Chapter

8 Functions and Graphing

What You'll Learn

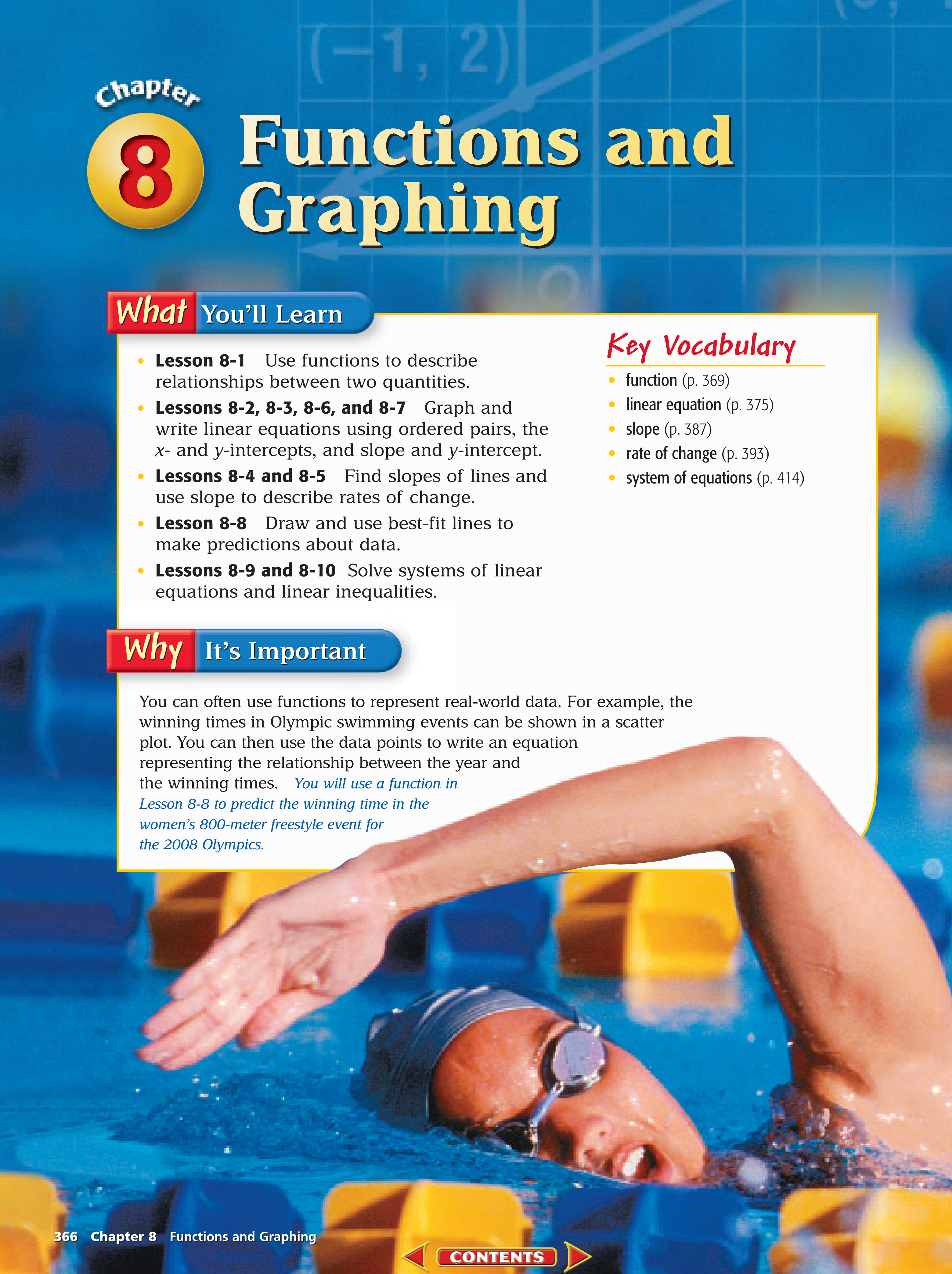
- **Lesson 8-1** Use functions to describe relationships between two quantities.
- **Lessons 8-2, 8-3, 8-6, and 8-7** Graph and write linear equations using ordered pairs, the x- and y-intercepts, and slope and y-intercept.
- **Lessons 8-4 and 8-5** Find slopes of lines and use slope to describe rates of change.
- **Lesson 8-8** Draw and use best-fit lines to make predictions about data.
- **Lessons 8-9 and 8-10** Solve systems of linear equations and linear inequalities.

Key Vocabulary

- function (p. 369)
- linear equation (p. 375)
- slope (p. 387)
- rate of change (p. 393)
- system of equations (p. 414)

Why It's Important

You can often use functions to represent real-world data. For example, the winning times in Olympic swimming events can be shown in a scatter plot. You can then use the data points to write an equation representing the relationship between the year and the winning times. *You will use a function in Lesson 8-8 to predict the winning time in the women's 800-meter freestyle event for the 2008 Olympics.*



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Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 8.

For Lesson 8-1 Relations

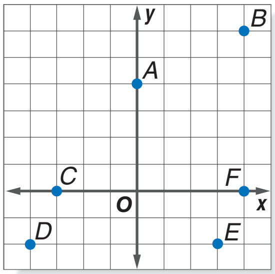
Express each relation as a table. Then determine the domain and range. *(For review, see Lesson 1-6.)*

1. $\{(0, 4), (-3, 3)\}$
2. $\{(-5, 11), (2, 1)\}$
3. $\{(6, 8), (7, 10), (8, 12)\}$
4. $\{(1, -9), (5, 12), (-3, -10)\}$
5. $\{(-8, 5), (7, -1), (6, 1), (1, -2)\}$

For Lesson 8-3 The Coordinate System

Use the coordinate grid to name the point for each ordered pair. *(For review, see Lesson 2-6.)*

6. $(-3, 0)$
7. $(3, -2)$
8. $(-4, -2)$
9. $(0, 4)$
10. $(4, 6)$
11. $(4, 0)$



For Lesson 8-10 Inequalities

For the given value, state whether the inequality is *true* or *false*. *(For review, see Lesson 7-3.)*

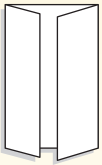
12. $8y \geq 25, y = 4$
13. $18 < t + 12, t = 10$
14. $n - 15 > 7, n = 20$
15. $5 \geq 2x + 3, x = 1$
16. $12 \leq \frac{2}{3}n, n = 9$
17. $\frac{1}{2}x - 5 < 0, x = 8$

FOLDABLES™ Study Organizer

Functions Make this Foldable to help you organize your notes. Begin with an 11" × 17" sheet of paper.

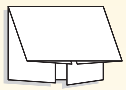
Step 1 Fold

Fold the short sides so they meet in the middle.



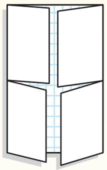
Step 2 Fold Again

Fold the top to the bottom.



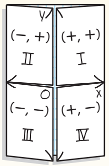
Step 3 Cut

Open. Cut along the second fold to make four tabs. Staple a sheet of grid paper inside.



Step 4 Label

Add axes as shown. Label the quadrants on the tabs.



Reading and Writing As you read and study the chapter, draw examples of functions on the grid paper and write notes under the tabs.

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Algebra Activity

A Preview of Lesson 8-1

Input and Output

In a *function*, there is a relationship between two quantities or sets of numbers. You start with an input value, apply a function rule of one or more operations, and get an output value. In this way, each input is assigned exactly one output.

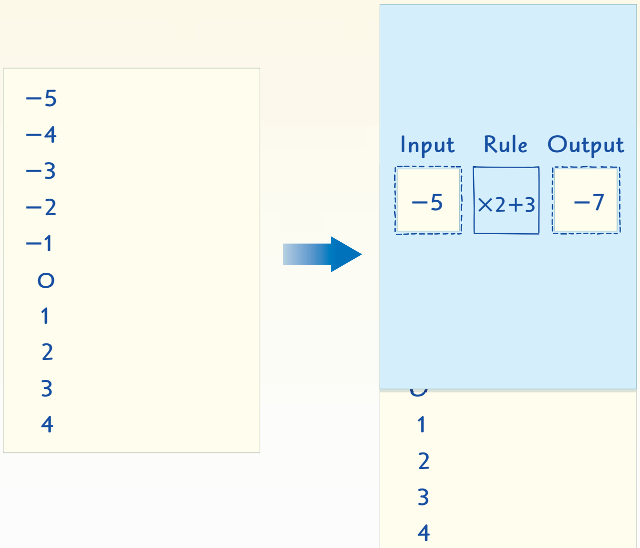
Collect the Data

Step 1 To make a *function machine*, draw three squares in the middle of a 3-by-5-inch index card, shown here in blue.

Step 2 Cut out the square on the left and the square on the right. Label the left "window" INPUT and the right "window" OUTPUT.

Step 3 Write a rule such as "$\times 2 + 3$" in the center square.

Step 4 On another index card, list the integers from -5 to 4 in a column close to the left edge.



Step 5 Place the function machine over the number column so that -5 is in the left window.

Step 6 Apply the rule to the input number. The output is $-5 \times 2 + 3$, or -7. Write -7 in the right window.

Make a Conjecture

1. Slide the function machine down so that the input is -4. Find the output and write the number in the right window. Continue this process for the remaining inputs.

2. Suppose x represents the input and y represents the output. Write an algebraic equation that represents what the function machine does.

3. Explain how you could find the input if you are given a rule and the corresponding output.

4. Determine whether the following statement is *true* or *false*. Explain.

The input values depend on the output values.

5. Write an equation that describes the relationship between the input value x and output value y in each table.

Input	Output
−1	−2
0	0
1	2
3	6

Input	Output
−2	2
−1	3
0	4
1	5

Extend the Activity

6. Write your own rule and use it to make a table of inputs and outputs. Exchange your table of values with another student. Use the table to determine each other's rule.

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8-1 Functions

What You'll Learn

- Determine whether relations are functions.
- Use functions to describe relationships between two quantities.

Vocabulary

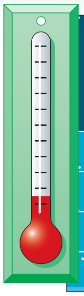
- function
- vertical line test

How can the relationship between actual temperatures and windchill temperatures be a function?

The table compares actual temperatures and windchill temperatures when the wind is blowing at 10 miles per hour.

a. On grid paper, graph the temperatures as ordered pairs (actual, windchill).

b. Describe the relationship between the two temperature scales.



Actual Temperature (°F)	Windchill Temperature (°F)
–10	–34
0	–22
10	–9
20	3

Source: *The World Almanac*

c. When the actual temperature is −20°F, which is the best estimate for the windchill temperature: −46°F, −28°F, or 0°F? Explain.

RELATIONS AND FUNCTIONS Recall that a relation is a set of ordered pairs. A **function** is a special relation in which each member of the domain is paired with *exactly* one member in the range.

Study Tip

Look Back
To review **relations**, **domain**, and **range**, see Lesson 1-6.

Relation	Diagram	Is the Relation a Function?
{(−10, −34), (0, −22), (10, −9), (20, 3)}	domain (*x*) range (*y*) −10 → −34 0 → −22 10 → −9 20 → 3	Yes, because each domain value is paired with exactly one range value.
{(−10, −34), (−10, −22), (10, −9), (20, 3)}	domain (*x*) range (*y*) −10 → −34 ↘ −22 10 → −9 20 → 3	No, because −10 in the domain is paired with two range values, −34 and −22.

Since functions are relations, they can be represented using ordered pairs, tables, or graphs.

Example 1 Ordered Pairs and Tables as Functions

Determine whether each relation is a function. Explain.

a. **{(−3, 1), (−2, 4), (−1, 7), (0, 10), (1, 13)}**

This relation is a function because each element of the domain is paired with exactly one element of the range.

b.

x	5	3	2	0	−4	−6
y	1	3	1	3	−2	2

This is a function because for each element of the domain, there is only one corresponding element in the range.

www.pre-alg.com/extra_examples

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Reading Math

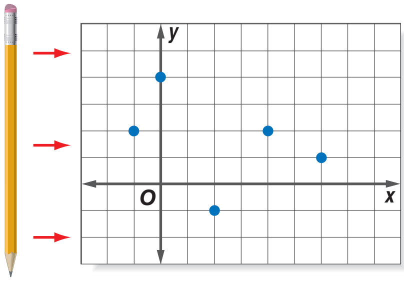
Function

Everyday Meaning: a relationship in which one quality or trait depends on another. Height is a function of age.

Math Meaning: a relationship in which a range value depends on a domain value. *y* is a function of *x*.

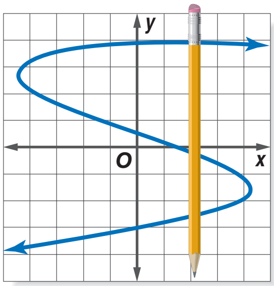
Another way to determine whether a relation is a function is to use the **vertical line test**. Use a pencil or straightedge to represent a vertical line.

Place the pencil at the left of the graph. Move it to the right across the graph. If, for each value of x in the domain, it passes through no more than one point on the graph, then the graph represents a function.



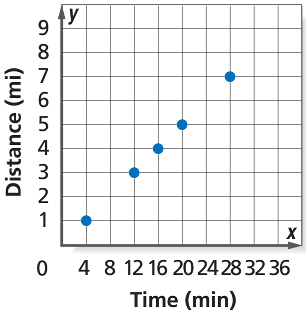
Example 2 Use a Graph to Identify Functions

Determine whether the graph at the right is a function. Explain your answer.



The graph represents a relation that is *not* a function because it does not pass the vertical line test. By examining the graph, you can see that when $x = 2$, there are three different y values.

DESCRIBE RELATIONSHIPS A function describes the relationship between two quantities such as time and distance. For example, the distance you travel on a bike depends on how long you ride the bike. In other words, *distance is a function of time*.



Example 3 Use a Function to Describe Data

SCUBA DIVING **The table shows the water pressure as a scuba diver descends.**

Depth (ft)	Water Pressure (lb/ft^3)
0	0
1	62.4
2	124.8
3	187.2
4	249.6
5	312.0

Source: www.infoplease.com

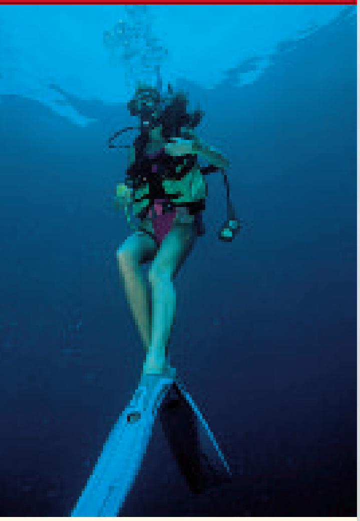
a. Do these data represent a function? Explain.

This relation is a function because at each depth, there is only one measure of pressure.

b. Describe how water pressure is related to depth.

Water pressure depends on the depth. As the depth increases, the pressure increases.

More About. . .



Scuba Diving

To prevent decompression sickness, or the "bends," it is recommended that divers ascend to the surface no faster than 30 feet per minute.

Source: www.mtsinai.org

✓ Concept Check In Example 3, what is the domain and what is the range?

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Check for Understanding

Concept Check

1. **Describe** three ways to represent a function. Show an example of each.

2. **Describe** two methods for determining whether a relation is a function.

3. **OPEN ENDED** Draw the graph of a relation that is not a function. Explain why it is not a function.

Guided Practice

Determine whether each relation is a function. Explain.

4. {(13, 5), (−4, 12), (6, 0), (13, 10)}

5. {(9.2, 7), (9.4, 11), (9.5, 9.5), (9.8, 8)}

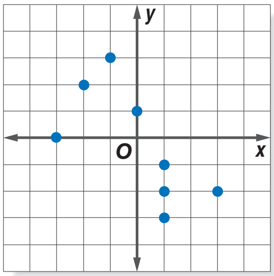
6.

