac{1}{3}b + 8 = \frac{1}{2}b - 4$

34. **GEOGRAPHY** The coastline of California is 46 miles longer than twice the length of Louisiana's coastline. It is also 443 miles longer than Louisiana's coastline. Find the lengths of the coastlines of California and Louisiana.
35. **CELLULAR PHONES** One cellular phone carrier charges \$29.75 a month plus \$0.15 a minute for local calls. Another carrier charges \$19.95 a month and \$0.29 a minute for local calls. For how many minutes is the cost of the plans the same?

The trends in attendance at various sporting events can be represented by equations. Visit www.pre-alg.com/webquest to continue work on your WebQuest project.

Standardized Test Practice

A B C D

36. An empty bucket is put under two faucets. If one faucet is turned on alone, the bucket fills in 6 minutes. If the other faucet is turned on alone, the bucket fills in 4 minutes. If both are turned on, how many seconds will it take to fill the bucket?
37. **CRITICAL THINKING** Three times the quantity $y + 7$ equals four times the quantity $y - 2$. What value of y makes the sentence true?
38. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How is solving equations with variables on each side like solving equations with variables on one side?

Include the following in your answer:

- examples of an equation with variables on each side and an equation with the variable on one side, and
- an explanation of how they are alike and how they are different.

39. Shoe World offers Olivia a temporary job during her spring break. The manager gives her a choice as to how she wants to be paid, but she must decide before she starts working. The choices are shown below.

	Pay per Hour	Pay for Each Dollar of Shoe Sales
Plan 1	\$3	15¢
Plan 2	\$4	10¢

Which equation shows what Olivia's sales would need to be in one hour to earn the same amount under either plan?

- (A) $3 + 0.15s = 4 + 0.10s$ (B) $3s + 0.15 = 4s + 0.10$
 (C) $3 + 0.10s = 4 + 0.15s$ (D) $3(s + 0.15) = 4(s + 0.10)$
40. What is the solution of $3x - 1 = x + 3$?
 (A) 1 (B) 2 (C) 3 (D) 4
41. **WEATHER** The formula $F = \frac{9}{5}C + 32$ is used for finding the Fahrenheit temperature when a Celsius temperature is known. Find the temperature where the Celsius and Fahrenheit scales are the same.

Extending the Lesson

Maintain Your Skills

Mixed Review

42. **PROBABILITY** What is the probability of randomly choosing the letter T from the letters in PITTSBURGH? (Lesson 6-9)
43. Find the percent of increase from \$80 to \$90. (Lesson 6-8)

ALGEBRA Solve each problem using an equation. (Lesson 6-7)

44. 14 is what percent of 20? 45. Find 36% of 18.
 46. 1.5 is 30% of what number? 47. Find 140% of 50.

Getting Ready for the Next Lesson

PREREQUISITE SKILL Use the Distributive Property to rewrite each expression as an equivalent algebraic expression.

(To review the **Distributive Property**, see Lesson 3-1.)

48. $4(x - 8)$ 49. $3(2a + 9)$ 50. $5(12 - x)$
 51. $2(1.2c + 14)$ 52. $8(-4k + 2.3)$ 53. $\frac{1}{2}(n - 9)$



Solving Equations with Grouping Symbols

What You'll Learn

- Solve equations that involve grouping symbols.
- Identify equations that have no solution or an infinite number of solutions.

Vocabulary

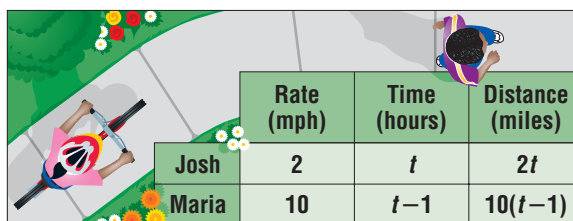
- null or empty set
- identity

Why

is the Distributive Property important in solving equations?

Josh starts walking at a rate of 2 mph. One hour later, his sister Maria starts on the same path on her bike, riding at 10 mph.

The table shows expressions for the distance each has traveled after a given time.



	Rate (mph)	Time (hours)	Distance (miles)
Josh	2	t	$2t$
Maria	10	$t - 1$	$10(t - 1)$

- What does t represent?
- Why is Maria's time shown as $t - 1$?
- Write an equation that represents the time when Maria catches up to Josh. (*Hint: They will have traveled the same distance.*)

SOLVE EQUATIONS WITH GROUPING SYMBOLS To find how many hours it takes Maria to catch up to Josh, you can solve the equation $2t = 10(t - 1)$. First, use the Distributive Property to remove the grouping symbols.

Study Tip

Look Back

To review the **Distributive Property**, see Lesson 3-1.

Example 1 Solve Equations with Parentheses

- Solve the equation $2t = 10(t - 1)$. Check your solution.

$$2t = 10(t - 1) \quad \text{Write the equation.}$$

$$2t = 10(t) - 10(1) \quad \text{Use the Distributive Property.}$$

$$2t = 10t - 10 \quad \text{Simplify.}$$

$$2t - 10t = 10t - 10t - 10 \quad \text{Subtract } 10t \text{ from each side.}$$

$$-8t = -10 \quad \text{Simplify.}$$

$$\frac{-8t}{-8} = \frac{-10}{-8} \quad \text{Divide each side by } -8.$$

$$t = \frac{5}{4} \text{ or } 1\frac{1}{4} \quad \text{Simplify.}$$

CHECK Josh traveled $\frac{2 \text{ miles}}{\text{hour}} \cdot \frac{5 \text{ hour}}{4}$ or $2\frac{1}{2}$ miles.

Maria traveled one hour less than Josh. She traveled

$$\frac{10 \text{ miles}}{\text{hour}} \cdot \frac{1 \text{ hour}}{4} \text{ or } 2\frac{1}{2} \text{ miles.}$$

Therefore, Maria caught up to Josh in $\frac{1}{4}$ hour, or 15 minutes.

Study Tip

Alternative Method

You can also solve the equation by subtracting $3a$ from each side first, then adding 20 to each side.

b. Solve $5(a - 4) = 3(a + 1.5)$.

$$5(a - 4) = 3(a + 1.5)$$

$$5a - 20 = 3a + 4.5$$

$$5a - 20 + 20 = 3a + 4.5 + 20$$

$$5a = 3a + 24.5$$

$$5a - 3a = 3a - 3a + 24.5$$

$$2a = 24.5$$

$$\frac{2a}{2} = \frac{24.5}{2}$$

$$a = 12.25$$

Write the equation.

Use the Distributive Property.

Add 20 to each side.

Simplify.

Subtract $3a$ from each side.

Simplify.

Divide each side by 2.

Simplify.

The solution is 12.25. Check your solution.



Concept Check

What property do you use to remove the grouping symbols from the equation $2(8 - a) = 4(a + 9)$?

Sometimes a geometric figure is described in terms of only one of its dimensions. To find the dimensions, you may have to solve an equation that contains grouping symbols.

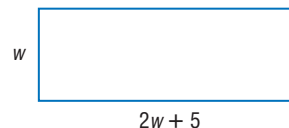
Example 2 Use an Equation to Solve a Problem

GEOMETRY The perimeter of a rectangle is 46 inches. Find the dimensions if the length is 5 inches greater than twice the width.

Words The length is 5 inches greater than twice the width.
The perimeter is 46 inches.

Variables Let w = the width.

Let $2w + 5$ = the length.



$$\underbrace{2 \text{ times length}} + \underbrace{2 \text{ times width}} = \underbrace{\text{perimeter}}$$

Equation $2(2w + 5) + 2w = 46$

Solve $2(2w + 5) + 2w = 46$.

$$2(2w + 5) + 2w = 46$$

Write the equation.

$$4w + 10 + 2w = 46$$

Use the Distributive Property.

$$6w + 10 = 46$$

Simplify.

$$6w + 10 - 10 = 46 - 10$$

Subtract 10 from each side.

$$6w = 36$$

Simplify.

$$w = 6$$

Mentally divide each side by 6.

Evaluate $2w + 5$ to find the length.

$$2(6) + 5 = 12 + 5 \text{ or } 17$$

Replace w with 6.

CHECK Add the lengths of the four sides.

$$6 + 17 + 6 + 17 = 46 \checkmark$$

The width is 6 inches. The length is 17 inches.

Study Tip

Look Back

To review **perimeter of a rectangle**, see Lesson 3-7.



NO SOLUTION OR ALL NUMBERS AS SOLUTIONS Some equations have *no* solution. That is, no value of the variable results in a true sentence. The solution set is the **null** or **empty set**, shown by the symbol $\emptyset$ or $\{\}$.

Example 3 *No Solution*

Solve $3x + \frac{1}{3} = 3x - \frac{1}{2}$.

$$3x + \frac{1}{3} = 3x - \frac{1}{2} \quad \text{Write the equation.}$$

$$3x - 3x + \frac{1}{3} = 3x - 3x - \frac{1}{2} \quad \text{Subtract } 3x \text{ from each side.}$$

$$\frac{1}{3} = -\frac{1}{2} \quad \text{Simplify.}$$

The sentence $\frac{1}{3} = -\frac{1}{2}$ is *never* true. So, the solution set is $\emptyset$.

Other equations may have every number as the solution. An equation that is true for every value of the variable is called an **identity**.

Example 4 *All Numbers as Solutions*

Solve $2(2x - 1) + 6 = 4x + 4$.

$$2(2x - 1) + 6 = 4x + 4 \quad \text{Write the equation.}$$

$$4x - 2 + 6 = 4x + 4 \quad \text{Use the Distributive Property.}$$

$$4x + 4 = 4x + 4 \quad \text{Simplify.}$$

$$4x + 4 - 4 = 4x + 4 - 4 \quad \text{Subtract 4 from each side.}$$

$$4x = 4x \quad \text{Simplify.}$$

$$x = x \quad \text{Mentally divide each side by 4.}$$

The sentence $x = x$ is *always* true. The solution set is all numbers.

Check for Understanding

Concept Check 1. List the steps you would take to solve the equation $2x + 3 = 4(x - 1)$.

2. **OPEN ENDED** Give an example of an equation that has no solution and an equation that is an identity.

Guided Practice Solve each equation. Check your solution.

3. $3(a - 5) = 18$

4. $32 = 4(x + 9)$

5. $2(d + 6) = 3d - 1$

6. $6(n - 3) = 4(n + 2.1)$

7. $12 - h = -h + 3$

8. $3(2g + 4) = 6(g + 2)$

Application 9. **GEOMETRY** The perimeter of a rectangle is 20 feet. The width is 4 feet less than the length. Find the dimensions of the rectangle. Then find its area.

Practice and Apply

Homework Help

For Exercises	See Examples
10–19, 24, 25, 28, 29	1
20–23, 26, 27, 30–33	3, 4 2

Extra Practice
See page 740.

Solve each equation. Check your solution.

10. $3(g - 3) = 6$

12. $5(2c + 7) = 80$

14. $3(a - 3) = 2(a + 4)$

16. $4(x - 2) = 3(1.5 + x)$

18. $2(3.5n + 6) = 2.5n - 2$

20. $4(f + 3) + 5 = 17 + 4f$

22. $8y - 3 = 5(y - 1) + 3y$

24. $\frac{1}{2}(2n - 5) = 4n - 1$

26. $-3(4b - 10) = \frac{1}{2}(-24b + 60)$

28. $\frac{d}{0.4} = 2d + 1.24$

11. $3(x + 1) = 21$

13. $6(3d + 5) = 75$

15. $3(s + 22) = 4(s + 12)$

17. $3(a - 1) = 4(a - 1.5)$

19. $4.2x - 9 = 3(1.2x + 4)$

21. $3n + 4 = 5(n + 2) - 2n$

23. $2(x - 5) = 4x - 2(x + 5)$

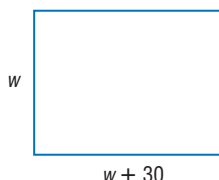
25. $y - 2 = \frac{1}{3}(y + 6)$

27. $\frac{3}{4}a + 4 = \frac{1}{4}(3a + 16)$

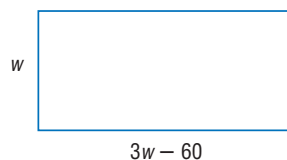
29. $\frac{a - 6}{12} = \frac{a - 2}{4}$

Find the dimensions of each rectangle. The perimeter is given.

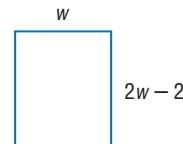
30. $P = 460$ ft



31. $P = 440$ yd



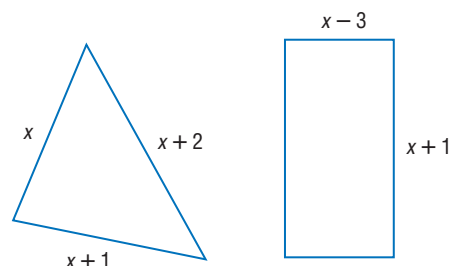
32. $P = 11$ m



33. **GEOMETRY** The perimeter of a rectangle is 32 feet. Find the dimensions if the length is 4 feet longer than three times the width. Then find the area of the rectangle.

34. **NUMBER THEORY** Three times the sum of three consecutive integers is 72. What are the integers?

35. **GEOMETRY** The triangle and the rectangle have the same perimeter. Find the dimensions of each figure. Then find the perimeter.



36. **BASKETBALL** Camilla has three times as many points as Lynn. Lynn has five more points than Kim. Camilla, Lynn, and Kim combined have twice as many points as Jasmine. If Jasmine has 25 points, how many points does each of the other three girls have?

37. **DECORATING** Suppose a rectangular room measures 15 feet long by 12 feet wide by 7 feet high and has two windows and two doors. Use the information at the left to find how many gallons of paint are needed to paint the room using two coats of paint.

More About...



Decorating

A gallon of paint covers about 350 square feet. Estimate the square footage by multiplying the combined wall lengths by wall height and subtracting 15 square feet for each window and door.

Source: *The Family Handyman Magazine Presents Handy Hints for Home, Yard, and Workshop*



38. **CRITICAL THINKING** An apple costs the same as 2 oranges. Together, an orange and a banana cost 10¢ more than an apple. Two oranges cost 15¢ more than a banana. What is the cost for one of each fruit?
39. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

Why is the Distributive Property important in solving equations?

Include the following in your answer:

- a definition of the Distributive Property, and
- a description of its use in solving equations.

Standardized Test Practice

A B C D

40. Which equation is equivalent to $2(3x - 1) = 10 + 2x$?
- (A) $8x - 2 = 10$ (B) $6x = 11 + 2x$
 (C) $4x - 2 = 10$ (D) $6x - 1 = 10 + 2x$
41. Car X leaves Northtown traveling at a steady rate of 55 mph. Car Y leaves 1 hour later following Car X, traveling at a steady rate of 60 mph. Which equation can be used to determine how long after Car X leaves Car Y will catch up?
- (A) $55x = 60x - 1$ (B) $55x = 60x$
 (C) $60x = 55(x - 1)$ (D) $55x = 60(x - 1)$

Maintain Your Skills

Mixed Review ALGEBRA Solve each equation. Check your solution. (Lesson 7-1)

42. $4x = 2x + 5$ 43. $3x + 5 = 7 - 2x$ 44. $1.5x + 9 = 3x - 3$

45. **PROBABILITY** Find the probability of choosing a girl's name at random from 20 girls' names and 30 boys' names. (Lesson 6-9)

Write each fraction as a decimal. Use a bar to show a repeating decimal.

(Lesson 5-1)

46. $\frac{4}{10}$ 47. $\frac{3}{8}$ 48. $-\frac{1}{3}$ 49. $3\frac{6}{25}$ 50. $4\frac{5}{11}$

Getting Ready for the Next Lesson

PREREQUISITE SKILL Evaluate each expression.

(To review evaluating expressions, see Lesson 1-3.)

51. $x - 12$, $x = 5$ 52. $b + 11$, $b = -15$ 53. $4a$, $a = -6$
 54. $2t + 8$, $t = -3$ 55. $\frac{24}{c}$, $c = -3$ 56. $\frac{3x}{4} + 2$, $x = 6$

Practice Quiz 1

Lessons 7-1 and 7-2

Define a variable and write an equation. Then solve. (Lesson 7-1)

1. Twice a number is 150 less than 5 times the number. What is the number?

Solve each equation. Check your solution. (Lessons 7-1 and 7-2)

2. $6y + 42 = 4y$ 3. $7m - 12 = 2.5m + 2$
 4. $8(p - 4) = 2(2p + 1)$ 5. $b + 2(b + 5) = 3(b - 1) + 13$



Reading Mathematics

Meanings of At Most and At Least

The phrases *at most* and *at least* are used in mathematics. In order to use them correctly, you need to understand their meanings.

Phrase	Meaning	Mathematical Symbol
at most	• no more than • less than or equal to	$\leq$
at least	• no less than • greater than or equal to	$\geq$

Here is an example of one common use of each phrase, its meaning, and a mathematical expression for the situation.

Verbal Expression	You can spend <i>at most</i> \$20.
Meaning	You can spend \$20 or any amount less than \$20.
Mathematical Expression	$s \leq 20$, where s represents the amount you spend.

Verbal Expression	A person must be <i>at least</i> 18 to vote.
Meaning	A person who is 18 years old or any age older than 18 may vote.
Mathematical Expression	$a \geq 18$, where a represents age.

Notice that the word *or* is part of the meaning in each case.

Reading to Learn

1. Write your own rule for remembering the meanings of *at most* and *at least*.

For each expression, write the meaning. Then write a mathematical expression using $\leq$ or $\geq$.

2. You need to earn at least \$50 to help pay for a class trip.
3. The sum of two numbers is at most 6.
4. You want to drive at least 250 miles each day.
5. You want to hike 4 hours each day at most.



7-3 Inequalities

Vocabulary

- inequality

What You'll Learn

- Write inequalities.
- Graph inequalities.

How can inequalities help you describe relationships?



If your age is *less than* 6, you eat free.



If your height is *more than* 40 inches, you can ride.



A speed of 35 or less is legal.

- Name three ages of children who can eat free at the restaurant. Does a child who is 6 years old eat free?
- Name three heights of children who can ride the ride at the amusement park. Can a child who is 40 inches tall ride?
- Name three speeds that are legal. Is a driver who is traveling at 35 mph driving at a legal speed?

WRITE INEQUALITIES A mathematical sentence that contains $<$ or $>$ is called an **inequality**.

Example 1 Write Inequalities with $<$ or $>$

Write an inequality for each sentence.

- a. Your age is less than 6 years.

Variable Let a represent age.

Inequality $a < 6$

- b. Your height is greater than 40 inches.

Variable Let h represent height.

Inequality $h > 40$

Some inequalities contain $\leq$ or $\geq$ symbols.

Example 2 Write Inequalities with $\leq$ or $\geq$

Write an inequality for each sentence.

- a. Your speed is less than or equal to 35 miles per hour.

Variable Let s represent speed.

Inequality $s \leq 35$

b. Your speed is greater than or equal to 55 miles per hour.

Variable Let s represent speed.

Inequality $s \geq 55$

The table below shows some common verbal phrases and the corresponding mathematical inequalities.

Study Tip

Inequalities

Notice that $\leq$ and $\geq$ combine the symbol $<$ or $>$ with part of the symbol for equals, $=$.

Concept Summary			Inequalities
$<$	$>$	$\leq$	$\geq$
- is less than - is fewer than	- is greater than - is more than - exceeds	- is less than or equal to - is no more than - is at most	- is greater than or equal to - is no less than - is at least

Example 3 Use an Inequality

NUTRITION A food can be labeled low fat only if it has no more than 3 grams of fat per serving. Write an inequality to describe low fat foods.