Domain	Range
−3	3
−1	−2
0	5
1	−4
2	3

7.

x	y
5	4
2	8
−7	9
2	12
5	14

8.



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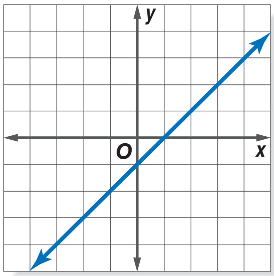


Application

WEATHER **For Exercises 10 and 11, use the table that shows how various wind speeds affect the actual temperature of 30°F.**

10. Do the data represent a function? Explain.

11. Describe how windchill temperatures are related to wind speed.

Wind Speed (mph)	Windchill Temperature (°F)
0	30
10	16
20	4
30	−2
40	−5

Source: *The World Almanac*

Practice and Apply

Homework Help

For Exercises	See Examples
12–19	1
20–23	2
24–27	3

Extra Practice
See page 741.

Determine whether each relation is a function. Explain.

12. {(−1, 6), (4, 2), (2, 36), (1, 6)}

13. {(−2, 3), (4, 7), (24, −6), (5, 4)}

14. {(9, 18), (0, 36), (6, 21), (6, 22)}

15. {(5, −4), (−2, 3), (5, −1), (2, 3)}

16.

Domain	Range
−4	−2
−2	1
0	2
3	1

17.

Domain	Range
−1	5
−2	5
−2	1
−6	1

Determine whether each relation is a function. Explain.

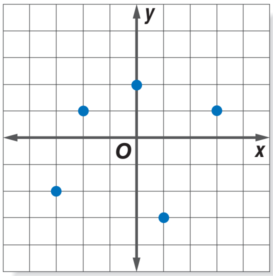
18.

x	y
−7	2
0	4
11	6
11	8
0	10

19.

x	y
14	5
15	10
16	15
17	20
18	25

20.



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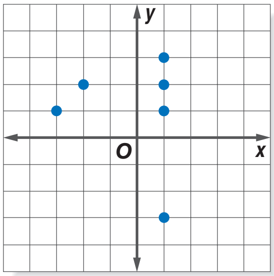

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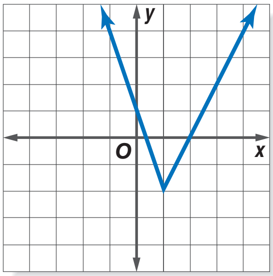

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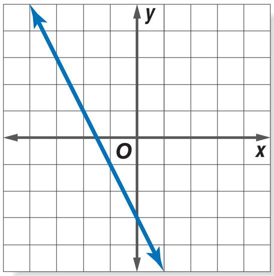
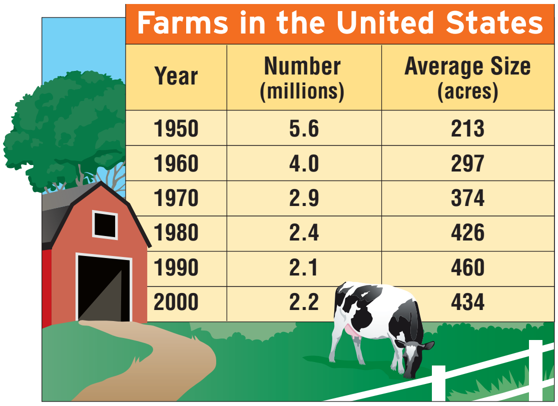


Study Tip

Trends

There may be general trends in sets of data. However, not every data point may follow the trend exactly.

FARMING For Exercises 24–27, use the table that shows the number and size of farms in the United States every decade from 1950 to 2000.



Farms in the United States

Year	Number (millions)	Average Size (acres)
1950	5.6	213
1960	4.0	297
1970	2.9	374
1980	2.4	426
1990	2.1	460
2000	2.2	434

Source: *The Wall Street Journal Almanac*

24. Is the relation (year, number of farms) a function? Explain.

25. Describe how the number of farms is related to the year.

26. Is the relation (year, average size of farms) a function? Explain.

27. Describe how the average size of farms is related to the year.

MEASUREMENTS For Exercises 28 and 29, use the data in the table.

Name	Foot Length (cm)	Height (cm)
Rosa	24	163
Tanner	28	182
Enrico	25	163
Jahad	24	168
Abbi	22	150
Cory	26	172

28. Do the data represent a function? Explain.

29. Is there any relation between foot length and height?

Tell whether each statement is *always*, *sometimes*, or *never* true. Explain.

30. A function is a relation.

31. A relation is a function.

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32. CRITICAL THINKING The *inverse* of any relation is obtained by switching the coordinates in each ordered pair of the relation.

a. Determine whether the inverse of the relation {(4, 0), (5, 1), (6, 2), (6, 3)} is a function.

b. Is the inverse of a function *always*, *sometimes*, or *never* a function? Give an example to explain your reasoning.

33. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How can the relationship between actual temperatures and windchill temperatures be a function?

Include the following in your answer:

- an explanation of how actual temperatures and windchill temperatures are related for a given wind speed, and
- a discussion about whether an actual temperature can ever have two corresponding windchill temperatures when the wind speed remains the same.

Standardized Test Practice

34. The relation {(2, 11), (−9, 8), (14, 1), (5, 5)} is *not* a function when which ordered pair is added to the set?

Ⓐ (8, −9) Ⓑ (6, 11) Ⓒ (0, 0) Ⓓ (2, 18)

35. Which statement is true about the data in the table?

x	y
−4	−4
2	16
5	8
10	−4
12	15

Ⓐ The data represent a function.

Ⓑ The data do not represent a function.

Ⓒ As the value of x increases, the value of y increases.

Ⓓ A graph of the data would not pass the vertical line test.

Maintain Your Skills

Mixed Review

Solve each inequality. Check your solution. *(Lessons 7-5 and 7-6)*

36. $4y > 24$ **37.** $\frac{a}{3} < -7$ **38.** $18 \geq -2k$

39. $2x + 5 < 17$ **40.** $2t - 3 \geq 1.4t + 6$ **41.** $12r - 4 > 7 + 12r$

Solve each problem by using the percent equation. *(Lesson 6-7)*

42. 10 is what percent of 50? **43.** What is 15% of 120?

44. Find 95% of 256. **45.** 46.5 is 62% of what number?

46. State whether the sequence 120, 100, 80, 60, … is *arithmetic*, *geometric*, or *neither*. Then write the next three terms of the sequence. *(Lesson 5-10)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Evaluate each expression if $x = 4$ and $y = -1$.**
*(To review **evaluating expressions**, see Lesson 1-3.)*

47. $3x + 1$ **48.** $2y$ **49.** $y + 6$

50. $-5x$ **51.** $2x - 8$ **52.** $3y - 4$

Graphing Calculator Investigation

A Preview of Lesson 8-2

Function Tables

You can use a TI-83 Plus graphing calculator to create function tables. By entering a function and the domain values, you can find the corresponding range values.

Use a function table to find the range of $y = 3n + 1$ if the domain is $\{-5, -2, 0, 0.5, 4\}$.

Step 1 *Enter the function.*

- The graphing calculator uses X for the domain values and Y for the range values. So, Y = 3X + 1 represents $y = 3n + 1$.
- Enter Y = 3X + 1 in the Y= list.

KEYSTROKES: Y= 3 X,T,θ,*n* + 1

Step 2 *Format the table.*

- Use TBLSET to select *Ask* for the independent variable and *Auto* for the dependent variable. Then you can enter any value for the domain.

KEYSTROKES: 2nd [TBLSET] ▼ ▼ ▶ ENTER ▼ ENTER

Step 3 *Find the range by entering the domain values.*

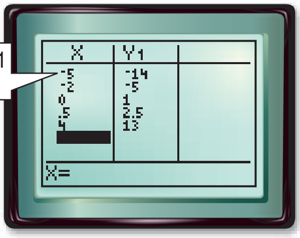
- Access the table.

KEYSTROKES: 2nd [TABLE]

- Enter the domain values.

KEYSTROKES: −5 ENTER −2 ENTER … 4 ENTER

The range is {−14, −5, 1, 2.5, 13}.



```
  X   | Y1  |
-5    | -14 |
-2    | -5  |
0     | 1   |
.5    | 2.5 |
4     | 13  |
X=
```

$y = 3(-5) + 1$
$= -14$

Exercises

Use the TABLE option on a graphing calculator to complete each exercise.

1. Consider the function $f(x) = -2x + 4$ and the domain values $\{-2, -1, 0, 1, 2\}$.
  a. Use a function table to find the range values.
  b. Describe the relationship between the X and Y values.
  c. If X is less than −2, would the value for Y be greater or less than 8? Explain.

2. Suppose you are using the formula $d = rt$ to find the distance d a car travels for the times t in hours given by {0, 1, 3.5, 10}.
  a. If the rate is 60 miles per hour, what function should be entered in the Y= list?
  b. Make a function table for the given domain.
  c. Between which two times in the domain does the car travel 150 miles?
  d. Describe how a function table can be used to better estimate the time it takes to drive 150 miles.

3. Serena is buying one packet of pencils for $1.50 and a number of fancy folders x for $0.40 each. The total cost y is given by $y = 1.50 + 0.40x$.
  a. Use a function table to find the total cost if Serena buys 1, 2, 3, 4, and 12 folders.
  b. Suppose plain folders cost $0.25 each. Enter $y = 1.50 + 0.25x$ in the Y= list as Y2. How much does Serena save if she buys pencils and 12 plain folders rather than pencils and 12 fancy folders?

www.pre-alg.com/other_calculator_keystrokes

8-2

Linear Equations in Two Variables

What You'll Learn

- Solve linear equations with two variables.
- Graph linear equations using ordered pairs.

Vocabulary

- linear equation

How can linear equations represent a function?

Peaches cost \$1.50 per can.

a. Complete the table to find the cost of 2, 3, and 4 cans of peaches.

b. On grid paper, graph the ordered pairs (number, cost). Then draw a line through the points.

Number of Cans (x)	$1.50x$	Cost (y)
1	1.50(1)	1.50
2		
3		
4		

c. Write an equation representing the relationship between number of cans x and cost y.

Study Tip

Input and Output

The variable for the input is called the **independent variable** because it can be any number. The variable for the output is called the **dependent variable** because it *depends* on the input value.

SOLUTIONS OF EQUATIONS Functions can be represented in words, in a table, as ordered pairs, with a graph, and with an equation.

An equation such as $y = 1.50x$ is called a linear equation. A **linear equation** in two variables is an equation in which the variables appear in separate terms and neither variable contains an exponent other than 1.

Solutions of a linear equation are ordered pairs that make the equation true. One way to find solutions is to make a table. Consider the equation $y = -x + 8$.

$y = -x + 8$			
x	$y = -x + 8$	y	(x, y)
-1	$y = -(-1) + 8$	9	$(-1, 9)$
0	$y = -(0) + 8$	8	$(0, 8)$
1	$y = -(1) + 8$	7	$(1, 7)$
2	$y = -(2) + 8$	6	$(2, 6)$

Step 1 Choose any convenient values for x.

Step 2 Substitute the values for x.

Step 3 Simplify to find the y values.

Step 4 Write the solutions as ordered pairs.

So, four solutions of $y = -x + 8$ are $(-1, 9)$, $(0, 8)$, $(1, 7)$, and $(2, 6)$.

Example 1 Find Solutions

Find four solutions of $y = 2x - 1$.

Choose four values for x. Then substitute each value into the equation and solve for y.

x	$y = 2x - 1$	y	(x, y)
0	$y = 2(0) - 1$	-1	$(0, -1)$
1	$y = 2(1) - 1$	1	$(1, 1)$
2	$y = 2(2) - 1$	3	$(2, 3)$
3	$y = 2(3) - 1$	5	$(3, 5)$

Four solutions are $(0, -1)$, $(1, 1)$, $(2, 3)$, and $(3, 5)$.

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Sometimes it is necessary to first rewrite an equation by solving for y.

Example 2 Solve an Equation for y

SHOPPING **Fancy goldfish x cost \$3, and regular goldfish y cost \$1. Find four solutions of $3x + y = 8$ to determine how many of each type of fish Tyler can buy for \$8.**

First, rewrite the equation by solving for y.

$3x + y = 8$ Write the equation.

$3x + y - 3x = 8 - 3x$ Subtract $3x$ from each side.

$y = 8 - 3x$ Simplify.

Choose four x values and substitute them into $y = 8 - 3x$. Four solutions are (0, 8), (1, 5), (2, 2), and (3, −1).

x	$y = 8 - 3x$	y	(x, y)
0	$y = 8 - 3(0)$	8	(0, 8)
1	$y = 8 - 3(1)$	5	(1, 5)
2	$y = 8 - 3(2)$	2	(2, 2)
3	$y = 8 - 3(3)$	−1	(3, −1)

(0, 8)	He can buy 0 fancy goldfish and 8 regular goldfish.
(1, 5)	He can buy 1 fancy goldfish and 5 regular goldfish.
(2, 2)	He can buy 2 fancy goldfish and 2 regular goldfish.
(3, −1)	This solution does not make sense, because there cannot be a negative number of goldfish.

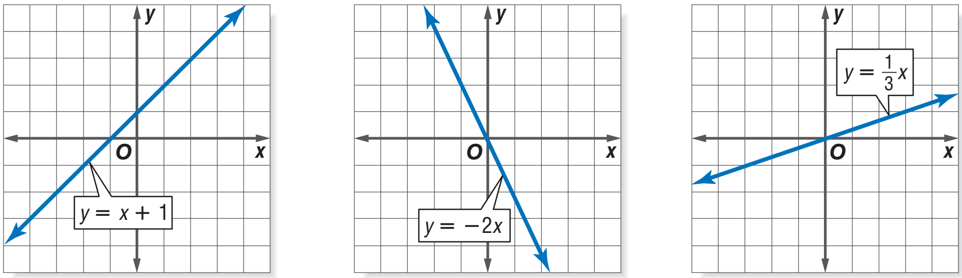
Study Tip

Choosing x Values
It is often convenient to choose 0 as an x value to find a value for y.

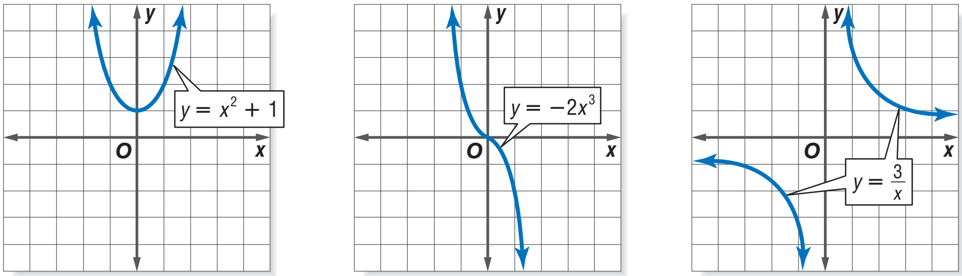
Concept Check Write the solution of $3x + y = 10$ if $x = 2$.

GRAPH LINEAR EQUATIONS A linear equation can also be represented by a graph. Study the graphs shown below.

Linear Equations



Nonlinear Equations



Notice that graphs of the linear equations are straight lines. This is true for all linear equations and is the reason they are called "linear." The coordinates of all points on a line are solutions to the equation.

CONTENTS

Study Tip

Plotting Points
You can also graph just two points to draw the line and then graph one point to check.

To graph a linear equation, find ordered pair solutions, plot the corresponding points, and draw a line through them. It is best to find at least three points.

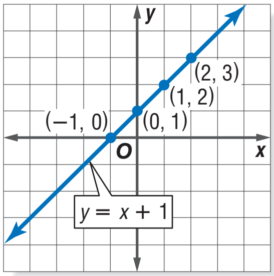
Example 3 Graph a Linear Equation

Graph $y = x + 1$ by plotting ordered pairs.

First, find ordered pair solutions. Four solutions are (−1, 0), (0, 1), (1, 2), and (2, 3).

x	$y = x + 1$	y	(x, y)
−1	$y = -1 + 1$	0	(−1, 0)
0	$y = 0 + 1$	1	(0, 1)
1	$y = 1 + 1$	2	(1, 2)
2	$y = 2 + 1$	3	(2, 3)

Plot these ordered pairs and draw a line through them. Note that the ordered pair for any point on this line is a solution of $y = x + 1$. The line is a complete graph of the function.



CHECK It appears from the graph that (−2, −1) is also a solution. Check this by substitution.

$y = x + 1$ Write the equation.

$-1 \stackrel{?}{=} -2 + 1$ Replace x with −2 and y with −1.

$-1 = -1$ ✓ Simplify.

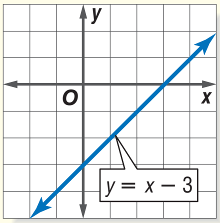
A linear equation is one of many ways to represent a function.

Concept Summary — Representing Functions

- **Words** The value of y is 3 less than the corresponding value of x.
- **Table**

x	y
0	−3
1	−2
2	−1
3	0

- **Graph**



- **Ordered Pairs** (0, −3), (1, −2), (2, −1), (3, 0)
- **Equation** $y = x - 3$

Check for Understanding

Concept Check

1. **Explain** why a linear equation has infinitely many solutions.
2. **OPEN ENDED** Write a linear equation that has (−2, 4) as a solution.

Guided Practice

3. Copy and complete the table. Use the results to write four solutions of $y = x + 5$. Write the solutions as ordered pairs.

x	$x + 5$	y
−3	−3 + 5	
−1		
0		
1		

Find four solutions of each equation. Write the solutions as ordered pairs.

4. $y = x + 8$ **5.** $y = 4x$ **6.** $y = 2x - 7$ **7.** $-5x + y = 6$

Graph each equation by plotting ordered pairs.

8. $y = x + 3$ **9.** $y = 2x - 1$ **10.** $x + y = 5$

Application

11. **SCIENCE** The distance y in miles that light travels in x seconds is given by $y = 186{,}000x$. Find two solutions of this equation and describe what they mean.

Practice and Apply

Homework Help

For Exercises	See Examples
12–25	1
26–29	2
30–41	3

Extra Practice
See page 742.

Copy and complete each table. Use the results to write four solutions of the given equation. Write the solutions as ordered pairs.

12. $y = x - 9$

x	$x - 9$	y
−1	−1 − 9	
0		
4		
7		

13. $y = 2x + 6$

x	$2x + 6$	y
−4	2(−4) + 6	
0		
2		
4		

Find four solutions of each equation. Write the solutions as ordered pairs.

14. $y = x + 2$ **15.** $y = x - 7$ **16.** $y = 3x$ **17.** $y = -5x$

18. $y = 2x - 3$ **19.** $y = 3x + 1$ **20.** $x + y = 9$ **21.** $x + y = -6$

22. $4x + y = 2$ **23.** $3x - y = 10$ **24.** $y = 8$ **25.** $x = -1$

MEASUREMENT **The equation $y = 0.62x$ describes the approximate number of miles y in x kilometers.**

26. Describe what the solution (8, 4.96) means.

27. About how many miles is a 10-kilometer race?

HEALTH **During a workout, a target heart rate y in beats per minute is represented by $y = 0.7(220 - x)$, where x is a person's age.**

28. Compare target heart rates of people 20 years old and 50 years old.

29. In which quadrant(s) would the graph of $y = 0.7(220 - x)$ make sense? Explain your reasoning.

Graph each equation by plotting ordered pairs.

30. $y = x + 2$ **31.** $y = x + 5$ **32.** $y = x - 4$ **33.** $y = -x - 6$

34. $y = -2x + 2$ **35.** $y = 3x - 4$ **36.** $x + y = 1$ **37.** $x - y = 6$

38. $2x + y = 5$ **39.** $3x - y = 7$ **40.** $x = 2$ **41.** $y = -3$

GEOMETRY **For Exercises 42–44, use the following information.**
The formula for the perimeter of a square with sides s units long is $P = 4s$.

42. Find three ordered pairs that satisfy this condition.

43. Draw the graph that contains these points.

44. Why do negative values of s make no sense?

CONTENTS

Determine whether each relation or equation is linear. Explain.

45.

x	y
−1	−2
0	0
1	2
2	4

46.

x	y
−1	1
0	0
1	1
2	4

47.

x	y
−1	−1
0	−1
1	−1
2	−1

48. $3x + y = 20$

49. $y = x^2$

50. $y = 5$

51. **CRITICAL THINKING** Compare and contrast the functions shown in the tables. (*Hint:* Compare the change in values for each column.)

x	y
−1	−2
0	0
1	2
2	4

x	y
−1	1
0	0
1	1
2	4

52. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can linear equations represent a function?

Include the following in your answer:

- a description of four ways that you can represent a function, and
- an example of a linear equation that could be used to determine the cost of x pounds of bananas that are $0.49 per pound.

Standardized Test Practice

53. Identify the equation that represents the data in the table.

x	y
−2	11
0	5
1	2
3	−4

Ⓐ $y = x + 5$ Ⓑ $y = -3x + 5$

Ⓒ $y = -5x + 1$ Ⓓ $y = x + 13$

54. The graph of $2x - y = 4$ goes through which pair of points?

Ⓐ $P(-2, -3), Q(0, 2)$ Ⓑ $P(-2, -1), Q(2, -3)$

Ⓒ $P(1, -2), Q(3, 2)$ Ⓓ $P(-3, 2), Q(0, -4)$

Maintain Your Skills

Mixed Review

Determine whether each relation is a function. Explain. *(Lesson 8-1)*

55. $\{(2, 3), (3, 4), (4, 5), (5, 6)\}$

56. $\{(0, 6), (-3, 9), (4, 9), (-2, 1)\}$

57. $\{(11, 8), (13, -2), (11, 21)\}$

58. $\{(-0.1, 5), (0, 10), (-0.1, -5)\}$

Solve each inequality and check your solution. Graph the solution on a number line. *(Lesson 7-6)*

59. $3x + 4 < 16$

60. $9 - 2d \leq 23$

61. Evaluate $a \div b$ if $a = \frac{4}{7}$ and $b = \frac{2}{3}$. *(Lesson 5-4)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL **In each equation, find the value of y when $x = 0$.**
*(To review **substitution**, see Lesson 1-5.)*

62. $y = 5x - 3$

63. $-x + y = 3$

64. $x + 2y = 12$

65. $4x - 5y = -20$

CONTENTS

Reading Mathematics

Language of Functions

Equations that are functions can be written in a form called *functional notation*, as shown below.

equation	**functional notation**
$y = 4x + 10$	$f(x) = 4x + 10$

Read $f(x)$ as f of x.

So, $f(x)$ is simply another name for y. Letters other than f are also used for names of functions. For example, $g(x) = 2x$ and $h(x) = -x + 6$ are also written in functional notation.

In a function, x represents the domain values, and $f(x)$ represents the range values.

$$\overset{\substack{\text{range}\\ \downarrow}}{f(x)} = \overset{\substack{\text{domain}\\ \downarrow}}{4x} + 10$$

$f(3)$ represents the element in the range that corresponds to the element 3 in the domain. To find $f(3)$, substitute 3 for x in the function and simplify.

Read $f(3)$ as f of 3.

$f(x) = 4x + 10$ Write the function.

$f(3) = 4(3) + 10$ Replace x with 3.

$f(3) = 12 + 10$ or 22 Simplify.

So, the functional value of f for $x = 3$ is 22.

Reading to Learn

1. **RESEARCH** Use the Internet or a dictionary to find the everyday meaning of the word *function*. Write a sentence describing how the everyday meaning relates to the mathematical meaning.
2. Write your own rule for remembering how the domain and the range are represented using functional notation.
3. Copy and complete the table below.

x	$f(x) = 3x + 5$	$f(x)$
0	$f(0) = 3(0) + 5$	
1		
2		
3		

4. If $f(x) = 4x - 1$, find each value.

   a. $f(2)$ **b.** $f(-3)$ **c.** $f\left(\frac{1}{2}\right)$

5. Find the value of x if $f(x) = -2x + 5$ and the value of $f(x)$ is -7.

CONTENTS

8-3 Graphing Linear Equations Using Intercepts

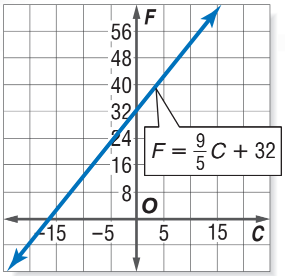
What You'll Learn

- Find the x- and y-intercepts of graphs.
- Graph linear equations using the x- and y-intercepts.

Vocabulary

- x-intercept
- y-intercept

How can intercepts be used to represent real-life information?



The relationship between the temperature in degrees Fahrenheit F and the temperature in degrees Celsius C is given by the equation $F = \frac{9}{5}C + 32$. This equation is graphed at the right.

a. Write the ordered pair for the point where the graph intersects the y-axis. What does this point represent?

b. Write the ordered pair for the point where the graph intersects the x-axis. What does this point represent?

Reading Math

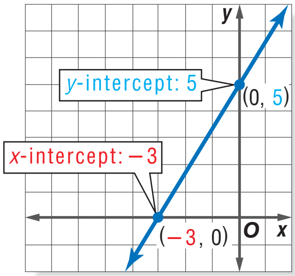
Intercept

Everyday Meaning: to interrupt or cut off

Math Meaning: the point where a coordinate axis crosses a line

FIND INTERCEPTS The **x-intercept** is the x-coordinate of a point where a graph crosses the x-axis. The y-coordinate of this point is 0.

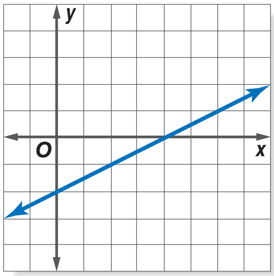
The **y-intercept** is the y-coordinate of a point where a graph crosses the y-axis. The x-coordinate of this point is 0.



Example 1 Find Intercepts From Graphs

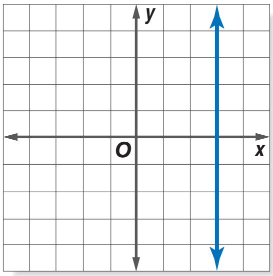
State the x-intercept and the y-intercept of each line.

a.



The graph crosses the x-axis at (4, 0). The x-intercept is 4. The graph crosses the y-axis at (0, −2). The y-intercept is −2.

b.



The graph crosses the x-axis at (3, 0). The x-intercept is 3. The graph does not cross the y-axis. There is no y-intercept.

Concept Check A graph passes through a point at (0, −10). Is −10 an x-intercept or a y-intercept?

You can also find the x-intercept and the y-intercept from an equation of a line.

Key Concept *Intercepts of Lines*

- To find the x-intercept, let $y = 0$ in the equation and solve for x.
- To find the y-intercept, let $x = 0$ in the equation and solve for y.

Example 2 Find Intercepts from Equations

Find the x-intercept and the y-intercept for the graph of $y = x - 6$.

To find the x-intercept, let $y = 0$.

$y = x - 6$ Write the equation.

$0 = x - 6$ Replace y with 0.

$6 = x$ Simplify.

The x-intercept is 6. So, the graph crosses the x-axis at (6, 0).

To find the y-intercept, let $x = 0$.

$y = x - 6$ Write the equation.

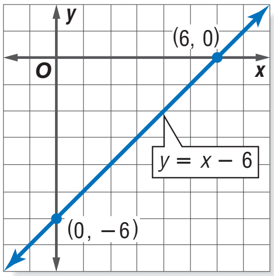
$y = 0 - 6$ Replace x with 0.

$y = -6$ Simplify.

The y-intercept is -6. So, the graph crosses the y-axis at $(0, -6)$.

GRAPH EQUATIONS You can use the x- and y-intercepts to graph equations of lines.

In Example 2, we determined that the graph of $y = x - 6$ passes through (6, 0) and $(0, -6)$. To draw the graph, plot these points and draw a line through them.



Example 3 Use Intercepts to Graph Equations

Graph $x + 2y = 4$ using the x- and y-intercepts.

Step 1

Find the x-intercept.

$x + 2y = 4$ Write the equation.

$x + 2(0) = 4$ Let $y = 0$.

$x = 4$ Simplify.

The x-intercept is 4, so the graph passes through (4, 0).

Step 2

Find the y-intercept.

$x + 2y = 4$ Write the equation.

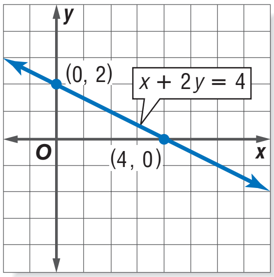
$0 + 2y = 4$ Let $x = 0$.

$y = 2$ Divide each side by 2.

The y-intercept is 2, so the graph passes through (0, 2).

Step 3

Graph the points at (4, 0) and (0, 2) and draw a line through them.



CHECK Choose some other point on the line and determine whether its ordered pair is a solution of $x + 2y = 4$.

Study Tip

Graphing Shortcuts

The ordered pairs of any two solutions can be used to graph a linear equation. However, it is often easiest to find the intercepts.

CONTENTS

More About. . .

Earth Science

The temperature of the air is about 3°F cooler for every 1000 feet increase in altitude.

Source: www.hot-air-balloons.com

Example 4 Intercepts of Real-World Data

EARTH SCIENCE **Suppose you take a hot-air balloon ride on a day when the temperature is 24°C at sea level. The equation $y = -6.6x + 24$ represents the temperature at x kilometers above sea level.**

a. Use the intercepts to graph the equation.

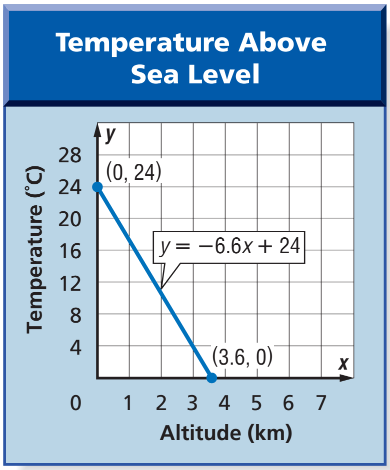
Step 1 Find the x-intercept.

$$
\begin{aligned}
y &= -6.6x + 24 && \text{Write the equation.}\\
0 &= -6.6x + 24 && \text{Replace } y \text{ with 0.}\\
0 - 24 &= -6.6x + 24 - 24 && \text{Subtract 24 from each side.}\\
\frac{-24}{-6.6} &= \frac{-6.6x}{-6.6} && \text{Divide each side by } -6.6.\\
3.6 &\approx x && \text{The } x\text{-intercept is approximately 3.6.}
\end{aligned}
$$

Step 2 Find the y-intercept.

$$
\begin{aligned}
y &= -6.6x + 24 && \text{Write the equation.}\\
y &= -6.6(0) + 24 && \text{Replace } x \text{ with 0.}\\
y &= 24 && \text{The } y\text{-intercept is 24.}
\end{aligned}
$$

Step 3 Plot the points with coordinates (3.6, 0) and (0, 24). Then draw a line through the points.



Temperature Above Sea Level

b. Describe what the intercepts mean.

The x-intercept 3.6 means that when the hot-air balloon is 3.6 kilometers above sea level, the temperature is 0°C. The y-intercept 24 means that the temperature at sea level is 24°C.

Some linear equations have just one variable. Their graphs are horizontal or vertical lines.