Words Grams of fat per serving is no more than 3.

Variable Let f = number of grams of fat per serving.

Inequality $f \leq 3$

The inequality is $f \leq 3$.

Inequalities with variables are open sentences. When the variable in an open sentence is replaced with a number, the inequality may be true or false.

Example 4 Determine Truth of an Inequality

For the given value, state whether each inequality is *true* or *false*.

a. $s - 7 < 5$, $s = 14$

$s - 7 < 5$ Write the inequality.

$14 - 7 \stackrel{?}{<} 5$ Replace s with 14.

$7 \nless 5$ Simplify.

This sentence is false.

b. $12 \geq \frac{a}{2} + 2$, $a = 20$

$12 \geq \frac{a}{2} + 2$ Write the inequality.

$12 \stackrel{?}{\geq} \frac{20}{2} + 2$ Replace a with 20.

$12 \stackrel{?}{\geq} 10 + 2$ Simplify.

$12 \geq 12$ Simplify.

Although the inequality $12 > 12$ is false, the equation $12 = 12$ is true.

Therefore, this sentence is true.

Reading Math

Symbols

Read the symbol $\nless$ as *is not less than*.

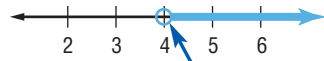


GRAPH INEQUALITIES Inequalities can be graphed on a number line. The graph helps you visualize the values that make the inequality true.

Example 5 *Graph Inequalities*

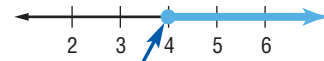
Graph each inequality on a number line.

a. $x > 4$



The open circle means the number 4 is *not* included in the graph.

b. $x \geq 4$



The closed circle means the number 4 is included in the graph.

c. $x < 4$

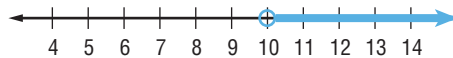


d. $x \leq 4$



Example 6 *Write an Inequality*

Write the inequality for the graph.



An open circle is on 10, so the point 10 is *not* included in the graph. The arrow points to the right, so the graph includes all numbers greater than 10. The inequality is $x > 10$.

Concept Check What symbols are used to write inequalities and what does each symbol mean?

Check for Understanding

Concept Check

1. **Explain** why a number line graph is a good way to represent an inequality.
2. **OPEN ENDED** Write four examples of inequalities using each of the symbols $<$, $>$, $\leq$, and $\geq$. Tell the meaning of each inequality.

Guided Practice

Write an inequality for each sentence.

3. A number increased by 14 is at least 25.
4. Five times some number is less than 65.

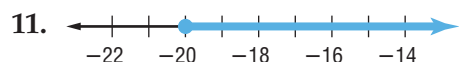
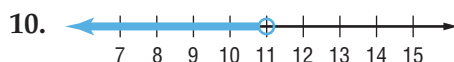
ALGEBRA For the given value, state whether the inequality is *true* or *false*.

5. $n + 4 > 6$, $n = 12$
6. $34 \leq 4r$, $r = 8$

Graph each inequality on a number line.

7. $n > 3$
8. $p \leq 5$
9. $x < 7$

Write the inequality for each graph.



- Application** 12. **SAFETY** The elevators in an office building have been approved for a maximum load of 3600 pounds. Write an inequality to describe a safe load.

Practice and Apply

Homework Help

For Exercises	See Examples
13–16	1, 2
17–22	4
23–34	5
35–40	6
41–43	3

Extra Practice
See page 740.

Write an inequality for each sentence.

13. More than 18,000 fans attended the Kings' opening hockey game at the Staples Center in Los Angeles.
14. Kyle's earnings at \$15 per hour were no more than \$60.
15. The 10-km race time of 86 minutes was at least twice as long as the winner's time.
16. A savings account decreased by \$75 is now less than \$500.

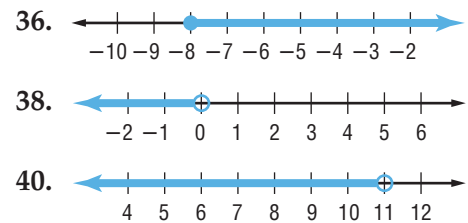
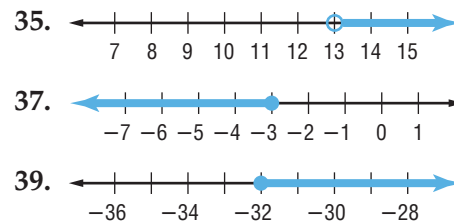
ALGEBRA For the given value, state whether each inequality is *true* or *false*.

17. $18 - x > 4$, $x = 12$
18. $14 + n < 23$, $n = 8$
19. $5k > 35$, $k = 7$
20. $16 \leq 3c$, $c = 8$
21. $\frac{x}{3} \geq 2$, $x = 9$
22. $\frac{14}{c} < 7$, $c = 2$

Graph each inequality on a number line.

23. $a > 4$
24. $x > 6$
25. $n < 11$
26. $x < 5$
27. $t \geq 9$
28. $b \geq 8$
29. $d \leq 5$
30. $w \leq 8$
31. $x > -4$
32. $n \geq -3$
33. $x \leq -5$
34. $x < -2$

Write the inequality for each graph.



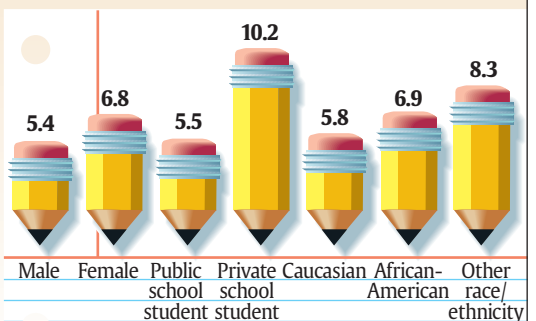
HOMEWORK For Exercises 41 and 42, use the graphic.

41. Inali spends at least an hour more than the average time spent by boys on homework each week. Write an inequality for Inali's homework time.
42. Anna usually spends no more than the average time spent by girls on homework each week. Write an inequality to represent Anna's homework time.

USA TODAY Snapshots®

High school homework time

Students ages 14–18 say they spent a weekly average 6.1 hours on homework last school year, down from 6.6 in 1996–97. Weekly average hours reported by these groups:



Source: The State of Our Nation's Youth 1998–1999, Horatio Alger Association

By Cindy Hall and Sam Ward, USA TODAY

More About...



Sports

Almost twice as many boys play high school football as basketball, but basketball is offered at more high schools.

Source: National Association of State High School Associations

43. **SPORTS** There are more than 30,000 high school basketball and track programs in the United States. If there are 14,600 track programs, write and solve an inequality to determine the number of basketball programs.
44. Find a value for x that satisfies the inequality $0.6 < x < 0.75$.
45. **CRITICAL THINKING** In Chapter 1, you studied the Symmetric and Transitive Properties of Equality. Restate these properties using inequalities. Are the properties true for inequalities? If a property is not true, give a counterexample.

46. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can inequalities help you describe relationships?

Include the following in your answer:

- real-life examples using the four inequality symbols, and
- an explanation of the relationships described by each inequality.

Standardized Test Practice

A B C D

47. Which inequality represents *a number decreased by 2 is at most 8*?
- (A) $n - 2$ (B) $n - 2 \leq 8$
(C) $n - 2 \geq 8$ (D) $n - 2 > 8$
48. Which of the following is an inequality?
- (A) $4 \leq x + 2$ (B) $x + 4 = 3$
(C) $x + 5 + y$ (D) $x - y$

Extending the Lesson

49. Graph the solutions for each compound inequality.
- a. $y < -2$ or $y > 3$. (*Hint: In a sentence, or means either part is true.*)
- b. $y \geq 0$ and $y \leq 5$ (*Hint: In a sentence, and means both parts must be true.*)

Maintain Your Skills

Mixed Review

ALGEBRA Solve each equation. Check your solution. (Lesson 7-2)

50. $2(3 + x) = 14$ 51. $63 = 9(2y - 3)$ 52. $3(n - 1) = 1.5(n + 2)$

53. **ALGEBRA** Four times a number minus 6 is equal to the sum of 3 times the number and 2. Define a variable and write an equation to find the number. (Lesson 7-1)

State whether each sequence is *arithmetic*, *geometric*, or *neither*. Then write the next three terms of each sequence. (Lesson 5-10)

54. $-4, -1, 2, 5, \dots$ 55. $-1, 2, -4, 8, \dots$ 56. $1, 2, 4, 7, \dots$

Getting Ready for the Next Lesson

PREREQUISITE SKILL Solve each equation.
(To review solving equations, see Lesson 3-3.)

57. $x + 19 = 32$ 58. $a + 7 = -3$ 59. $26 + c = 19$
60. $44 - c = 26$ 61. $y - 9.7 = 10.1$ 62. $r - 1.6 = -0.6$

Solving Inequalities by Adding or Subtracting

What You'll Learn

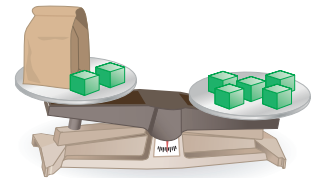
- Solve inequalities by using the Addition and Subtraction Properties of Inequality.

How is solving an inequality similar to solving an equation?

On the balance at the right, the paper bag may contain some blocks.

The blocks and bag on the scale model an inequality because the two sides are not equal.

The model shows the inequality $x + 2 < 5$. The side with the bag and 2 blocks weighs less than the side with 5 blocks.



$$x + 2 < 5$$

- How many blocks would be in the bag if the left side balanced the right side? (Assume that the paper bag weighs nothing.)
- Explain how you determined your answer to part a.
- What numbers of blocks can be in the bag to make the left side weigh *less than* the right side?
- Write an inequality to represent your answer to part c.

SOLVE INEQUALITIES BY ADDING OR SUBTRACTING Solving an inequality means finding values for the variable that make the inequality true. In the example above, any number less than 3 is a solution. The solution is written as the inequality $x < 3$.

You can solve inequalities by using the Addition and Subtraction Properties of Inequalities.

Study Tip

Inequalities

When you add or subtract any number from each side of an inequality, the inequality symbol remains the same.

Key Concept

Addition and Subtraction Properties

- Words** When you add or subtract the same number from each side of an inequality, the inequality remains true.
- Symbols** For all numbers a , b , and c ,
 - if $a > b$, then $a + c > b + c$ and $a - c > b - c$.
 - if $a < b$, then $a + c < b + c$ and $a - c < b - c$.
- Examples**

$2 < 4$	$6 > 3$
$2 + 3 < 4 + 3$	$6 - 4 > 3 - 4$
$5 < 7$	$2 > -1$

These properties are also true for $a \geq b$ and $a \leq b$.



Concept Check

How are these properties similar to the Properties of Equality?

Study Tip

Checking Solutions

Try a number less than 7 to show that it is *not* a solution.

Example 1 Solve an Inequality Using Subtraction

Solve $x + 3 > 10$. Check your solution.

$$x + 3 > 10 \quad \text{Write the inequality.}$$

$$x + 3 - 3 > 10 - 3 \quad \text{Subtract 3 from each side.}$$

$$x > 7 \quad \text{Simplify.}$$

To check your solution, try any number greater than 7.

CHECK $x + 3 > 10$ Write the inequality.

$$8 + 3 > 10 \quad \text{Replace } x \text{ with 8.}$$

$$11 > 10 \quad \checkmark \quad \text{This statement is true.}$$

Any number greater than 7 will make the statement true. Therefore, the solution is $x > 7$.

Example 2 Solve an Inequality Using Addition

Solve $-6 \geq n - 5$. Check your solution.

$$-6 \geq n - 5 \quad \text{Write the inequality.}$$

$$-6 + 5 \geq n - 5 + 5 \quad \text{Add 5 to each side.}$$

$$-1 \geq n \quad \text{Simplify.}$$

CHECK You can check your result by replacing n in the original inequality with a number less than or equal to -1 .

The solution is $-1 \geq n$ or $n \leq -1$.

Example 3 Graph Solutions of Inequalities

Solve $a + \frac{1}{2} < 2$. Graph the solution on a number line.

$$a + \frac{1}{2} < 2 \quad \text{Write the inequality.}$$

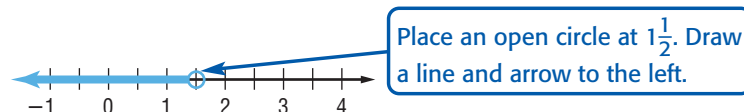
$$a + \frac{1}{2} - \frac{1}{2} < 2 - \frac{1}{2} \quad \text{Subtract } \frac{1}{2} \text{ from each side.}$$

$$a < \frac{4}{2} - \frac{1}{2} \quad \text{Rename 2 as a fraction with a denominator of 2.}$$

$$a < \frac{3}{2} \text{ or } 1\frac{1}{2} \quad \text{Simplify.}$$

The solution is $a < 1\frac{1}{2}$. Check your solution.

Graph the solution.



Concept Check

Does an inequality have only one solution? Explain.

Example 4 Use an Inequality to Solve a Problem

STATE FAIRS Antonio has \$18 to ride go-carts and play games at the State Fair. If the go-carts cost \$5.50, what is the most he can spend on games?

Explore We need to find the greatest amount of money Antonio can spend on games.

Plan Let x represent the amount Antonio can spend on games. Write an inequality to represent the problem. Recall that *at most* means *less than or equal to*.

$$\begin{array}{ccccccc} \text{Cost of} & & \text{cost of} & & \text{must be less} & & \text{total} \\ \text{go-cart} & \text{plus} & \text{games} & & \text{than or equal to} & & \text{amount.} \\ 5.50 & + & x & & \leq & & 18 \end{array}$$

Solve $5.5 + x \leq 18$ Write the inequality. ($5.50 = 5.5$)
 $5.5 - 5.5 + x \leq 18 - 5.5$ Subtract 5.5 from each side.
 $x \leq 12.5$ Simplify.

Examine Check by choosing an amount less than or equal to \$12.50, say, \$10. Then Antonio would spend \$5.50 + \$10 or \$15.50 in all. Since $\$15.50 < \18 , the answer is reasonable.

So, the most Antonio can spend on games is \$12.50.

More About...



State Fairs

More than three million fairgoers are greeted each year by Big Tex, the symbol of the State Fair of Texas.

Source: www.bigtex.com

Check for Understanding

Concept Check

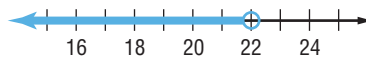
1. **Explain** when you would use addition and when you would use subtraction to solve an inequality.
2. **FIND THE ERROR** Dylan and Jada are using the statement *a minus three is greater than or equal to 15* to find values of a .

$$\begin{array}{l} \text{Dylan} \\ a - 3 \geq 15 \\ a - 3 + 3 \geq 15 + 3 \\ a \geq 18 \end{array}$$

$$\begin{array}{l} \text{Jada} \\ a - 3 = 15 \\ a - 3 + 3 = 15 + 3 \\ a = 18 \end{array}$$

Who is correct? Explain your reasoning.

3. **OPEN ENDED** Make up a problem whose solution is graphed below.



Guided Practice

Solve each inequality. Check your solution.

4. $x + 3 < 8$

5. $14 + y \geq 7$

6. $-13 \geq 9 + b$

7. $a - 5 > 6$

8. $c - (-2) \leq 3$

9. $-5 < t - 2$

Solve each inequality. Then graph the solution on a number line.

10. $h + 4 > 4$

11. $x - 6 \leq 4$

Application

12. **SAVINGS** Chris is saving money for a ski trip. He has \$62.50, but his goal is to save at least \$100. What is the least amount Chris needs to save to reach his goal?



Practice and Apply

Homework Help

For Exercises	See Examples
13–18, 25–28	1
19–24, 29, 30	2
31–42	3
43–46	4

Extra Practice
See page 740.

Solve each inequality. Check your solution.

- | | | |
|-----------------------|--------------------------------------|---------------------------------------|
| 13. $p + 7 < 9$ | 14. $t + 6 > -3$ | 15. $-14 \geq 8 + b$ |
| 16. $16 > -11 + k$ | 17. $3 \geq -2 + y$ | 18. $25 < n + (-12)$ |
| 19. $r - 5 \leq 2$ | 20. $a - 6 < 13$ | 21. $j - 8 \leq -12$ |
| 22. $-8 > h - 1$ | 23. $22 > w - (-16)$ | 24. $-30 \leq d + (-5)$ |
| 25. $1 + y \leq 2.4$ | 26. $2.9 < c + 7$ | 27. $f + (-4) \geq 1.4$ |
| 28. $z + (-2) > -3.8$ | 29. $b - \frac{3}{4} < 2\frac{1}{2}$ | 30. $g - 1\frac{2}{3} > 2\frac{1}{6}$ |

Solve each inequality. Then graph the solution on a number line.

- | | | |
|------------------------------|------------------------------|----------------------------|
| 31. $n + 4 < 9$ | 32. $t + 7 > 12$ | 33. $p + (-5) > -3$ |
| 34. $-3 + z > 2$ | 35. $-13 \geq x - 8$ | 36. $-32 \geq a + (-5)$ |
| 37. $33 \leq m - (-6)$ | 38. $k + 9 \geq -21$ | 39. $1\frac{1}{4} + b < 3$ |
| 40. $3 \leq \frac{1}{2} + a$ | 41. $4 \geq s - \frac{2}{3}$ | 42. $-\frac{3}{4} < w - 1$ |

43. **TRANSPORTATION** A certain minivan has a maximum carrying capacity of 1100 pounds. If the luggage weighs 120 pounds, what is the maximum weight allowable for passengers?

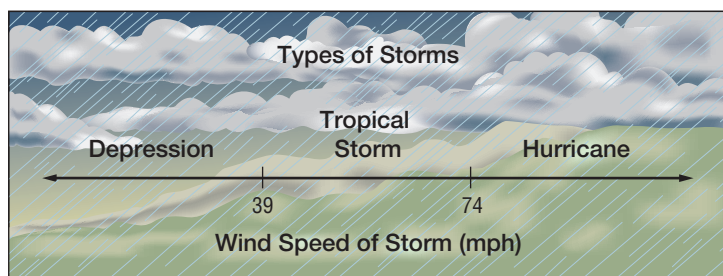
44. **BIOLOGY** Female killer whales usually weigh more than 3000 pounds and are up to 19 feet long. Suppose a female whale is 12 feet long. Write and solve an inequality to find how much longer the whale could grow.

Source: www.seaworld.org



Online Research Data Update Are there any whales in the world that are heavier or longer than the killer whale? Visit www.pre-alg.com/data_update to learn more.

WEATHER For Exercises 45 and 46, use the diagram below.



45. A hurricane has winds that are at least 74 miles per hour. Suppose a tropical storm has winds that are 42 miles per hour. Write and solve an inequality to find how much the winds must increase before the storm becomes a hurricane.
46. A *major storm* has wind speeds that are at least 110 miles per hour. Write and solve an inequality that describes how much greater these wind speeds are than the slowest hurricane.
47. **CRITICAL THINKING** Is it *always*, *sometimes*, or *never* true that $x - 1 < x$? Explain your answer.

Career Choices



Marine Biologist

Marine biologists study the behavior, diseases, and life processes of salt-water organisms.



Online Research

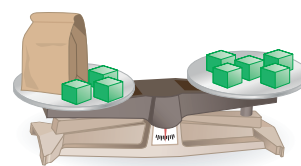
For information about a career as a marine biologist, visit: www.pre-alg.com/careers

48. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How is solving an inequality similar to solving an equation?