Reading Math

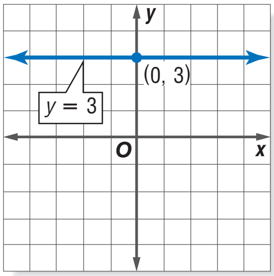
- $y = 3$ can be read *for all x, y = 3.*
- $x = -2$ can be read *for all y, x = −2.*

Example 5 Horizontal and Vertical Lines

Graph each equation using the x- and y-intercepts.

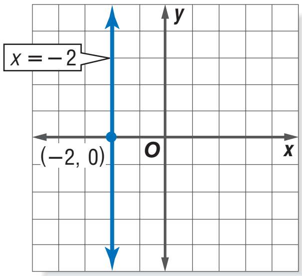
a. $y = 3$

Note that $y = 3$ is the same as $0x + y = 3$. The y-intercept is 3, and there is no x-intercept.



b. $x = -2$

Note that $x = -2$ is the same as $x + 0y = -2$. The x-intercept is −2, and there is no y-intercept.



Check for Understanding

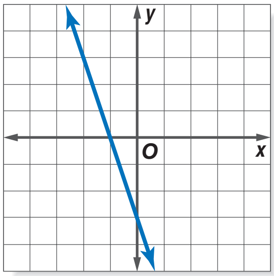
Concept Check

1. **Explain** how to find the x- and y-intercepts of a line given its equation.
2. **OPEN ENDED** Sketch the graph of a function whose x- and y-intercepts are both negative. Label the intercepts.

Guided Practice

State the x-intercept and the y-intercept of each line.

3.



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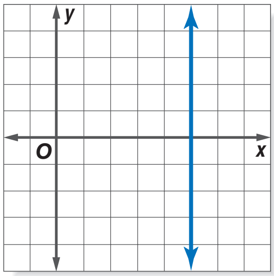


Find the x-intercept and the y-intercept for the graph of each equation.

5. $y = x + 4$

6. $y = 7$

7. $2x + 3y = 6$

Graph each equation using the x- and y-intercepts.

8. $y = x + 1$

9. $x - 2y = 6$

10. $x = -1$

Application

11. **BUSINESS** A lawn mowing service charges a base fee of \$3, plus \$6 per hour for labor. This can be represented by $y = 6x + 3$, where y is the total cost and x is the number of hours. Graph this equation and explain what the y-intercept represents.

Practice and Apply

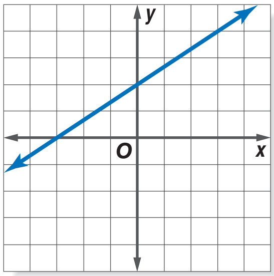
Homework Help

For Exercises	See Examples
12–15	1
16–24	2
25–33	3, 5
34, 35	4

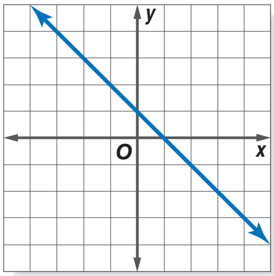
Extra Practice
See page 742.

State the x-intercept and the y-intercept of each line.

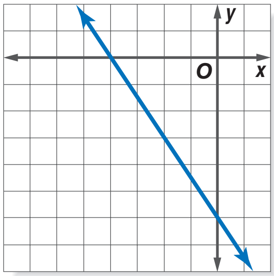
12.



13.



14.



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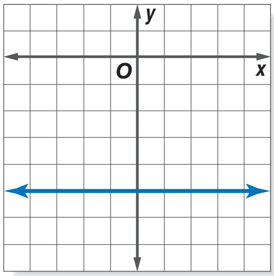


Find the x-intercept and the y-intercept for the graph of each equation.

16. $y = x - 1$

17. $y = x + 5$

18. $x = 9$

19. $y + 4 = 0$

20. $y = 2x + 10$

21. $x - 2y = 8$

22. $y = -3x - 12$

23. $4x + 5y = 20$

24. $6x + 7y = 12$

CONTENTS

Graph each equation using the x- and y-intercepts.

25. $y = x + 2$ **26.** $y = x - 3$ **27.** $x + y = 4$

28. $y = 5x + 5$ **29.** $y = -2x + 4$ **30.** $x + 2y = -6$

31. $y = -2$ **32.** $x - 3 = 0$ **33.** $3x + 6y = 18$

34. **CATERING** For a luncheon, a caterer charges $8 per person, plus a setup fee of $24. The total cost of the luncheon y can be represented by $y = 8x + 24$, where x is the number of people. Graph the equation and explain what the y-intercept represents.

35. **MONEY** Jasmine has $18 to buy books at the library used book sale. Paperback books cost $3 each. The equation $y = 18 - 3x$ represents the amount of money she has left over if she buys x paperback books. Graph the equation and describe what the intercepts represent.

36. **GEOMETRY** The perimeter of a rectangle is 50 centimeters. This can be given by the equation $50 = 2\ell + 2w$, where ℓ is the length and w is the width. Name the x- and y-intercepts of the equation and explain what they mean.

37. **CRITICAL THINKING** Explain why you cannot graph $y = 2x$ by using intercepts only. Then draw the graph.

38. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can intercepts be used to represent real-life information?

Include the following in your answer:

- a graph showing a decrease in temperature, with the x-axis representing time and the y-axis representing temperature, and
- an explanation of what the intercepts mean.

Standardized Test Practice

39. What is the x-intercept of the graph of $y = 8x - 32$?

Ⓐ -4 Ⓑ 4 Ⓒ -32 Ⓓ 32

40. The graph of which equation does *not* have a y-intercept of 3?

Ⓐ $2x + 3y = 9$ Ⓑ $4x + y = 3$ Ⓒ $x + 3y = 6$ Ⓓ $x - 2y = -6$

Maintain Your Skills

Mixed Review

Find four solutions of each equation. *(Lesson 8-2)*

41. $y = 2x + 7$ **42.** $y = -3x + 1$ **43.** $4x - y = -5$

Determine whether each relation is a function. *(Lesson 8-1)*

44. $\{(2, 12), (4, -5), (-3, -4), (11, 0)\}$

45. $\{(-4.2, 17), (-4.3, 16), (-4.3, 15), (-4.3, 14)\}$

Solve each inequality. *(Lesson 7-4)*

46. $y + 3 < 5$ **47.** $-2 + n > 10$ **48.** $7 \leq x + 8$

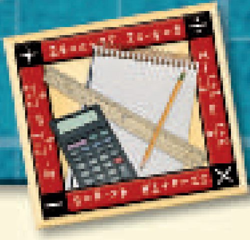
49. Express 0.028 as a percent. *(Lesson 6-4)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Subtract.** *(To review* ***subtracting integers****, see Lesson 2-3.)*

50. $-11 - 13$ **51.** $15 - 31$ **52.** $-26 - (-26)$ **53.** $9 - (-16)$

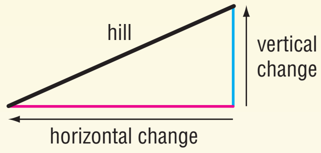
CONTENTS



Algebra Activity

A Preview of Lesson 8-4

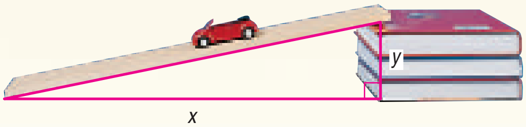
It's All Downhill



The steepness, or *slope*, of a hill can be described by a ratio.

$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} \begin{array}{l} \leftarrow \text{height} \\ \leftarrow \text{length} \end{array}$$

Collect the Data



Step 1 Use posterboard or a wooden board, tape, and three or more books to make a "hill."

Step 2 Measure the height y and length x of the hill to the nearest $\frac{1}{2}$ inch or $\frac{1}{4}$ inch. Record the measurements in a table like the one below.

Hill	Height y (in.)	Length x (in.)	Car Distance (in.)	Slope $\frac{y}{x}$
1				
2				
3				

Step 3 Place a toy car at the top of the hill and let it roll down. Measure the distance from the bottom of the ramp to the back of the car when it stops. Record the distance in the table.

Step 4 For the second hill, increase the height by adding one or two more books. Roll the car down and measure the distance it rolls. Record the dimensions of the hill and the distance in the table.

Step 5 Take away two or three books so that hill 3 has the least height. Roll the car down and measure the distance it rolls. Record the dimensions of the hill and the distance in the table.

Step 6 Find the slopes of hills 1, 2, and 3 and record the values in the table.

Analyze the Data

1. How did the slope change when the height increased and the length decreased?

2. How did the slope change when the height decreased and the length increased?

3. **MAKE A CONJECTURE** On which hill would a toy car roll the farthest—a hill with slope $\frac{18}{25}$ or $\frac{25}{18}$? Explain by describing the relationship between slope and distance traveled.

Extend the Activity

4. Make a fourth hill. Find its slope and predict the distance a toy car will go when it rolls down the hill. Test your prediction by rolling a car down the hill.

CONTENTS

8-4 Slope

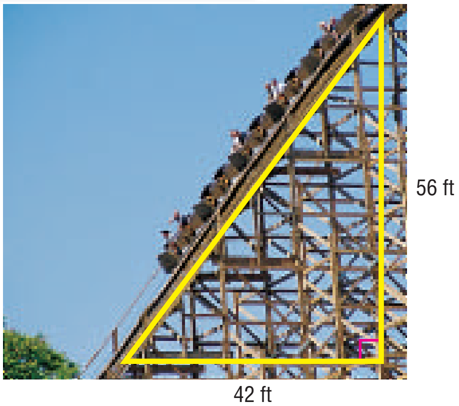
What You'll Learn

- Find the slope of a line.

Vocabulary

- slope

How is slope used to describe roller coasters?



Some roller coasters can make you feel heavier than a shuttle astronaut feels on liftoff. This is because the speed and steepness of the hills increase the effects of gravity.

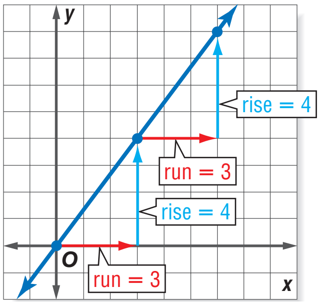
a. Use the roller coaster to write the ratio $\frac{\text{height}}{\text{length}}$ in simplest form.

b. Find the ratio of a hill that has the same length but is 14 feet higher than the hill above. Is this hill steeper or less steep than the original?

SLOPE **Slope** describes the steepness of a line. It is the ratio of the *rise*, or the vertical change, to the *run*, or the horizontal change.

$$\text{slope} = \frac{\text{rise}}{\text{run}} \quad \begin{array}{l} \leftarrow \text{vertical change} \\ \leftarrow \text{horizontal change} \end{array}$$

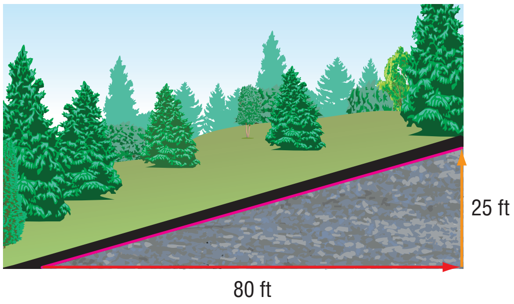
$$= \frac{4}{3}$$



Note that the slope is the same for any two points on a straight line.

Example 1 Use Rise and Run to Find Slope

Find the slope of a road that rises 25 feet for every horizontal change of 80 feet.



$$\text{slope} = \frac{\text{rise}}{\text{run}} \qquad \text{Write the formula.}$$

$$= \frac{25 \text{ ft}}{80 \text{ ft}} \qquad \text{rise} = 25 \text{ ft, run} = 80 \text{ ft}$$

$$= \frac{5}{16} \qquad \text{Simplify.}$$

The slope of the road is $\frac{5}{16}$ or 0.3125.

Concept Check What is the slope of a ramp that rises 2 inches for every horizontal change of 24 inches?

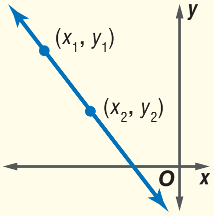
CONTENTS

You can also find the slope by using the coordinates of any two points on a line.

Key Concept

Slope

- **Words** The slope m of a line passing through points at (x_1, y_1) and (x_2, y_2) is the ratio of the difference in y-coordinates to the corresponding difference in x-coordinates.
- **Symbols** $m = \frac{y_2 - y_1}{x_2 - x_1}$, where $x_2 \neq x_1$
- **Model**



The slope of a line may be positive, negative, zero, or undefined.

Study Tip

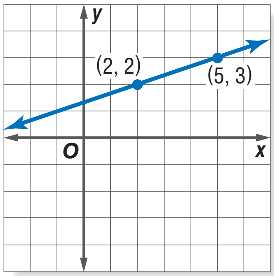
Choosing Points

- Any two points on a line can be chosen as (x_1, y_1) and (x_2, y_2).
- The coordinates of both points must be used in the same order.

Check: In Example 2, let $(x_1, y_1) = (5, 3)$ and let $(x_2, y_2) = (2, 2)$, then find the slope.

Example 2 Positive Slope

Find the slope of the line.



$m = \frac{y_2 - y_1}{x_2 - x_1}$ Definition of slope

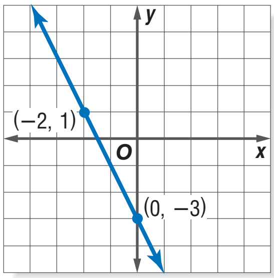
$m = \frac{3 - 2}{5 - 2}$ $(x_1, y_1) = (2, 2)$, $(x_2, y_2) = (5, 3)$

$m = \frac{1}{3}$

The slope is $\frac{1}{3}$.

Example 3 Negative Slope

Find the slope of the line.



$m = \frac{y_2 - y_1}{x_2 - x_1}$ Definition of slope

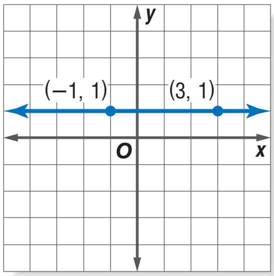
$m = \frac{-3 - 1}{0 - (-2)}$ $(x_1, y_1) = (-2, 1)$, $(x_2, y_2) = (0, -3)$

$m = \frac{-4}{2}$ or -2

The slope is -2.

Example 4 Zero Slope

Find the slope of the line.



$m = \frac{y_2 - y_1}{x_2 - x_1}$ Definition of slope

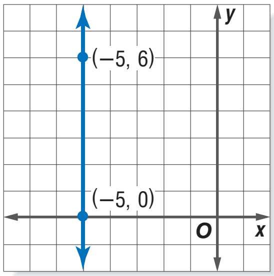
$m = \frac{1 - 1}{3 - (-1)}$ $(x_1, y_1) = (-1, 1)$, $(x_2, y_2) = (3, 1)$

$m = \frac{0}{4}$ or 0

The slope is 0.

Example 5 Undefined Slope

Find the slope of the line.



$m = \frac{y_2 - y_1}{x_2 - x_1}$ Definition of slope

$m = \frac{0 - 6}{-5 - (-5)}$ $(x_1, y_1) = (-5, 6)$, $(x_2, y_2) = (-5, 0)$

$m = \cancel{\frac{-6}{0}}$

Division by 0 is undefined. So, the slope is undefined.

The steepness of real-world inclines can be compared by using slope.

Standardized Test Practice

Example 6 Compare Slopes

Multiple-Choice Test Item

There are two major hills on a hiking trail. The first hill rises 6 feet vertically for every 42-foot run. The second hill rises 10 feet vertically for every 98-foot run. Which statement is true?

- Ⓐ The first hill is steeper than the second hill.
- Ⓑ The second hill is steeper than the first hill.
- Ⓒ Both hills have the same steepness.
- Ⓓ You cannot determine which hill is steeper.

Test-Taking Tip

Make a Drawing
Whenever possible, make a drawing that displays the given information. Then use the drawing to estimate the answer.

Read the Test Item To compare steepness of the hills, find the slopes.

Solve the Test Item

first hill

$\text{slope} = \frac{\text{rise}}{\text{run}}$

$= \frac{6 \text{ ft}}{42 \text{ ft}}$ rise = 6 ft, run = 42 ft

$= \frac{1}{7}$ or about 0.14

second hill

$\text{slope} = \frac{\text{rise}}{\text{run}}$

$= \frac{10 \text{ ft}}{98 \text{ ft}}$ rise = 10 ft, run = 98 ft

$= \frac{5}{49}$ or about 0.10

$0.14 > 0.10$, so the first hill is steeper than the second. The answer is A.

Check for Understanding

Concept Check

1. **Describe** your own method for remembering whether a horizontal line has 0 slope or an undefined slope.
2. **OPEN ENDED** Draw a line whose slope is $-\frac{1}{4}$.
3. **FIND THE ERROR** Mike and Chloe are finding the slope of the line that passes through $Q(-2, 8)$ and $R(11, 7)$.

Mike

$m = \frac{8 - 7}{-2 - 11}$

Chloe

$m = \frac{7 - 8}{11 - 2}$

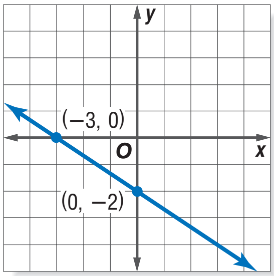
Who is correct? Explain your reasoning.

Guided Practice

4. Find the slope of a line that decreases 24 centimeters vertically for every 30-centimeter horizontal increase.

Find the slope of each line.

5.



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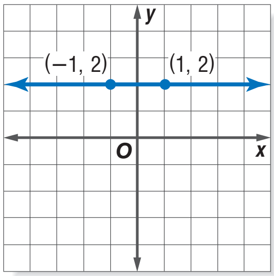


Find the slope of the line that passes through each pair of points.

7. $A(3, 4)$, $B(4, 6)$

8. $J(-8, 0)$, $K(-8, 10)$

9. $P(7, -1)$, $Q(9, -1)$

10. $C(-6, -4)$, $D(-8, -3)$

Standardized Test Practice

11. Which bike ramp is the steepest?

Ⓐ 1

Ⓑ 2

Ⓒ 3

Ⓓ 4

Bike Ramp	Height (ft)	Length (ft)
1	6	8
2	10	4
3	5	3
4	8	4

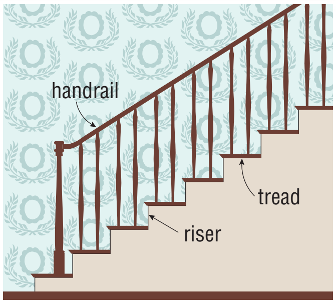
Practice and Apply

Homework Help

For Exercises	See Examples
12, 13	1
14–25	2–5
26, 27	6

Extra Practice
See page 742.

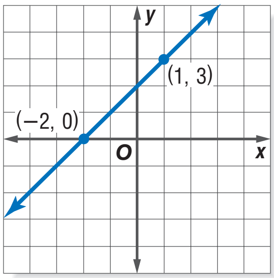
12. **CARPENTRY** In a stairway, the slope of the handrail is the ratio of the riser to the tread. If the tread is 12 inches long and the riser is 8 inches long, find the slope.



13. **HOME REPAIR** The bottom of a ladder is placed 4 feet away from a house and it reaches a height of 16 feet on the side of the house. What is the slope of the ladder?

Find the slope of each line.

14.



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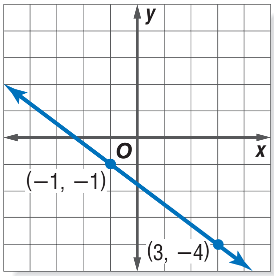

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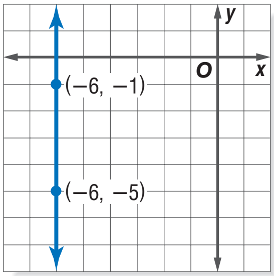

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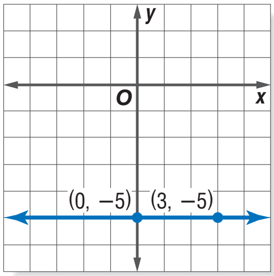


Find the slope of the line that passes through each pair of points.

18. $A(1, -3), B(5, 4)$

19. $Y(4, -3), Z(5, -2)$

20. $D(5, -1), E(-3, 4)$

21. $J(-3, 6), K(-5, 9)$

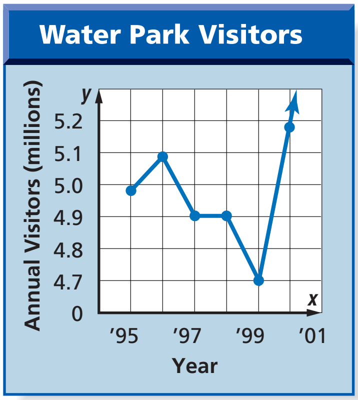
22. $N(2, 6), P(-1, 6)$

23. $S(-9, -4), T(-9, 8)$

24. $F(0, 1.6), G(0.5, 2.1)$

25. $W\left(3\frac{1}{2}, 5\frac{1}{4}\right), X\left(2\frac{1}{2}, 6\right)$

ENTERTAINMENT **For Exercises 26 and 27, use the graph.**



Source: *Amusement Business*

26. Which section of the graph shows the greatest increase in attendance? Describe the slope.

27. What happened to the attendance at the water park from 1996–1997? Describe the slope of this part of the graph.

28. **CRITICAL THINKING** The graph of a line goes through the origin (0, 0) and $C(a, b)$. State the slope of this line and explain how it relates to the coordinates of point C.

29. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

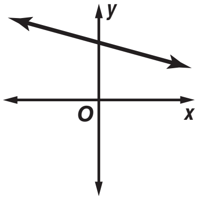
How is slope used to describe roller coasters?

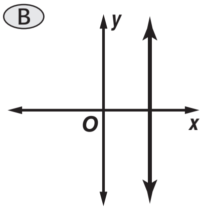
Include the following in your answer:

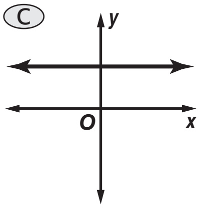
- a description of slope, and
- an explanation of how changes in rise or run affect the steepness of a roller coaster.

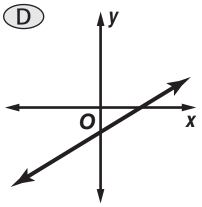
Standardized Test Practice

30. Identify the graph that has a positive slope.

A


B


C


D


31. What is the slope of line LM given $L(9, -2)$ and $M(3, -5)$?

A $\frac{1}{2}$ B $-\frac{1}{2}$ C 5 D $-\frac{1}{3}$

Maintain Your Skills

Mixed Review

Find the x-intercept and the y-intercept for the graph of each equation. *(Lesson 8-3)*

32. $y = x + 8$

33. $y = -3x + 6$

34. $4x - y = 12$

Find four solutions of each equation. Write the solutions as ordered pairs. *(Lesson 8-2)*

35. $y = 2x + 5$

36. $y = -3x$

37. $x + y = 7$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Rewrite $y = kx$ by replacing k with each given value.** *(To review **substitution**, see Lesson 1-3.)*

38. $k = 5$

39. $k = -2$

40. $k = 0.25$

41. $k = \frac{1}{3}$

Algebra Activity

A Preview of Lesson 8-5

Slope and Rate of Change

In this activity, you will investigate the relationship between slope and rate of change.

Collect the Data

Step 1 On grid paper, make a coordinate grid of the first quadrant. Label the x-axis *Number of Measures* and label the y-axis *Height of Water (cm)*.

Step 2 Pour water into a drinking glass or a beaker so that it is more than half full.

Step 3 Use a ruler to find the initial height of the water and record the measurement in a table.

Step 4 Remove a tablespoon of water from the glass or beaker and record the new height in your table.

Step 5 Repeat Step 4 so that you have six measures.

Step 6 Fill the glass or beaker again so that it has the same initial height as in Step 3.

Step 7 Repeat Steps 4 and 5, using a $\frac{1}{8}$-cup measuring cup.

Analyze the Data

1. On the coordinate grid, graph the ordered pairs (number of measures, height of water) for each set of data. Draw a line through each set of points. Label the lines 1 and 2, respectively.
2. Compare the steepness of the two graphs. Which has a steeper slope?
3. What does the height of the water depend on?
4. What happens as the number of measures increases?
5. Did you empty the glass at a faster rate using a tablespoon or a $\frac{1}{8}$-cup? Explain.

Make a Conjecture

6. Describe the relationship between slope and the rate at which the glass was emptied.
7. What would a graph look like if you emptied a glass using a teaspoon? a $\frac{1}{4}$ cup? Explain.

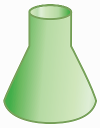
Extend the Activity

8. Water is emptied at a constant rate from containers shaped like the ones shown below. Draw a graph of the water level in each of the containers as a function of time.

a.

b.

c.



CONTENTS

8-5 Rate of Change

What You'll Learn

- Find rates of change.
- Solve problems involving direct variation.

Vocabulary

- rate of change
- direct variation
- constant of variation

How are slope and speed related?

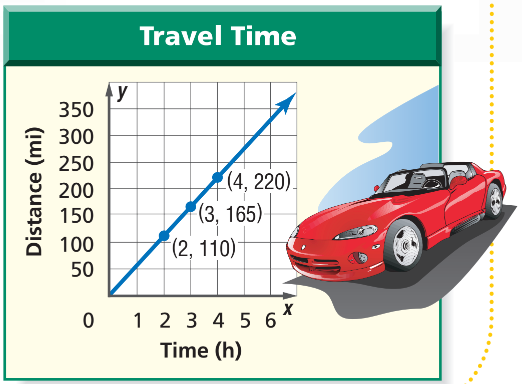
A car traveling 55 miles per hour goes 110 miles in 2 hours, 165 miles in 3 hours, and 220 miles in 4 hours, as shown.

a. For every 1-hour increase in time, what is the change in distance?

b. Find the slope of the line.

c. **Make a conjecture** about the relationship between slope of the line and speed of the car.

Travel Time



RATE OF CHANGE A change in one quantity with respect to another quantity is called the **rate of change**. Rates of change can be described using slope.

$$\text{slope} = \frac{\text{change in } y}{\text{change in } x}$$

$$= \frac{55 \text{ mi}}{1 \text{ h}} \text{ or } 55 \text{ mi/h}$$

Time (h) x	Distance (mi) y
2	110
3	165
4	220
5	275

+1 +1 +1

+55 +55 +55

Each time x increases by 1, y increases by 55.

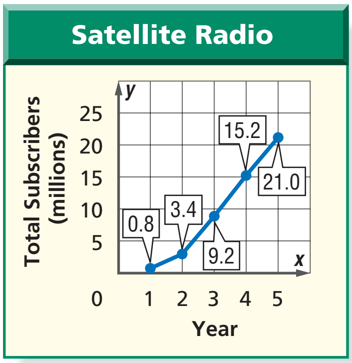
The slope of the line is the speed of the car.

You can find rates of change from an equation, a table of values, or a graph.

Example 1 Find a Rate of Change

TECHNOLOGY **The graph shows the expected growth of subscribers to satellite radio for the first five years that it is introduced. Find the expected rate of change from Year 2 to Year 5.**

Satellite Radio



Source: The Yankee Group

rate of change = slope

$$= \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Definition of slope}$$

$$= \frac{21.0 - 3.4}{5 - 2} \quad \begin{matrix}\leftarrow \text{change in subscribers} \\ \leftarrow \text{change in time}\end{matrix}$$

$$\approx 5.9 \quad \text{Simplify.}$$

So, the expected rate of change in satellite radio subscribers is an increase of about 5.9 million people per year.

CONTENTS

The steepness of slopes is also important in describing rates of change.

Example 2 Compare Rates of Change

GEOMETRY **The table shows how the perimeters of an equilateral triangle and a square change as side lengths increase. Compare the rates of change.**

Side Length x	Perimeter y	
	Triangle	Square
0	0	0
2	6	8
4	12	16

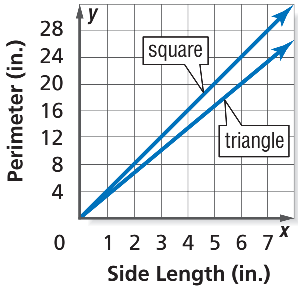
$$\text{triangle rate of change} = \frac{\text{change in } y}{\text{change in } x}$$

$$= \frac{6}{2} \text{ or } 3$$ For each side length increase of 2, the perimeter increases by 6.

$$\text{square rate of change} = \frac{\text{change in } y}{\text{change in } x}$$

$$= \frac{8}{2} \text{ or } 4$$ For each side length increase of 2, the perimeter increases by 8.

The perimeter of a square increases at a faster rate than the perimeter of a triangle. A steeper slope on the graph indicates a greater rate of change for the square.



Study Tip

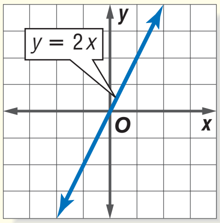
Slopes

- Positive slopes represent a rate of increase.
- Negative slopes represent a rate of decrease.
- Steeper slopes represent greater rates of change.
- Less steep slopes represent a smaller rate of change.

DIRECT VARIATION A special type of linear equation that describes rate of change is called a **direct variation**. The graph of a direct variation always passes through the origin and represents a proportional situation.

Key Concept Direct Variation

- **Words** A direct variation is a relationship such that as x increases in value, y increases or decreases at a constant rate k.
- **Symbols** $y = kx$, where $k \neq 0$
- **Example** $y = 2x$
- **Model**



In the equation $y = kx$, k is called the **constant of variation**. It is the slope, or rate of change. We say that y *varies directly with* x.

Example 3 Write a Direct Variation Equation

Suppose y varies directly with x and $y = -6$ when $x = 2$. Write an equation relating x and y.

Step 1 Find the value of k.

$y = kx$	Direct variation
$-6 = k(2)$	Replace y with -6 and x with 2.
$-3 = k$	Simplify.

Step 2 Use k to write an equation.

$y = kx$	Direct variation
$y = -3x$	Replace k with -3.

So, a direct variation equation that relates x and y is $y = -3x$.

CONTENTS

The direct variation $y = kx$ can be written as $k = \frac{y}{x}$. In this form, you can see that the ratio of y to x is the same for any corresponding values of y and x.

Example 4 Use Direct Variation to Solve Problems

POOLS **The height of the water as a pool is being filled is recorded in the table below.**

a. Write an equation that relates time and height.

Step 1 Find the ratio of y to x for each recorded time. These are shown in the third column of the table. The ratios are approximately equal to 0.4.

Time (min) x	Height (in.) y	$k = \frac{y}{x}$
5	2.0	0.40
10	3.75	0.38
15	5.5	0.37
20	7.5	0.38

To the nearest tenth, $k \approx 0.4$.

Step 2 Write an equation.

$y = kx$ Direct variation

$y = 0.4x$ Replace k with 0.4.

So, a direct variation equation that relates the time x and the height of the water y is $y = 0.4x$.

b. Predict how long it will take to fill the pool to a height of 48 inches.

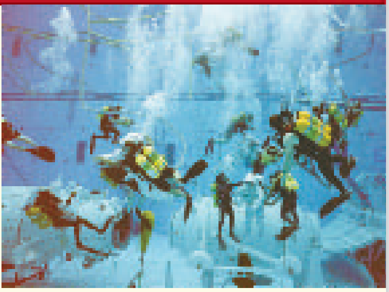
$y = 0.4x$ Write the direct variation equation.

$48 = 0.4x$ Replace y with 48.

$120 = x$ Divide each side by 0.4.

It will take about 120 minutes, or 2 hours to fill the pool.

More About. . .