Include the following in your answer:

- a description of what would happen if 3 blocks were removed from each side of the scale modeled at the right, and,
- a sentence that compares removing 3 blocks from each side of a scale and subtracting 3 from each side of an inequality.



$$x + 3 > 5$$

Standardized Test Practice

A B C D

49. Which inequality represents a temperature that is equal to or less than 42° ?
- (A) $t \geq 42$ (B) $t > 42$ (C) $t \leq 42$ (D) $t < 42$
50. Trevor has \$25 to spend on a T-shirt and shorts for gym class. The shorts cost \$14. Based on the inequality $14 + t \leq 25$, where t represents the cost of the T-shirt, what is the most Trevor can spend on the T-shirt?
- (A) \$9 (B) \$10.99 (C) \$11 (D) \$11.50

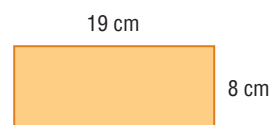
Maintain Your Skills

Mixed Review ALGEBRA For the given value, state whether each inequality is *true* or *false*.
(Lesson 7-3)

51. $x - 5 > 4$, $x = 9$ 52. $9 + a \leq 3$, $a = -7$
53. $\frac{x}{2} \geq 8$, $x = 4$ 54. $6n < -4$, $n = -1$

55. **GEOMETRY** The perimeter of a rectangle is 24 centimeters. Find the dimensions if the length is 3 more than twice the width. (Lesson 7-2)

56. **GEOMETRY** Find the perimeter and area of the rectangle at the right.
(Lesson 3-7)



ALGEBRA Use the Distributive Property to rewrite each expression.
(Lesson 3-1)

57. $4(2 + 8)$ 58. $-2(n + 6)$
59. $5(x - 3.5)$ 60. $(9 - d)(-3c)$

Find each difference. (Lesson 2-3)

61. $-15 - (-12)$ 62. $8 - (-5)$
63. $-9 - 6$ 64. $27 - 45$

Getting Ready for the Next Lesson

PREREQUISITE SKILL Solve each equation.
(To review *solving equations*, see Lesson 3-4.)

65. $-7x = 14$ 66. $-3y = -27$
67. $5x = -20$ 68. $\frac{d}{-3} = -6$
69. $\frac{c}{-4} = 12$ 70. $\frac{a}{2} = -8$



Solving Inequalities by Multiplying or Dividing

What You'll Learn

- Solve inequalities by multiplying or dividing by a positive number.
- Solve inequalities by multiplying or dividing by a negative number.

How are inequalities used in studying space?

An astronaut in a space suit weighs about 300 pounds on Earth, but only 50 pounds on the moon because of weaker gravity.

$$\underbrace{\text{weight on Earth}}_{300} > \underbrace{\text{weight on moon}}_{50}$$

If the astronaut and space suit each weighed half as much, would the inequality still be true? That is, would the astronaut's weight still be greater on Earth?

- Divide each side of the inequality $300 > 50$ by 2. Is the inequality still true? Explain by using an inequality.
- Would the weight of 5 astronauts be greater on Pluto or on Earth? Explain by using an inequality.

Location	Weight of Astronaut (lb)
Earth	300
Moon	50
Pluto	67
Mars	113
Neptune	407
Jupiter	796



MULTIPLY OR DIVIDE BY A POSITIVE NUMBER The application above demonstrates how you can solve inequalities by using the Multiplication and Division Properties of Inequalities.

Key Concept

Multiplication and Division Properties

- Words** When you multiply or divide each side of an inequality by the same positive number, the inequality remains true.
- Symbols** For all numbers a , b , and c , where $c > 0$,
 - if $a > b$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$.
 - if $a < b$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$.
- Examples**

$2 < 6$	$3 > -9$
$4(2) < 4(6)$	$\frac{3}{3} > \frac{-9}{3}$
$8 < 24$	$1 > -3$

Study Tip

Positive Number

The inequality $c > 0$ means that c is a positive number.

These properties are also true for $a \geq b$ and $a \leq b$.

Example 1 Multiply or Divide by a Positive Number

- a. Solve $8x \leq 40$. Check your solution.

$$8x \leq 40 \quad \text{Write the inequality.}$$

$$\frac{8x}{8} \leq \frac{40}{8} \quad \text{Divide each side by 8.}$$

$$x \leq 5 \quad \text{Simplify.}$$

The solution is $x \leq 5$. You can check this solution by substituting 5 or a number less than 5 into the inequality.

- b. Solve $\frac{d}{2} > 7$. Check your solution.

$$\frac{d}{2} > 7 \quad \text{Write the inequality.}$$

$$2\left(\frac{d}{2}\right) > 2(7) \quad \text{Multiply each side by 2.}$$

$$d > 14 \quad \text{Simplify.}$$

The solution is $d > 14$. You can check this solution by substituting a number greater than 14 into the inequality.

Standardized Test Practice

A B C D

Example 2 Write an Inequality

Multiple-Choice Test Item

Ling earns \$8 per hour in the summer working at the zoo. Which inequality can be used to find how many hours he must work in a week to earn at least \$120?

- (A) $8x < 120$ (B) $8x \leq 120$ (C) $8x > 120$ (D) $8x \geq 120$

Test-Taking Tip

Before taking a standardized test, review the meanings of phrases like *at least* and *at most*.

Read the Test Item

You are to write an inequality to represent a real-world problem.

Solve the Test Item

Let x represent the number of hours worked.

Amount earned per hour	times	number of hours	is at least	amount earned each week.
$\underbrace{\hspace{1.5cm}}$	$\cdot$	$\underbrace{\hspace{1.5cm}}$	$\geq$	$\underbrace{\hspace{1.5cm}}$
8		x		120

The answer is D.

MULTIPLY OR DIVIDE BY A NEGATIVE NUMBER What happens when each side of an inequality is multiplied or divided by a negative number?

$$\begin{aligned} -6 &< 11 \\ -1(-6) &\stackrel{?}{<} -1(11) && \text{Multiply each side by } -1. \\ 6 &\stackrel{?}{<} -11 && \text{This inequality is false.} \end{aligned}$$

$$\begin{aligned} 10 &> 5 \\ \frac{10}{-5} &\stackrel{?}{>} \frac{5}{-5} && \text{Divide each side by } -5. \\ -2 &\stackrel{?}{>} -1 && \text{This inequality is false.} \end{aligned}$$



The inequalities $6 < -11$ and $-2 > -1$ are both false. However, they would both be true if the inequality symbols were reversed. That is, change $<$ to $>$ and change $>$ to $<$.

$$6 > -11 \quad \text{true} \qquad -2 < -1 \quad \text{true}$$

This investigation suggests the following properties.

Study Tip

Negative Number

The inequality $c < 0$ means that c is a negative number.

Key Concept

Multiplication and Division Properties

- **Words** When you multiply or divide each side of an inequality by the same negative number, the inequality symbol must be reversed for the inequality to remain true.
- **Symbols** For all numbers a , b , and c , where $c < 0$,
 1. if $a > b$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$.
 2. if $a < b$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$.
- **Examples**

$7 > 1$		$-4 < 16$
$-2(7) < -2(1)$	Reverse the symbols.	$\frac{-4}{-4} > \frac{16}{-4}$
$-14 < -2$		$1 > -4$

These properties are also true for $a \geq b$ and $a \leq b$.



Concept Check

Explain why it is necessary to reverse the symbol when you multiply each side of an inequality by a negative number.

Example 3 Multiply or Divide by a Negative Number

Solve each inequality and check your solution. Then graph the solution on a number line.

a. $\frac{x}{-3} \leq 4$

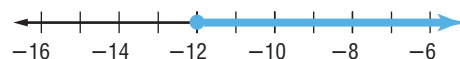
$$\frac{x}{-3} \leq 4 \quad \text{Write the inequality.}$$

$$-3\left(\frac{x}{-3}\right) \geq -3(4) \quad \text{Multiply each side by } -3 \text{ and reverse the symbol.}$$

$$x \geq -12 \quad \text{Check this result.}$$

CHECK You can check your result by replacing x in the original inequality with a number greater than -12 .

Graph the solution, $x \geq -12$.



b. $-7x > -56$

$$-7x > -56 \quad \text{Write the inequality.}$$

$$\frac{-7x}{-7} < \frac{-56}{-7} \quad \text{Divide each side by } -7 \text{ and reverse the symbol.}$$

$$x < 8 \quad \text{Check this result.}$$

Graph the solution, $x < 8$.



Check for Understanding

Concept Check

1. List the steps you would use to solve $\frac{y}{-12} < 6$.
2. **OPEN ENDED** Write an inequality that can be solved using the Division Property of Inequality, where the inequality symbol is *not* reversed.
3. **FIND THE ERROR** Brittany and Tamika each solved $-45 \geq 9k$.

Brittany

$$\begin{aligned} -45 &\geq 9k \\ \frac{-45}{9} &\leq \frac{9k}{9} \\ -5 &\leq k \end{aligned}$$

Tamika

$$\begin{aligned} -45 &\geq 9k \\ \frac{-45}{9} &\geq \frac{9k}{9} \\ -5 &\geq k \end{aligned}$$

Who is correct? Explain your reasoning.

Guided Practice

Solve each inequality and check your solution. Then graph the solution on a number line.

4. $2x < 8$

5. $3x \geq -6$

6. $-4t > -20$

7. $\frac{a}{5} > 10$

8. $-8 > \frac{k}{-0.4}$

9. $\frac{m}{-7} \leq 1.2$

10. $-\frac{s}{3} \leq -3.5$

11. $36 \geq -\frac{1}{2}y$

12. $-273 \geq -13z$

Standardized Test Practice

A B C D

13. **EARNINGS** Julia delivers pizzas on weekends. Her average tip is \$1.50 for each pizza that she delivers. How many pizzas must she deliver to earn at least \$20 in tips?

(A) 10

(B) 13

(C) 14

(D) 20

Practice and Apply

Homework Help

For Exercises

14–21, 32, 33
22–31, 34–37
42, 43

See Examples

1
3
2

Extra Practice
See page 741.