Pools

The Johnson Space Center in Houston, Texas, has a 6.2 million gallon pool used to train astronauts for space flight. It is 202 feet long, 102 feet wide, and 40 feet deep.

Source: www.jsc.nasa.gov

Check for Understanding

Concept Check

1. **Describe** how slope, rate of change, and constant of variation are related by using $y = 60x$ as a model.
2. **OPEN ENDED** Draw a line that shows a 2-unit increase in y for every 1-unit increase in x. State the rate of change.
3. **FIND THE ERROR** Justin and Carlos are determining how to find rate of change from the equation $y = 4x + 5$.

Justin

The rate of change is the slope of its graph.

Carlos

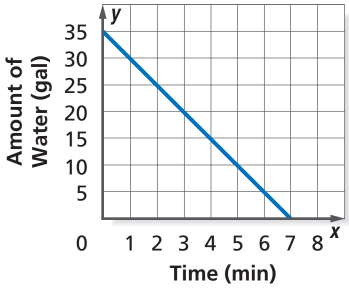
There is no rate of change because the equation is not a direct variation.

Who is correct? Explain your reasoning.

Guided Practice

Find the rate of change for each linear function.

4.



5.

Time (h) x	Wage ($) y
0	0
1	12
2	24
3	36

Suppose y varies directly with x. Write an equation relating x and y.

6. $y = 5$ when $x = -15$

7. $y = 24$ when $x = 4$

Application

8. **PHYSICAL SCIENCE** The length of a spring varies directly with the amount of weight attached to it. When a 25-gram weight is attached, a spring stretches to 8 centimeters.

a. Write a direct variation equation relating the weight x and the length y.

b. Estimate the length of a spring that has a 60-gram weight attached.

Practice and Apply

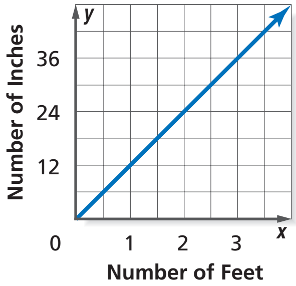
Homework Help

For Exercises	See Examples
9–12	1
13	2
14, 15	3
16, 17	4

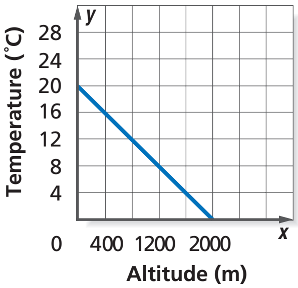
Extra Practice
See page 743.

Find the rate of change for each linear function.

9.



10.



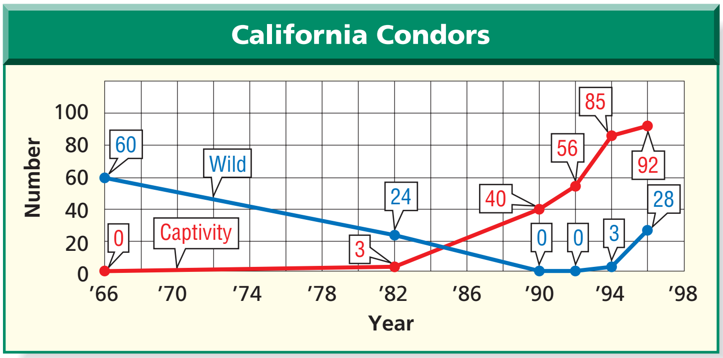
11.

Time (min)	Temperature (°F)
x	y
0	58
1	56
2	54
3	52

12.

Time (h)	Distance (mi)
x	y
0.0	0
0.5	25
1.5	75
3.0	150

13. **ENDANGERED SPECIES** The graph shows the populations of California condors in the wild and in captivity.



Source: Los Angeles Zoo

Write several sentences that describe how the populations have changed since 1966. Include the rate of change for several key intervals.

Online Research **Data Update** What has happened to the condor population since 1996? Visit www.pre-alg.com/data_update to learn more.

Suppose y varies directly with x. Write an equation relating x and y.

14. $y = 8$ when $x = 4$

15. $y = -30$ when $x = 6$

16. $y = 9$ when $x = 24$

17. $y = 7.5$ when $x = 10$

18. **FOOD COSTS** The cost of cheese varies directly with the number of pounds bought. If 2 pounds cost \$8.40, find the cost of 3.5 pounds.

19. **CONVERTING MEASUREMENTS** The number of centimeters in a measure varies directly as the number of inches. Write a direct variation equation that could be used to convert inches to centimeters.

Measure in Inches x	Measure in Centimeters y
1	2.54
2	5.08
3	7.62

20. **CRITICAL THINKING** Describe the rate of change for a graph that is a horizontal line and a graph that is a vertical line.

21. *WRITING IN MATH* Answer the question that was posed at the beginning of the lesson.

How are slope and speed related?

Include the following in your answer:

- a drawing of a graph showing distance versus time, and
- an explanation of how slope changes when speed changes.

Standardized Test Practice

22. A graph showing an increase in sales over time would have a(n)

Ⓐ positive slope. Ⓑ negative slope.

Ⓒ undefined slope. Ⓓ slope of 0.

23. Choose an equation that does *not* represent a direct variation.

Ⓐ $y = x$ Ⓑ $y = 1$ Ⓒ $y = -5x$ Ⓓ $y = 0.9x$

Maintain Your Skills

Mixed Review

Find the slope of the line that passes through each pair of points. *(Lesson 8-4)*

24. $Q(-4, 4), R(3, 5)$ **25.** $A(2, 6), B(-1, 0)$

Graph each equation using the x- and y-intercepts. *(Lesson 8-3)*

26. $y = x + 5$ **27.** $y = -x + 1$ **28.** $2x + y = 4$

29. Estimate 20% of 72. *(Lesson 6-6)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Solve each equation for y.**

(To review ***solving equations for a variable****, see Lesson 8-2.)*

30. $x + y = 6$ **31.** $3x + y = 1$ **32.** $-x + 5y = 10$

Practice Quiz 1 *Lessons 8-1 through 8-5*

Determine whether each relation is a function. Explain. *(Lesson 8-1)*

1. $\{(0, 5), (1, 2), (1, -3), (2, 4)\}$ **2.** $\{(-6, 3.5), (-3, 4.0), (0, 4.5), (3, 5.0)\}$

Graph each equation using ordered pairs. *(Lesson 8-2)*

3. $y = x - 4$ **4.** $y = 2x + 3$

Find the x-intercept and the y-intercept for the graph of each equation. *(Lesson 8-3)*

5. $y = x + 9$ **6.** $x + 2y = 12$ **7.** $4x - 5y = 20$

Find the slope of the line that passes through each pair of points. *(Lessons 8-4 and 8-5)*

8. $(1, 4), (0, 0)$ **9.** $(-2, 4), (3, -6)$ **10.** $(0, 2), (5, 2)$

CONTENTS

8-6 Slope-Intercept Form

What You'll Learn

- Determine slopes and y-intercepts of lines.
- Graph linear equations using the slope and y-intercept.

Vocabulary

- slope-intercept form

How can knowing the slope and y-intercept help you graph an equation?

Copy the table.

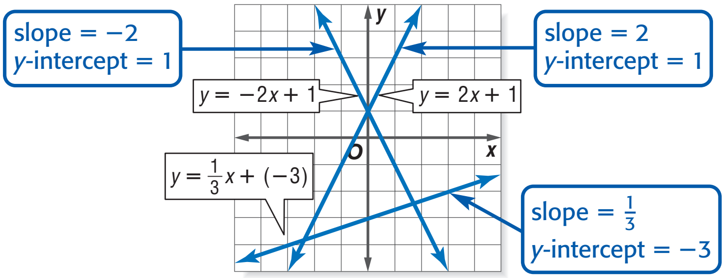
Equation	Slope	y-intercept
$y = 2x + 1$		
$y = \frac{1}{3}x - 3$		
$y = -2x + 1$		

a. On the same coordinate plane, use ordered pairs or intercepts to graph each equation in a different color.

b. Find the slope and the y-intercept of each line. Complete the table.

c. Compare each equation with the value of its slope and y-intercept. What do you notice?

SLOPE AND y-INTERCEPT



All the equations above are written in the form $y = mx + b$, where m is the slope and b is the y-intercept. This is called **slope-intercept form**.

$$y = \underset{\text{slope}}{m}x + \underset{y\text{-intercept}}{b}$$

Example 1 Find the Slope and y-Intercept

State the slope and the y-intercept of the graph of $y = \frac{3}{5}x - 7$.

$y = \frac{3}{5}x - 7$ Write the original equation.

$y = \frac{3}{5}x + (-7)$ Write the equation in the form $y = mx + b$.

$\uparrow \qquad \uparrow$

$y = mx + b$ $m = \frac{3}{5}$, $b = -7$

The slope of the graph is $\frac{3}{5}$, and the y-intercept is -7.

Study Tip

Different Forms

Both equations below are written in slope-intercept form.

$y = x + (-2)$

$y = x - 2$

Concept Check What is the slope of $y = 8x + 6$?

CONTENTS

Sometimes you must first write an equation in slope-intercept form before finding the slope and y-intercept.

Example 2 Write an Equation in Slope-Intercept Form

State the slope and the y-intercept of the graph of $5x + y = 3$.

$$
\begin{aligned}
5x + y &= 3 && \text{Write the original equation.}\\
5x + y - 5x &= 3 - 5x && \text{Subtract } 5x \text{ from each side.}\\
y &= \underset{\uparrow}{-5}x + \underset{\uparrow}{3} && \text{Write the equation in slope-intercept form.}\\
y &= mx + b && m = -5,\ b = 3
\end{aligned}
$$

The slope of the graph is -5, and the y-intercept is 3.

GRAPH EQUATIONS You can use the slope-intercept form of an equation to easily graph a line.

Example 3 Graph an Equation

Graph $y = -\frac{1}{2}x - 4$ using the slope and y-intercept.

Step 1 Find the slope and y-intercept.

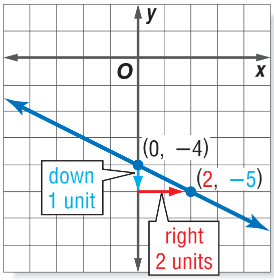
slope $= -\frac{1}{2}$ y-intercept $= -4$

Step 2 Graph the y-intercept point at $(0, -4)$.

Step 3 Write the slope $-\frac{1}{2}$ as $\frac{-1}{2}$. Use it to locate a second point on the line.

$$m = \frac{-1}{2} \begin{array}{l} \leftarrow \text{change in } y\text{: down 1 unit} \\ \leftarrow \text{change in } x\text{: right 2 units} \end{array}$$

Another point on the line is at $(2, -5)$.



Step 4 Draw a line through the two points.

Example 4 Graph an Equation to Solve a Problem

BUSINESS A T-shirt company charges a design fee of \$24 for a pattern and then sells the shirts for \$12 each. The total cost y can be represented by the equation $y = 12x + 24$, where x represents the number of T-shirts.

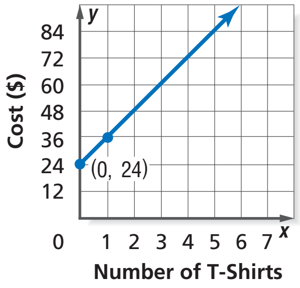
a. Graph the equation.

First, find the slope and the y-intercept.

slope $= 12$

y-intercept $= 24$

Plot the point at $(0, 24)$. Then go up 12 and right 1. Connect these points.



b. Describe what the y-intercept and the slope represent.

The y-intercept 24 represents the design fee. The slope 12 represents the cost per T-shirt, which is the rate of change.

Career Choices

Business Owner

Business owners must understand the factors that affect cost and profit. Graphs are a useful way for them to display this information.

Online Research

For information about a career as a business owner, visit: www.pre-alg.com/careers

www.pre-alg.com/extra_examples

Check for Understanding

Concept Check

1. **State** the value that tells you how many units to go up or down from the y-intercept if the slope of a line is $\frac{a}{b}$.

2. **OPEN ENDED** Draw the graph of a line that has a y-intercept but no x-intercept. What is the slope of the line?

3. **FIND THE ERROR** Carmen and Alex are finding the slope and y-intercept of $x + 2y = 8$.

Carmen

slope = 2

y-intercept = 8

Alex

slope = $-\frac{1}{2}$

y-intercept = 4

Who is correct? Explain your reasoning.

Guided Practice

State the slope and the y-intercept for the graph of each equation.

4. $y = x + 8$
5. $x + y = 0$
6. $x + 3y = 6$

Graph each equation using the slope and y-intercept.

7. $y = \frac{1}{4}x + 1$
8. $3x + y = 2$
9. $x - 2y = 4$

Application

BUSINESS **Mrs. Allison charges $25 for a basic cake that serves 12 people. A larger cake costs an additional $1.50 per serving. The total cost can be given by $y = 1.5x + 25$, where x represents the number of additional slices.**

10. Graph the equation.
11. Explain what the y-intercept and the slope represent.

Practice and Apply

Homework Help

For Exercises	See Examples
12–17	1, 2
18–31	3
32–34	4

Extra Practice
See page 743.

State the slope and the y-intercept for the graph of each equation.

12. $y = x + 2$
13. $y = 2x - 4$
14. $x + y = -3$
15. $2x + y = -3$
16. $5x + 4y = 20$
17. $y = 4$

Graph each line with the given slope and y-intercept.

18. slope = 3, y-intercept = 1
19. slope = $-\frac{3}{2}$, y-intercept = -1

Graph each equation using the slope and y-intercept.

20. $y = x + 5$
21. $y = -x + 6$
22. $y = 2x - 3$
23. $y = \frac{3}{4}x + 2$
24. $x + y = -3$
25. $x + y = 0$
26. $-2x + y = -1$
27. $5x + y = -3$
28. $x - 3y = -6$
29. $2x + 3y = 12$
30. $3x + 4y = 12$
31. $y = -3$

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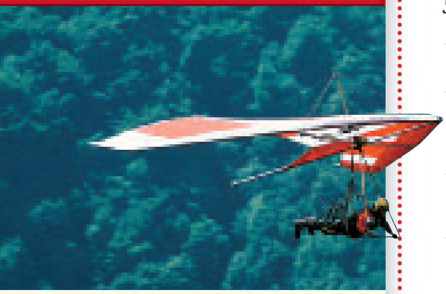
More About. . .



Hang Gliding

In the summer, hang gliders in the western part of the United States achieve altitudes of 5000 to 10,000 feet and fly for over 100 miles.

Source: www.ushga.org

HANG GLIDING **For Exercises 32–34, use the following information.** The altitude in feet y of a hang glider who is slowly landing can be given by $y = 300 - 50x$, where x represents the time in minutes.

32. Graph the equation using the slope and y-intercept.

33. State the slope and y-intercept of the graph of the equation and describe what they represent.

34. Name the x-intercept and describe what it represents.

35. **CRITICAL THINKING** What is the x-intercept of the graph of $y = mx + b$? Explain how you know.

36. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can knowing the slope and y-intercept help you graph an equation?

Include the following in your answer:

- a description of how the slope-intercept form of an equation gives information, and
- an explanation of how you could write an equation for a line if you know the slope and y-intercept.

Standardized Test Practice

37. Which is $2x + 3y = 6$ written in slope-intercept form?

Ⓐ $y = -\frac{2}{3}x - 2$

Ⓑ $y = -\frac{2}{3}x + 2$

Ⓒ $y = -\frac{3}{2}x + 2$

Ⓓ $y = \frac{3}{2}x - 2$

38. What is the slope and y-intercept of the graph of $-x + 2y = 6$?

Ⓐ $-\frac{1}{2}, 1$

Ⓑ $1, 6$

Ⓒ $1, 3$

Ⓓ $\frac{1}{2}, 3$

Maintain Your Skills

Mixed Review

Suppose y varies directly with x. Write an equation relating x and y for each pair of values. *(Lesson 8-5)*

39. $y = -36$ when $x = 9$

40. $y = 5$ when $x = 25$

Find the slope of the line that passes through each pair of points. *(Lesson 8-4)*

41. $A(3, 1), B(6, 7)$

42. $J(-2, 5), K(8, 5)$

43. $Q(2, 4), R(0, -4)$

44. Solve $4(r - 3) = 8$. *(Lesson 7-2)*

45. Six times a number is 28 more than twice the number. Write an equation and find the number. *(Lesson 7-1)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Simplify.** *(To review **order of operations**, see Lesson 1-2.)*

46. $2(18) - 1$

47. $(-2 - 4) \div 10$

48. $-1(6) + 8$

49. $5 - 8(-3)$

50. $(9 + 6) \div 3$

51. $3 - (-2)(4)$

www.pre-alg.com/self_check_quiz

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Graphing Calculator Investigation

A Follow-Up of Lesson 8-6

Families of Graphs

A graphing calculator is a valuable tool when investigating characteristics of linear functions. Before graphing, you must create a viewing window that shows both the *x*- and *y*-intercepts of the graph of a function.

You can use the standard viewing window $[-10, 10]$ scl: 1 by $[-10, 10]$ scl: 1 or set your own minimum and maximum values for the axes and the scale factor by using the WINDOW option.

The tick marks on the *x* scale and on the *y* scale are 1 unit apart.

$[-10, 10]$ scl: 1 by $[-10, 10]$ scl: 1

The *x*-axis goes from −10 to 10.

The *y*-axis goes from −10 to 10.

You can use a TI-83 Plus graphing calculator to enter several functions and graph them at the same time on the same screen. This is useful when studying a **family of graphs**. A family of linear graphs is related by having the same slope or the same *y*-intercept.

Graph $y = 3x - 2$ and $y = 3x + 4$ in the standard viewing window and describe how the graphs are related.

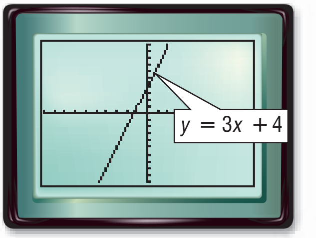
Step 1 Graph $y = 3x + 4$ in the standard viewing window.

- Clear any existing equations from the Y= list.

KEYSTROKES: `Y=` `CLEAR`

- Enter the equation and graph.

KEYSTROKES: `Y=` 3 `X,T,θ,n` `+` 4 `ZOOM` 6



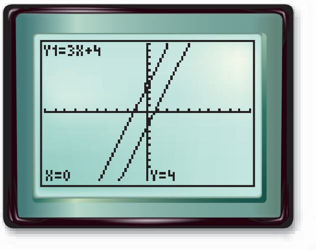
Step 2 Graph $y = 3x - 2$.

- Enter the function $y = 3x - 2$ as Y2 with $y = 3x + 4$ already existing as Y1.

KEYSTROKES: `Y=` 3 `X,T,θ,n` `−` 2

- Graph both functions in the standard viewing window.

KEYSTROKES: `ZOOM` 6



Y1=3X+4

X=0 Y=4

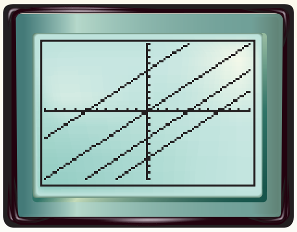
The first function graphed is Y1 or $y = 3x + 4$. The second function graphed is Y2 or $y = 3x - 2$. Press `TRACE`. Move along each function using the right and left arrow keys. Move from one function to another using the up and down arrow keys. The graphs have the same slope, 3, but different *y*-intercepts at 4 and −2.

www.pre-alg.com/other_calculator_keystrokes

Exercises

Graph $y = 2x - 5$, $y = 2x - 1$, and $y = 2x + 7$.

1. Compare and contrast the graphs.
2. How does adding or subtracting a constant c from a linear function affect its graph?
3. Write an equation of a line whose graph is parallel to $y = 3x - 5$, but is shifted up 7 units.
4. Write an equation of the line that is parallel to $y = 3x - 5$ and passes through the origin.
5. Four functions with a slope of 1 are graphed in the standard viewing window, as shown at the right. Write an equation for each, beginning with the left-most graph.

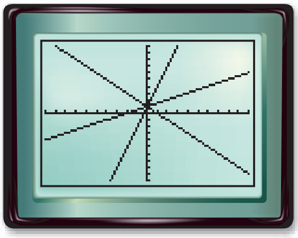


Clear all functions from the Y= menu and graph $y = \frac{1}{3}x$, $y = \frac{3}{4}x$, $y = x$, and $y = 4x$ in the standard viewing window.

6. How does the steepness of a line change as the coefficient for x increases?
7. Without graphing, determine whether the graph of $y = 0.4x$ or the graph of $y = 1.4x$ has a steeper slope. Explain.

Clear all functions from the Y= menu and graph $y = -4x$ and $y = 4x$.

8. How are these two graphs different?
9. How does the sign of the coefficient of x affect the slope of a line?
10. Clear Y2. Then with $y = -4x$ as Y1, enter $y = -x$ as Y2 and $y = -\frac{1}{2}x$ as Y3. Graph the functions and draw the three graphs on grid paper. How does the steepness of the line change as the absolute value of the coefficient of x increases?
11. The graphs of $y = 3x + 1$, $y = \frac{1}{2}x + 1$, and $y = -x + 1$ are shown at the right. Draw the graphs on the same coordinate grid and label each graph with its equation.



12. Describe the similarities and differences between the graph of $y = 2x - 3$ and the graph of each equation listed below.
    - **a.** $y = 2x + 3$
    - **b.** $y = -2x - 3$
    - **c.** $y = 0.5x + 3$
13. Write an equation of a line whose graph lies between the graphs of $y = -3x$ and $y = -6x$.

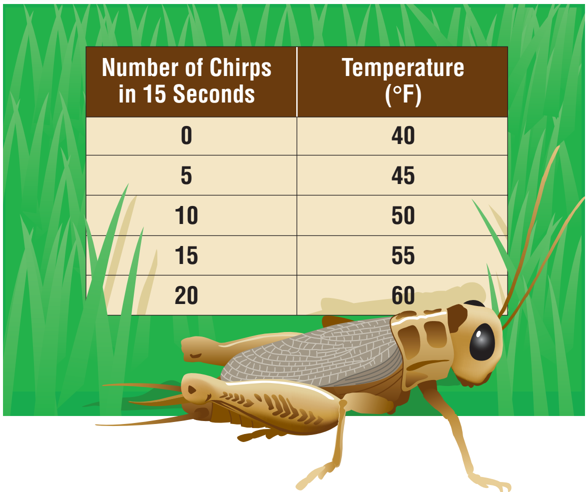
8-7 Writing Linear Equations

What You'll Learn

- Write equations given the slope and y-intercept, a graph, a table, or two points.

How can you model data with a linear equation?

You can determine the approximate outside temperature by counting the chirps of crickets, as shown in the table.



Number of Chirps in 15 Seconds	Temperature (°F)
0	40
5	45
10	50
15	55
20	60

a. Graph the ordered pairs (chirps, temperature). Draw a line through the points.

b. Find the slope and the y-intercept of the line. What do these values represent?

c. Write an equation in the form $y = mx + b$ for the line. Then translate the equation into a sentence.

WRITE EQUATIONS There are many different methods for writing linear equations. If you know the slope and y-intercept, you can write the equation of a line by substituting these values in $y = mx + b$.

Example 1 Write Equations From Slope and y-Intercept

Write an equation in slope-intercept form for each line.

a. slope = 4, y-intercept = −8

$y = mx + b$ Slope-intercept form

$y = 4x + (-8)$ Replace m with 4 and b with −8.

$y = 4x - 8$ Simplify.

b. slope = 0, y-intercept = 5

$y = mx + b$ Slope-intercept form

$y = 0x + 5$ Replace m with 0 and b with 5.

$y = 5$ Simplify.

c. slope = $-\frac{1}{2}$, y-intercept = 0

$y = mx + b$ Slope-intercept form

$y = -\frac{1}{2}x + 0$ Replace m with $-\frac{1}{2}$ and b with 0.

$y = -\frac{1}{2}x$ Simplify.

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You can also write equations from a graph.

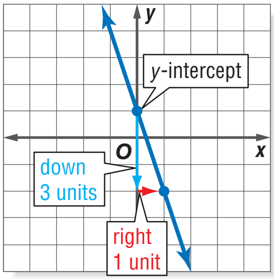
Study Tip

Check Equation

To check, choose another point on the line and substitute its coordinates for x and y in the equation.

Example 2 Write an Equation From a Graph

Write an equation in slope-intercept form for the line graphed.



The y-intercept is 1. From (0, 1), you can go down 3 units and right 1 unit to another point on the line. So, the slope is $\frac{-3}{1}$, or -3.

$y = mx + b$ Slope-intercept form

$y = -3x + 1$ Replace m with -3 and b with 1.

In Lesson 8-3, you explored the relationship between altitude and temperature. You can write an equation for this relationship and use it to make predictions.

Example 3 Write an Equation to Solve a Problem

EARTH SCIENCE **On a summer day, the temperature at altitude 0, or sea level, is 30°C. The temperature decreases 2°C for every 305 meters increase in altitude.**

a. Write an equation to show the relationship between altitude x and temperature y.

Words Temperature decreases 2°C for every 305 meters increase in altitude.

Variables Let x = the altitude and let y = the temperature.

Equations Use $m = \frac{\text{change in } y}{\text{change in } x}$ and $y = mx + b$.

Step 1

Find the slope m.

$m = \frac{\text{change in } y}{\text{change in } x}$ ← change in temperature / ← change in altitude

$= \frac{-2}{305}$ ← decrease of −2°C / ← increase of 305 m

≈ -0.007 Simplify.

Step 2

Find the y-intercept b.

$(x, y) = (\text{altitude, temperature})$

$= (0, b)$

When the altitude is 0, or sea level, the temperature is 30°C. So, the y-intercept is 30.

Study Tip

Use a Table

Translate the words into a table of values to help clarify the meaning of the slope.

	Alt. (m)	Temp. (°C)	
	0	30	
+305	305	28	−2
+305	610	26	−2

Step 3

Write the equation.

$y = mx + b$ Slope-intercept form

$y = -0.007x + 30$ Replace m with -0.007 and b with 30.

So, the equation that represents this situation is $y = -0.007x + 30$.

b. Predict the temperature for an altitude of 2000 meters.

$y = -0.007x + 30$ Write the equation.

$y = -0.007(2000) + 30$ Replace x with 2000.

$y \approx 16$ Simplify.

So, at an altitude of 2000 meters, the temperature is about 16°C.

You can also write an equation for a line if you know the coordinates of two points on a line.

Example 4 Write an Equation Given Two Points

Write an equation for the line that passes through (−2, 5) and (2, 1).

Step 1 Find the slope m.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Definition of slope

$m = \frac{5 - 1}{-2 - 2}$ or -1 $(x_1, y_1) = (-2, 5)$, $(x_2, y_2) = (2, 1)$

Step 2 Find the y-intercept b. Use the slope and the coordinates of either point.

$y = mx + b$ Slope-intercept form

$5 = -1(-2) + b$ Replace (x, y) with $(-2, 5)$ and m with -1.

$3 = b$ Simplify.

Step 3 Substitute the slope and y-intercept.

$y = mx + b$ Slope-intercept form

$y = -1x + 3$ Replace m with -1 and b with 3.

$y = -x + 3$ Simplify.

Example 5 Write an Equation From a Table

Use the table of values to write an equation in slope-intercept form.

x	y
−5	6
5	−2
10	−6
15	−10

Step 1 Find the slope m. Use the coordinates of any two points.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Definition of slope

$m = \frac{-2 - 6}{5 - (-5)}$ or $-\frac{4}{5}$ $(x_1, y_1) = (-5, 6)$, $(x_2, y_2) = (5, -2)$

Step 2 Find the y-intercept b. Use the slope and the coordinates of any point.

$y = mx + b$ Slope-intercept form

$6 = -\frac{4}{5}(-5) + b$ Replace (x, y) with $(-5, 6)$ and m with $-\frac{4}{5}$.

$2 = b$ Simplify.

Step 3 Substitute the slope and y-intercept.

$y = mx + b$ Slope-intercept form

$y = -\frac{4}{5}x + 2$ Replace m with $-\frac{4}{5}$ and b with 2.

CHECK $y = -\frac{4}{5}x + 2$ Write the equation.

$-10 \overset{?}{=} -\frac{4}{5}(15) + 2$ Replace (x, y) with the coordinates of another point, $(15, -10)$.

$-10 = -10$ ✓ Simplify.

Study Tip

Alternate Strategy
If a table includes the y-intercept, simply use this value and the slope to write an equation.

x	y
−5	6
0	2

y-intercept = 2

Check for Understanding

Concept Check

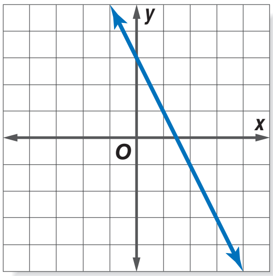
1. **Explain** how to write the equation of a line if you are given a graph.
2. **OPEN ENDED** Choose a slope and y-intercept. Then graph the line.

Guided Practice

Write an equation in slope-intercept form for each line.

3. slope = $\frac{1}{2}$, y-intercept = 1
4. slope = 0, y-intercept = −7

5.



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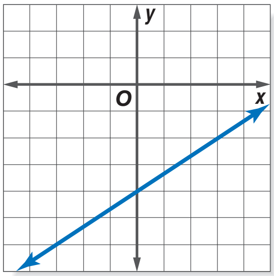


Write an equation in slope-intercept form for the line passing through each pair of points.

7. (2, 2) and (4, 3)
8. (3, −4) and (−1, 4)
9. Write an equation in slope-intercept form to represent the table of values.

x	−4	0	4	8
y	−4	−1	2	5

Application

10. **PICNICS** It costs $50 plus $10 per hour to rent a park pavilion.
    - **a.** Write an equation in slope-intercept form that shows the cost y for renting the pavilion for x hours.
    - **b.** Find the cost of renting the pavilion for 8 hours.

Practice and Apply

Homework Help

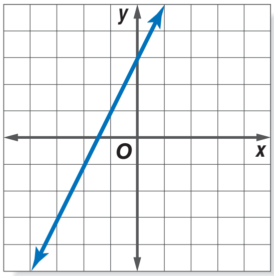
For Exercises	See Examples
11–16	1
17–22	2
23–28	4
29, 30	5
31–33	3

Extra Practice
See page 743.

Write an equation in slope-intercept form for each line.

11. slope = 2, y-intercept = 6
12. slope = −4, y-intercept = 1
13. slope = 0, y-intercept = 5
14. slope = 1, y-intercept = −2
15. slope = $-\frac{1}{3}$, y-intercept = 8
16. slope = $\frac{2}{5}$, y-intercept = 0

17.



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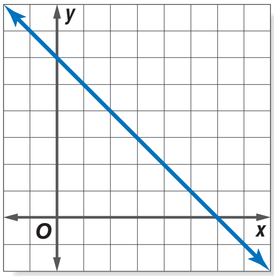

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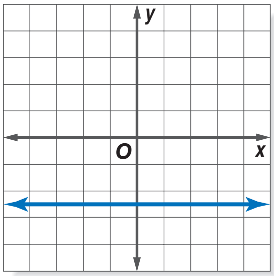

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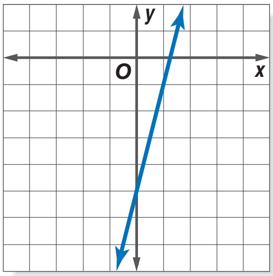

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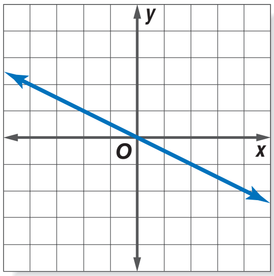

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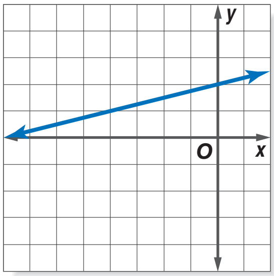


www.pre-alg.com/self_check_quiz

CONTENTS

Write an equation in slope-intercept form for the line passing through each pair of points.