Solve each inequality and check your solution. Then graph the solution on a number line.

14. $4x < 4$

15. $7y > 63$

16. $13a \geq -26$

17. $-15 \leq 5b$

18. $144 < 12d$

19. $15 \geq 3t$

20. $\frac{p}{6} > 5$

21. $7 \geq \frac{h}{14}$

22. $-3m > -33$

23. $-8z \leq -24$

24. $18 > -2g$

25. $-8 \leq -4w$

26. $6 > \frac{x}{-7}$

27. $\frac{r}{-2} < -2$

28. $\frac{y}{-3} < -7$

29. $\frac{k}{-2} < 9$

30. $-6a > -78$

31. $-25t \leq 400$

32. $\frac{y}{4} \geq 2.4$

33. $\frac{n}{5} \leq 0.8$

34. $-5 \leq \frac{c}{-4.5}$

35. $-19 > \frac{y}{-0.3}$

36. $-\frac{1}{3}x \geq -9$

37. $-36 < -\frac{1}{2}b$

38. **SOCCER** Tomás wants to spend less than \$100 for a new soccer ball and shoes. The ball costs \$24.

- a. Write an inequality to represent the amount left for shoes.
- b. What amount can he spend on shoes?



39. **SWIMMING** Nicole swims 40 meters per minute, and she wants to swim at least 2000 meters this morning.
- Write an inequality to represent how long she should swim.
 - How many minutes should she swim?
40. **CRITICAL THINKING** The product of an integer and -7 is less than -84 . Find the least integer that meets this condition.
41. **WRITING IN MATH** Answer the following question that was posed at the beginning of the lesson.

How are inequalities used in studying space?

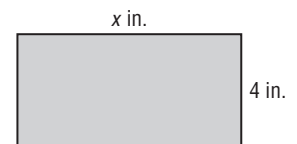
Include the following in your answer:

- inequalities comparing the weight of two astronauts on Mars and on the moon, and
- an explanation of how the Multiplication and Division Properties of Inequality can be used to compare planets' gravities.



42. Which number is *not* a possible length of the rectangle if the area is less than 36 square inches?

- (A) 6 (B) 7
(C) 8 (D) 9



43. **GRID IN** Jessica is putting water into a 20-gallon fish tank using a 2-quart pitcher. How many pitchers of water will she need to fill the tank?

Maintain Your Skills

Mixed Review ALGEBRA Solve each inequality. Check your solution. (Lesson 7-4)

44. $-4 + x > 23$ 45. $c + 18 \leq -2$ 46. $6 > n - 10$

47. Write an inequality for 2 times a number is at most 14. (Lesson 7-3)

Find each product. Write in simplest form. (Lesson 5-3)

48. $\frac{1}{8} \cdot \frac{3}{4}$ 49. $-\frac{3}{7} \cdot \frac{5}{9}$ 50. $2\frac{1}{2} \cdot \left(-\frac{5}{6}\right)$ 51. $\frac{ab}{2} \cdot \frac{4}{bc}$

Getting Ready for the Next Lesson PREREQUISITE SKILL ALGEBRA Solve each equation.
(To review *two-step equations*, see Lesson 3-5.)

52. $2x + 3 = 9$ 53. $5a - 6 = 14$ 54. $3n - 8 = -26$
55. $\frac{t}{3} + 5 = 2$ 56. $\frac{c}{4} - 1 = 4$ 57. $\frac{d}{2} + 3 = 19$

Practice Quiz 2

Lessons 7-3 through 7-5

Graph each inequality on a number line. (Lesson 7-3)

1. $x < -3$ 2. $y \geq 5$

Solve each inequality. Check your solution. (Lessons 7-4 and 7-5)

3. $a - 26 \leq 14$ 4. $46 + k > -8$ 5. $115 \leq -9 + n$ 6. $2.5 > 5r$
7. $\frac{r}{5} < -45$ 8. $-\frac{s}{8} < -80$ 9. $-12g \geq -84$ 10. $5w \geq -2$

7-6

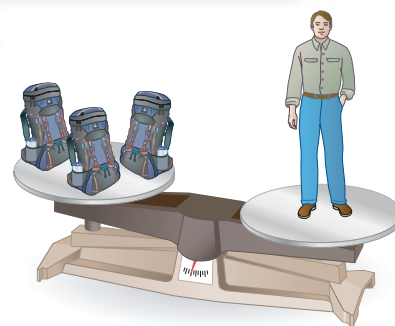
Solving Multi-Step Inequalities

What You'll Learn

- Solve inequalities that involve more than one operation.

How are multi-step inequalities used in backpacking?

Nearly 10 million Americans go backpacking each year. According to a fitness magazine, to avoid injury, three times the weight of your backpack and its contents should be less than your body weight.



- Write an inequality that represents the relationship between body weight and a safe total backpack and contents weight.
- Suppose you weigh 120 pounds and your empty backpack weighs 5 pounds. Write an inequality that represents the maximum weight you can safely carry in the backpack.

INEQUALITIES WITH MORE THAN ONE OPERATION Some inequalities involve more than one operation. To solve the inequality, work backward to undo the operations, just as you did in solving multi-step equations.

Example 1 Solve a Two-Step Inequality

Solve $6x + 15 > 9$ and check your solution. Graph the solution on a number line.

$$6x + 15 > 9$$

Write the inequality.

$$6x + 15 - 15 > 9 - 15$$

Subtract 15 from each side.

$$6x > -6$$

Simplify.

$$x > -1$$

Mentally divide each side by 6.

To check your solution, try 0, a number greater than -1 .

CHECK $6x + 15 > 9$

Write the inequality.

$$6(0) + 15 > 9$$

Replace x with 0.

$$0 + 15 > 9$$

Simplify.

$$15 > 9 \checkmark$$

The solution checks.

Graph the solution, $x > -1$.



Study Tip

Common Misconception

Do not reverse the inequality sign just because there is a negative sign in the inequality. Only reverse the sign when you multiply or divide by a negative number.

Remember that you must reverse the inequality symbol if you multiply or divide each side of an inequality by a negative number.

Example 2 Reverse the Inequality Symbol

Solve $10 - 3a \leq 25 + 2a$ and check your solution. Graph the solution on a number line.

	$10 - 3a \leq 25 + 2a$	Write the inequality.
	$10 - 3a - 2a \leq 25 + 2a - 2a$	Subtract $2a$ from each side.
	$10 - 5a \leq 25$	Simplify.
	$10 - 10 - 5a \leq 25 - 10$	Subtract 10 from each side.
	$-5a \leq 15$	Simplify.
Reverse the inequality in this step.	$\frac{-5a}{-5} \geq \frac{15}{-5}$	Divide each side by -5 and change $\leq$ to $\geq$.
	$a \geq -3$	Simplify.

CHECK

$10 - 3a \leq 25 + 2a$	Try -2 , a number greater than -3 .
$10 - 3(-2) \stackrel{?}{\leq} 25 + 2(-2)$	Replace a with -2 .
$10 + 6 \stackrel{?}{\leq} 25 - 4$	Simplify.
$16 \leq 21 \checkmark$	The solution checks.

Graph the solution, $a \geq -3$.



More About...



Backpacking

One of the most popular treks for backpackers is Yosemite National Park in California's High Sierra. It is possible to hike there for two weeks and never see a road!

Source: www.gorp.com

Example 3 Inequalities with Grouping Symbols

BACKPACKING A person weighing 126 pounds has a 6-pound backpack. Refer to the application at the beginning of page 355. What is the maximum weight for the contents of the pack?

Let c represent the weight of the contents of the pack.

Words	$\frac{3}{3}$	times	weight of pack and contents	should be less than	body weight.
Inequality	3	$\cdot$	$(6 + c)$	$<$	126

Solve the inequality.

$3(6 + c) < 126$	Write the inequality.
$18 + 3c < 126$	Use the Distributive Property.
$18 + 3c - 18 < 126 - 18$	Subtract 18 from each side.
$3c < 108$	Simplify.
$\frac{3c}{3} < \frac{108}{3}$	Divide each side by 3.
$c < 36$	Simplify.

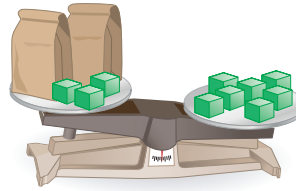
The weight of the contents should be less than 36 pounds.

Concept Check How do you solve an inequality with more than one operation?

Check for Understanding

Concept Check

- OPEN ENDED** Explain how to check the solution of an inequality.
- Write** an inequality for the model at the right. Then solve the inequality.



- FIND THE ERROR** Jerome and Ryan are beginning to solve $2(2y + 3) > y + 1$.

Jerome

$$\begin{aligned} 2(2y + 3) &> y + 1 \\ 4y + 6 &> y + 1 \end{aligned}$$

Ryan

$$\begin{aligned} 2(2y + 3) &> y + 1 \\ 4y + 3 &> y + 1 \end{aligned}$$

Who is correct? Explain your reasoning.

Guided Practice

Solve each inequality and check your solution. Then graph the solution on a number line.

- $3x + 4 \leq 31$
- $y + 1 \geq 4y + 4$
- $-6.1n \geq 3.9n + 5$
- $-3(b - 1) > 18$
- $2n + 5 > 11 - n$
- $16 - 2c < 14$
- $-4 \leq \frac{x}{4} - 6$
- $\frac{1}{2}(2d + 3) < -8$

Application

- MONEY** Dante's telephone company charges \$10 a month plus \$0.05 for every minute or part of a minute. Dante wants his monthly bill to be under \$30. What is the greatest number of minutes he can talk?

Practice and Apply

Homework Help

For Exercises	See Examples
13–18, 25, 26	1
19, 20, 27, 28	2
21–24, 29, 30, 33–36	3

Extra Practice
See page 741.

Solve each inequality and check your solution. Then graph the solution on a number line.

- $2x + 8 > 24$
- $3 + 4c > -13$
- $3x - 2 > 10 - x$
- $4 - 3k \leq 19$
- $2(n + 3) < -4$
- $8 + 3b \leq 2(9 - b)$
- $2 + 0.3y \geq 11$
- $\frac{1}{2}(6 - c) > 5$
- $4y - 1 \leq 5$
- $9 + 2p \leq 15$
- $c - 1 < 3c + 5$
- $16 - 4n > 20$
- $2(d + 1) > 16$
- $\frac{m}{2} + 9 \geq 5$
- $0.5a - 1.4 \leq 2.1$
- $\frac{2}{3}(9 - x) < 3$
- Four times a number less 6 is greater than two times the same number plus 8. For what number or numbers is this true?
- One-half of the sum of a number and 6 is less than 25. What is the number?



Solve each inequality and check your solution. Graph the solution on a number line.

31. $1.3n + 6.7 \geq 3.1n - 1.4$

32. $-5a + 3 > 3a + 23$

33. $-5(t + 4) \geq 3(t - 4)$

34. $8x - (x - 5) > x + 17$

35. $\frac{c + 8}{4} < \frac{5 - c}{9}$

36. $\frac{2(n + 1)}{7} \geq \frac{n + 4}{5}$

For Exercises 37–40, write and solve an inequality.

37. **CANDY** You buy some candy bars at \$0.55 each and one newspaper for \$0.35. How many candy bars can you buy with \$2?

38. **SCHOOL** Nate has scores of 85, 91, 89, and 93 on four tests. What is the least number of points he can get on the fifth test to have an average of at least 90?

39. **SALES** You earn \$2.00 for every magazine subscription you sell plus a salary of \$10 each week. How many subscriptions do you need to sell each week to earn at least \$40 each week?

40. **REAL ESTATE** A real estate agent receives a monthly salary of \$1500 plus a 4% commission on every home sold. For what amount of monthly sales will the agent earn at least \$5000?

41. **CAR RENTAL** The costs for renting a car from Able Car Rental and from Baker Car Rental are shown in the table. For what mileage does Baker have the better deal? Use the inequality $30 + 0.05x > 20 + 0.10x$. Explain why this inequality works.

Rental Car Costs		
	Cost per Day	Cost per Mile
Able Car Rental	\$30	\$0.05
Baker Car Rental	\$20	\$0.10



42. **HIKING** You hike along the Appalachian Trail at 3 miles per hour. You stop for one hour for lunch. You want to walk at least 18 miles. How many hours should you expect to spend on the trail?

43. **PHONE SERVICES** Miko was asked by FoneCom to sign up for their service at \$15 per month plus \$0.10 per minute. Miko currently has BestPhone service at \$20 per month plus \$0.05 per minute. Miko figures that her monthly bill would be more with FoneCom. For how many minutes per month does she use the phone?

44. **FUND-RAISERS** The Booster Club sells football programs for \$1 each. The costs to make the programs are \$60 for page layout plus \$0.20 for printing each program. If they print 400 programs, how many programs must the Club sell to make at least \$200 profit?

45. **CRITICAL THINKING** Assume that k is an integer. Solve the inequality $10 - 2|k| > 4$.



Career Choices

Real Estate Agent

Real estate agents help people with one of the most important financial decisions of their lives—buying and selling a home. All states require prospective agents to pass a written test, which usually contains a section on mathematics.

Online Research

For information about a career as a real estate agent, visit: www.pre-alg.com/careers

46. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are multi-step inequalities used in backpacking?

Include the following in your answer:

- an explanation of what multi-step inequalities are, and
- a solution of the inequality you wrote for part **b** on page 355.



47. Which inequality represents *five more than twice a number is less than ten*?

- (A) $(5 + 2)n < 10$ (B) $2n - 5 < 10$
(C) $10 < 2n + 5$ (D) $5 + 2n < 10$

48. Enola's scores on the first five science tests are shown in the table. Which inequality represents the score she must receive on the sixth test to have an average score of more than 88?

Test	Score
1	85
2	84
3	90
4	95
5	88

- (A) $s \geq 86$ (B) $s \leq 88$
(C) $s < 88$ (D) $s > 86$

Extending the Lesson

49. The sum of three times a number and 5 lies between -10 and 8 . Solve the *compound inequality* $-10 < 3x + 5 < 8$ to find the solution(s). (Hint: Any operation must be done to all three parts of the inequality.)

Maintain Your Skills

Mixed Review ALGEBRA Solve each inequality. Check your solution. (Lessons 7-4 and 7-5)

50. $20 < -9 + k$ 51. $22 \leq -15 + y$
52. $6x < -27$ 53. $-5n \geq -25$
54. $\frac{n}{-4} \leq -11$ 55. $\frac{a}{-3} > 6.2$

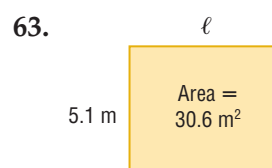
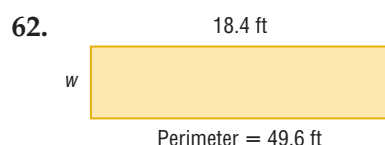
56. If 12 of the 20 students in a class are boys, what percent are boys?
(Lesson 6-5)

57. Write $\frac{1}{200}$ as a percent. (Lesson 6-4)

Express each ratio as a unit rate. (Lesson 6-1)

58. \$5 for 2 loaves of bread 59. 200 miles on 12 gallons
60. 24 meters in 4 seconds 61. 9 monthly issues for \$11.25

GEOMETRY Find the missing dimension in each rectangle. (Lesson 3-7)



Study Guide and Review

Vocabulary and Concept Check

identity (p. 336)

inequality (p. 340)

null or empty set (p. 336)

Determine whether each statement is *true* or *false*. If false, replace the underlined word or number to make a true statement.

- When an equation has no solution, the solution set is the null set.
- The inequality $n + 8 - 8 \geq 14 - 8$ demonstrates the Subtraction Property of Inequality.
- An equation that is true for every value of the variable is called an inequality.
- The inequality $\frac{x}{4}(4) < 7(4)$ demonstrates the Division Property of Inequality.
- A mathematical sentence that contains $<$ or $>$ is called an empty set.
- When the final result in solving an equation is $5 = -8$, the solution set is the null set.
- When the final result in solving an equation is $x = x$, the solution set is all numbers.
- To solve $3(x + 5) = 10$, use the Distributive Property to remove the parentheses.
- The symbol $\geq$ means is less than or equal to.
- A closed circle on a number line indicates that the point is included in the solution set for the inequality.

Lesson-by-Lesson Review

7-1

See pages
330–333.

Solving Equations with Variables on Each Side

Concept Summary

- Use the Addition or Subtraction Property of Equality to isolate the variables on one side of an equation.

Example

Solve $7x = 3x - 12$.

$$7x = 3x - 12$$

Write the equation.

$$7x - 3x = 3x - 3x - 12$$

Subtract $3x$ from each side.

$$4x = -12$$

Simplify.

$$x = -3$$

Mentally divide each side by 4.

Exercises Solve each equation. Check your solution.

See Example 1 on page 330.

11. $2a + 9 = 5a$

12. $x - 4 = 3x$

13. $3y - 8 = y$

14. $19t = 26 + 6t$

15. $2 + 7n = 8 + n$

16. $5 + 6t = 10t - 7$

17. $-r + 4.2 = 8.8r + 14$

18. $12 + 1.5x = 9x$

19. $5b - 1 = 2.5b - 4$

7-2 Solving Equations with Grouping SymbolsSee pages
334–338.**Concept Summary**

- Use the Distributive Property to remove the grouping symbols.

ExampleSolve $2(x + 3) = 15$.

$$2(x + 3) = 15 \quad \text{Write the equation.}$$

$$2x + 6 = 15 \quad \text{Use the Distributive Property.}$$

$$2x = 9 \quad \text{Subtract 6 from each side and simplify.}$$

$$x = 4.5 \quad \text{Divide each side by 2 and simplify.}$$

Exercises Solve each equation. See Examples 1–4 on pages 334–336.

20. $4(k + 1) = 16$

21. $2(n - 5) = 8$

22. $11 + 2q = 2(q + 4)$

23. $\frac{1}{2}(t + 8) = \frac{3}{4}t$

24. $4(x + 2.5) = 3(7 + x)$

25. $3(x + 1) - 5 = 3x - 2$

7-3 InequalitiesSee pages
340–344.**Concept Summary**

- An inequality is a mathematical sentence that contains $<$, $>$, $\leq$, or $\geq$.

ExampleState whether $n + 11 < 14$ is *true* or *false* for $n = 5$.

$$n + 11 < 14 \quad \text{Write the inequality.}$$

$$5 + 11 \stackrel{?}{<} 14 \quad \text{Replace } n \text{ with } 5.$$

$$16 \nless 14 \quad \text{Simplify.} \quad \text{The sentence is false.}$$

Exercises For the given value, state whether each inequality is *true* or *false*.
See Example 4 on page 341.

26. $x + 4 > 9$, $x = 12$

27. $15 \leq 5n$, $n = 3$

28. $3n + 1 \geq 14$, $n = 4$

7-4 Solving Inequalities by Adding or SubtractingSee pages
345–349.**Concept Summary**

- Solving an inequality means finding values for the variable that make the inequality true.

ExampleSolve $x - 7 < 3$. Graph the solution on a number line.

$$x - 7 < 3 \quad \text{Write the inequality.}$$

$$x - 7 + 7 < 3 + 7 \quad \text{Add 7 to each side.}$$

$$x < 10 \quad \text{Simplify.}$$

**Exercises** Solve each inequality. Graph the solution on a number line.
See Examples 1 and 2 on page 346.

29. $b - 9 \geq 8$

30. $x + 4.8 \leq 2$

31. $t + \frac{1}{2} < 4$

- Extra Practice, see pages 739–741.
- Mixed Problem Solving, see page 764.

7-5

See pages 350–354.

Solving Inequalities by Multiplying or Dividing**Concept Summary**

- When you multiply or divide each side of an inequality by a positive number, the inequality symbol remains the same.
- When you multiply or divide each side of an inequality by a negative number, the inequality symbol must be reversed.

Examples**1** Solve $\frac{a}{3} > 2$. Graph the solution on a number line.

$$\frac{a}{3} > 2$$

Write the inequality.

$$3\left(\frac{a}{3}\right) > 3(2)$$

Multiply each side by 3.

$$a > 6$$

Simplify.

The solution is $a > 6$.**2** Solve $-2n \geq 26$. Graph the solution on a number line.

$$-2n \geq 26$$

Write the inequality.

$$\frac{-2n}{-2} \leq \frac{26}{-2}$$

Divide each side by -2 and reverse the symbol.

$$n \leq -13$$

Simplify.

The solution is $n \leq -13$.**Exercises** Solve each inequality. Graph the solution on a number line.

See Examples 1 and 3 on pages 351 and 352.

32. $\frac{n}{4} < 6$

33. $\frac{k}{1.7} \leq 3$

34. $0.5x > 3.2$

35. $-56 \geq 8y$

36. $9 > \frac{x}{-4}$

37. $-\frac{5}{6}a \leq 2$

7-6

See pages 355–359.

Solving Multi-Step Inequalities**Concept Summary**

- To solve an inequality that involves more than one operation, work backward to undo the operations.

ExampleSolve $4t + 7 < -5$.

$$4t + 7 < -5$$

Write the inequality.

$$4t + 7 - 7 < -5 - 7$$

Subtract 7 from each side.

$$4t < -12$$

Simplify.

$$t < -3$$

Mentally divide each side by 4.

The solution is $t < -3$.**Exercises** Solve each inequality. See Examples 1 and 2 on pages 355 and 356.

38. $2x - 3 > 19$

39. $5n + 4 \leq 24$

40. $6 \geq \frac{r}{7} + 1$

41. $\frac{t}{-2} + 15 < 21$

42. $3(a + 8.4) > 30$

43. $\frac{1}{4} + 2b < 13 + 5b$

Vocabulary and Concepts

1. **State** when to use an open circle and a closed circle in graphing an inequality.
2. **Describe** what happens to an inequality when each side is multiplied or divided by a negative number.

Skills and Applications

Solve each equation. Check your solution.

3. $7x - 3 = 10x$

4. $p - 9 = 4p$

5. $2.3n - 8 = 1.2n + 3$

6. $\frac{3}{8}y - 5 = \frac{5}{8}y - 3$

7. $6 + 2(x - 4) = 2(x - 1)$

8. $2(6 - 5d) = 8$

9. $8(2x - 9) = 4(5 + 4x)$

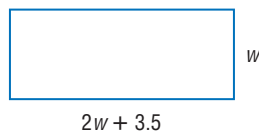
10. $4(a + 3) = 20$

11. $\frac{1}{3}(9b + 1) = b - 1$

Define a variable and write an equation to find each number. Then solve.

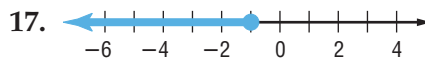
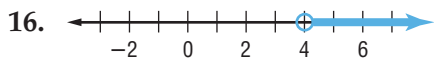
12. Eight more than three times a number equals four less than the number.
13. The product of a number and five is twelve more than the number.

14. **GEOMETRY** The perimeter of the rectangle is 22 feet. Find the dimensions of the rectangle.



15. **SHOPPING** The cost of purchasing four shirts is at least \$120. Write an inequality to describe this situation.

Write the inequality for each graph.



Solve each inequality and check your solution. Then graph the solution on a number line.

18. $-4 \geq p - 2$

19. $3x \geq 15$

20. $-42 < -0.6x$

21. $c - 3 \leq 4c + 9$

22. $7(3 - 2b) > 5b + 2$

23. $\frac{1}{2}(a + 4) > \frac{1}{4}(a - 8)$

24. **SALES** The Cookie Factory has a fixed cost of \$300 per month plus \$0.45 for each cookie sold. Each cookie sells for \$0.95. How many cookies must be sold during one month for the profit to be at least \$100?

25. **STANDARDIZED TEST PRACTICE** Danny earns \$6.50 per hour working at a movie theater. Which inequality can be used to find how many hours he must work each week to earn at least \$100 a week?

(A) $6.50h < 100$

(B) $6.50h > 100$

(C) $6.50h \leq 100$

(D) $6.50h \geq 100$



Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. A delivery service calculates the cost c of shipping a package with the equation $c = 0.30w + 6$, where w is the weight of the package in pounds. Your package weighs at least 8 pounds. What is the lowest possible cost to ship your package? (Lesson 3-7)

(A) \$6.30 (B) \$8.40
(C) \$14.30 (D) \$30.00

2. The school band traveled on two buses with 36 students on each bus. At a lunch stop, two-thirds of the students on the first bus ate at Hamburger Haven, and the others ate at Taco Time. Three-fourths of the students on the second bus ate at Hamburger Haven. How many students in all ate at Hamburger Haven? (Lesson 5-3)

(A) 24 (B) 48
(C) 51 (D) 102

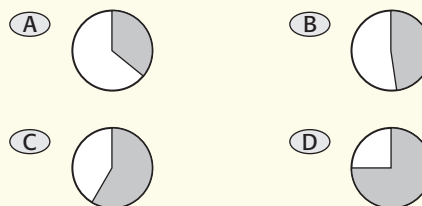
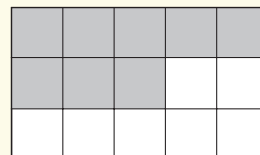
3. Shanté earned \$360 last summer. She spent $\frac{5}{9}$ of her earnings. How much money did she have left? (Lesson 5-3)

(A) \$40 (B) \$160
(C) \$200 (D) \$320

4. While exercising, Luke's heart is beating at 170 beats per minute. If he maintains this rate, about how many times will his heart beat in one hour? (Lesson 6-1)

(A) 1000 (B) 5000
(C) 10,000 (D) 100,000

5. Which of the circles has approximately the same fractional part shaded as that of the rectangle below? (Lesson 6-4)



6. Which of the following statements is true? (Lesson 6-4)

(A) $0.4 > 40\%$ (B) $0.04 = 40\%$
(C) $40\% \leq 0.04$ (D) $40\% > 0.04$

7. A survey at the MegaMall showed that 15% of visitors attend a movie while at the mall. If 8700 people are at the mall, how many of these visitors are likely to attend a movie there? (Lesson 6-7)

(A) 580 (B) 870
(C) 1305 (D) 5800

8. Last year there were 1536 students at Cortéz Middle School. This year there are 5% more students. About how many students attend Cortéz this year? (Lesson 6-9)

(A) 1550 (B) 1600
(C) 1650 (D) 1700

9. If $5(x + 2) = 40$, what is the value of x ? (Lesson 7-2)

(A) 4 (B) 6
(C) 8 (D) 10

10. Which of the following inequalities is equivalent to $\frac{x}{3} < 5$? (Lesson 7-5)

(A) $x < \frac{5}{3}$ (B) $x < 2$
(C) $x > 2$ (D) $x < 15$

Test-Taking Tip

Questions 9 and 10

When an item requires you to solve an equation or inequality, plug in your solution to the original problem in order to check your answer.

Part 2 Short Response/Grid In

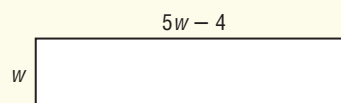
Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

11. What is the value of $16 + 18 \div 2 \times 3$? (Lesson 1-2)
12. In 5 days, the stock market fell 25 points. What integer expresses the average change in the stock market per day? (Lesson 2-5)
13. Write $\frac{1}{5 \times 5 \times 5 \times 5}$ using a negative exponent. (Lesson 4-7)
14. Find $\frac{5}{12} - \frac{3}{8}$. (Lesson 5-4)
15. The high temperatures for five days in April are shown in the table below. What was the median high temperature? (Lesson 5-8)

High Temperatures	
Monday	45°
Tuesday	62°
Wednesday	57°
Thursday	41°
Friday	53°

16. To mix a certain color of paint, Alexis combines 5 liters of white paint, 2 liters of red paint, and 1 liter of blue paint. What is the ratio of white paint to the total amount of paint? (Lesson 6-1)
17. A box contains 42 pencils. Some are yellow, some are red, some are white, and some are black. If the probability of randomly selecting a red pencil is $\frac{3}{7}$, how many red pencils are in the box? (Lesson 6-2)
18. A city received a federal grant of \$350 million to build a light-rail system which actually cost \$625 million. What percent of the total cost was paid for with the Federal grant? Round to the nearest percent. (Lesson 6-7)

19. A leather jacket is on sale for 40% off the original price. The sale price is \$64 less than the original price. What was the original price of the jacket? (Lesson 6-8)
20. Find x if $8x - 12 = 5x + 6$. (Lesson 7-1)
21. Find the width w of the rectangle below if its perimeter is 88 meters. (Lesson 7-2)



22. Dakota earns \$8 per hour working at a landscaping company and wants to earn at least \$1200 this summer. What is the minimum number of hours he will have to work? (Lesson 7-5)

Part 3 Extended Response

Record your answers on a sheet of paper. Show your work.

23. At a post office, a customer bought an equal number of the following stamps: 1¢, 22¢, and 34¢. She also mailed a package that required \$2.80 in postage. The total bill was \$14. (Lesson 3-6)
 - a. Write an equation that describes this situation.
 - b. What does the variable in your equation represent?
 - c. Solve the equation. Show your work.
 - d. Write a sentence describing what the solution represents.
24. A magazine publisher collected data on subscription renewals and found that each year 3 out of 50 subscribers do *not* renew. The magazine currently has 24,000 subscribers. (Lesson 6-6)
 - a. What percent of subscribers do *not* renew their subscriptions?
 - b. What percent of subscribers per year do renew their subscriptions?
 - c. How many subscribers will likely renew their subscriptions this year?