23. (−2, −1) and (1, 2) **24.** (−4, 3) and (4, −1) **25.** (0, 0) and (−1, 1)

26. (4, 2) and (−8, −16) **27.** (8, 7) and (−9, 7) **28.** (5, −6) and (3, 2)

Write an equation in slope-intercept form for each table of values.

29.

x	−1	0	1	2
y	−7	−3	1	5

30.

x	−3	−1	1	3
y	7	5	3	1

More About. . .

Sound

Coyotes communicate by using different barks and howls. Because of the way in which sound travels, a coyote is usually not in the area from which the sound seems to be coming.

Source: www.livingdesert.org

SOUND **For Exercises 31 and 32, use the table that shows the distance that sound travels through dry air at 0°C.**

Time(s)	Distance (ft)
x	y
0	0
1	1088
2	2176
3	3264

31. Write an equation in slope-intercept form to represent the data in the table. Describe what the slope means.

32. Estimate the number of miles that sound travels through dry air in one minute.

33. **CRITICAL THINKING** A CD player has a pre-sale price of \$$c$. Kim buys it at a 30% discount and pays 6% sales tax. After a few months, she sells it for \$$d$, which was 50% of what she paid originally.

a. Express d as a function of c.

b. How much did Kim sell it for if the pre-sale price was \$50?

34. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can you model data with a linear equation?

Include the following in your answer:

- an explanation of how to find the y-intercept and slope by using a table.

Standardized Test Practice

35. Which equation is of a line that passes through (2, −2) and (0, 2)?

Ⓐ $y = -3x$ Ⓑ $y = -2x + 3$ Ⓒ $y = -2x + 2$ Ⓓ $y = -3x + 2$

36. Which equation represents the table of values?

x	−4	−8	−12	−16
y	6	8	10	12

Ⓐ $y = -2x + 4$ Ⓑ $y = 2x + 6$

Ⓒ $y = -\frac{1}{2}x + 4$ Ⓓ $y = -x - 6$

Maintain Your Skills

Mixed Review

State the slope and the y-intercept for the graph of each equation. *(Lesson 8-6)*

37. $y = 6x + 7$ **38.** $y = -x + 4$ **39.** $-3x + y = -2$

40. Suppose y varies directly as x and $y = 14$ when $x = 35$. Write an equation relating x and y. *(Lesson 8-5)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL *(To review* ***scatter plots****, see Lesson 1-7.)*

41. State whether a scatter plot containing the following set of points would show a *positive*, *negative*, or *no* relationship.
(0, 15), (2, 20), (4, 36), (5, 44), (4, 32), (3, 30), (6, 50)

8-8 Best-Fit Lines

What You'll Learn

- Draw best-fit lines for sets of data.
- Use best-fit lines to make predictions about data.

Vocabulary

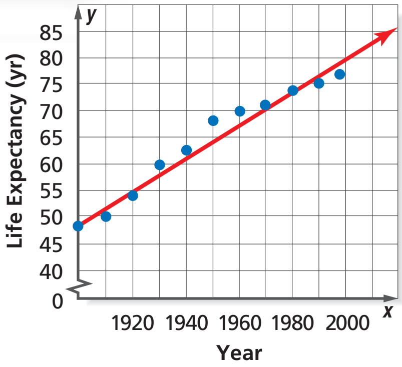
- best-fit line

How can a line be used to predict life expectancy for future generations?

The scatter plot shows the number of years people in the United States are expected to live, according to the year they were born.

a. Use the line drawn through the points to predict the life expectancy of a person born in 2010.

b. What are some limitations in using a line to predict life expectancy?



Source: *The World Almanac*

Study Tip

Estimation

Drawing a best-fit line using the method in this lesson is an estimation. Therefore, it is possible to draw different lines to approximate the same data.

BEST-FIT LINES When real-life data are collected, the points graphed usually do not form a straight line, but may approximate a linear relationship. A best-fit line can be used to show such a relationship. A **best-fit line** is a line that is very close to most of the data points.

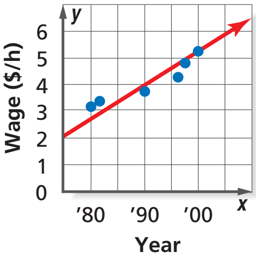
Example 1 *Make Predictions from a Best-Fit Line*

MONEY **The table shows the changes in the minimum wage since 1980.**

a. Make a scatter plot and draw a best-fit line for the data.

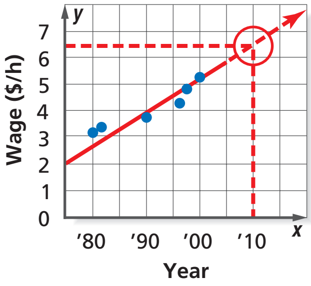
Year	Wage ($/h)
1980	3.10
1981	3.35
1990	3.80
1996	4.25
1997	4.75
2000	5.15

Draw a line that best fits the data.



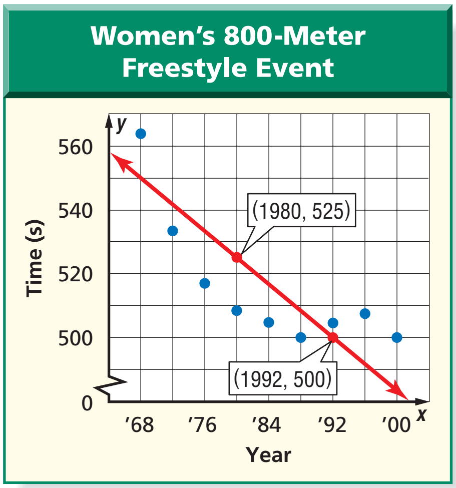
b. Use the best-fit line to predict the minimum wage for the year 2010.

Extend the line so that you can find the y value for an x value of 2010. The y value for 2010 is about 6.4. So, a prediction for the minimum wage in 2010 is approximately $6.40.



PREDICTION EQUATIONS You can also make predictions from the equation of a best-fit line.

Example 2 Make Predictions from an Equation


Women's 800-Meter Freestyle Event

SWIMMING **The scatter plot shows the winning Olympic times in the women's 800-meter freestyle event from 1968 through 2000.**

a. Write an equation in slope-intercept form for the best-fit line.

Step 1

First, select two points on the line and find the slope. Notice that the two points on the best-fit line are not original data points. We have chosen (1980, 525) and (1992, 500).

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Definition of slope

$= \frac{525 - 500}{1980 - 1992}$ $(x_1, y_1) = (1992, 500)$, $(x_2, y_2) = (1980, 525)$

≈ -2.1 Simplify.

Step 2

Next, find the y-intercept.

$y = mx + b$ Slope-intercept form

$525 = -2.1(1980) + b$ Replace (x, y) with (1980, 525) and m with -2.1.

$4683 \approx b$ Simplify.

Step 3

Write the equation.

$y = mx + b$ Slope-intercept form

$y = -2.1x + 4683$ Replace m with -2.1 and b with 4683.

b. Predict the winning time in the women's 800-meter freestyle event in the year 2008.

$y = -2.1x + 4683$ Write the equation of the best-fit line.

$y = -2.1(2008) + 4683$ Replace x with 2008.

$y \approx 466.2$ Simplify.

A prediction for the winning time in the year 2008 is approximately 466.2 seconds or 7 minutes, 46.2 seconds.

Check for Understanding

Concept Check

1. **Explain** how to use a best-fit line to make a prediction.

2. **OPEN ENDED** Make a scatter plot with at least ten points that appear to be somewhat linear. Draw two different lines that could approximate the data.

CONTENTS

Guided Practice

TECHNOLOGY **For Exercises 3 and 4, use the table that shows the number of U.S. households with Internet access.**

3. Make a scatter plot and draw a best-fit line.

4. Use the best-fit line to predict the number of U.S. households that will have Internet access in 2005.

Year	Number of Households (millions)
1995	9.4
1996	14.7
1997	21.3
1998	27.3
1999	32.7
2000	36.0

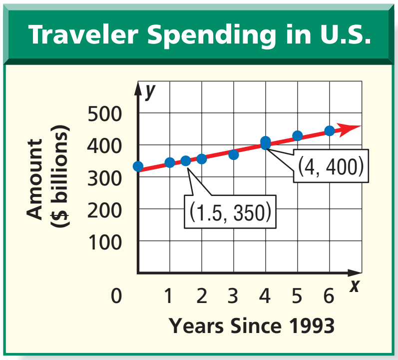
Source: *Wall Street Journal Almanac*

Application

SPENDING **For Exercises 5 and 6, use the best-fit line that shows the billions of dollars spent by travelers in the United States.**

5. Write an equation in slope-intercept form for the best-fit line.

6. Use the equation to predict how much money travelers will spend in 2008.



Practice and Apply

Homework Help

For Exercises	See Examples
7, 8, 12, 13	1
9–11, 14–17	2

Extra Practice
See page 744.

ENTERTAINMENT **For Exercises 7 and 8, use the table that shows movie attendance in the United States.**

7. Make a scatter plot and draw a best-fit line.

8. Use the best-fit line to predict movie attendance in 2005.

Year	Attendance (millions)
1993	1244
1994	1292
1995	1263
1996	1339
1997	1388
1998	1475
1999	1460

Source: *Wall Street Journal Almanac*

PRESSURE **For Exercises 9–11, use the table that shows the approximate barometric pressure at various altitudes.**

9. Make a scatter plot of the data and draw a best-fit line.

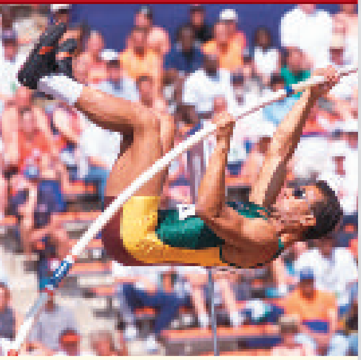
10. Write an equation for the best-fit line and use it to estimate the barometric pressure at 60,000 feet. Is the estimation reasonable? Explain.

11. Do you think that a line is the best model for this data? Explain.

Altitude (ft)	Barometric Pressure (in. mercury)
0	30
5000	25
10,000	21
20,000	14
30,000	9
40,000	6
50,000	3

Source: *New York Public Library Science Desk Reference*

More About. . .



Pole Vaulting

In 1964, thirteen competitors broke or equaled the previous Olympic pole vault record a total of 36 times. This was due to the new fiberglass pole.

Source: *Chance*

POLE VAULTING **For Exercises 12 and 13, use the table that shows the men's winning Olympic pole vault heights to the nearest inch.**

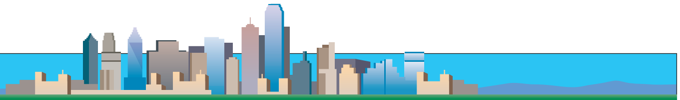
Year	Height (in.)
1976	217
1980	228
1984	226
1988	232
1992	228
1996	233
2000	232

Source: *The World Almanac*

12. Make a scatter plot and draw a best-fit line.

13. Use the best-fit line to predict the winning pole vault height in the 2008 Olympics.

EARTH SCIENCE **For Exercises 14–17, use the table that shows the latitude and the average temperature in July for five cities in the United States.**



City	Latitude (°N)	Average July High Temperature (°F)
Chicago, IL	41	73
Dallas, TX	32	85
Denver, CO	39	74
New York, NY	40	77
Duluth, MN	46	66

Source: *The World Almanac*

14. Make a scatter plot of the data and draw a best-fit line.

15. Describe the relationship between latitude and temperature shown by the graph.

16. Write an equation for the best-fit line you drew in Exercise 14.

17. Use your equation to estimate the average July temperature for a location with latitude 50° north.

18. **CRITICAL THINKING** The table at the right shows the percent of public schools in the United States with Internet access. Suppose you use (Year, Percent of Schools) to write a linear equation describing the data. Then you use (Years Since 1996, Percent of Schools) to write an equation. Is the slope or y-intercept of the graphs of the equations the same? Explain.

Year	Years Since 1996	Percent of Schools
1996	0	65
1997	1	78
1998	2	89
1999	3	95
2000	4	98

Source: National Center for Education Statistics

WebQuest

Best-fit lines can help make predictions about recreational activities. Visit www.pre-alg.com/webquest to continue work on your WebQuest project.

19. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can a line be used to predict life expectancy for future generations?

Include the following in your answer:

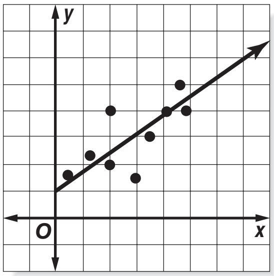
- a description of a best-fit line, and
- an explanation of how lines can represent sets of data that are not exactly linear.

Standardized Test Practice

20. Use the best-fit line at the right to predict the value of y when $x = 7$.

Ⓐ 4 Ⓑ 6

Ⓒ 0 Ⓓ 7



21. Choose the correct statement about best-fit lines.

Ⓐ A best-fit line is close to most of the data points.

Ⓑ A best-fit line describes the exact coordinates of each point in the data set.

Ⓒ A best-fit line always has a positive slope.

Ⓓ A best-fit line must go through at least two of the data points.

Maintain Your Skills

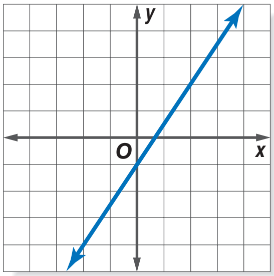
Mixed Review

Write an equation in slope-intercept form for each line. *(Lesson 8-7)*

22. slope = 3, y-intercept = 5

23. slope = −2, y-intercept = 2

24.



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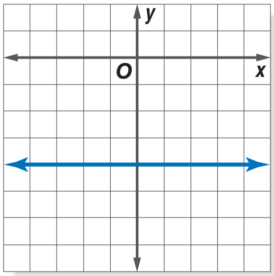


Graph each equation using the slope and y-intercept. *(Lesson 8-6)*

26. $y = x - 2$

27. $y = -x + 3$

28. $y = \frac{1}{2}x$

Solve each inequality and check your solution. *(Lesson 7-6)*

29. $3n - 11 \leq 10$

30. $8(x + 1) < 16$

31. $5d + 2 > d - 4$

32. SCHOOL Zach has no more than twelve days to complete his science project. Write an inequality to represent this sentence. *(Lesson 7-3)*

Solve each proportion. *(Lesson 6-2)*

33. $\frac{a}{3} = \frac{16}{24}$

34. $\frac{5}{10} = \frac{15}{x}$

35. $\frac{2}{16} = \frac{n}{36}$

36. Evaluate xy if $x = \frac{8}{9}$ and $y = \frac{12}{30}$. Write in simplest form. *(Lesson 5-3)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find the value of y in each equation by substituting the given value of x.** *(To review **substitution**, see Lesson 1-2.)*

37. $y = x + 1; x = 2$

38. $y = x + 5; x = -1$

39. $y = x - 4; x = 3$

40. $x + y = 2; x = 0$

41. $x + y = -1; x = 1$

42. $x + y = 0; x = 4$

CONTENTS

8-9 Solving Systems of Equations

What You'll Learn

- Solve systems of linear equations by graphing.
- Solve systems of linear equations by substitution.

Vocabulary

- system of equations
- substitution

How can a system of equations be used to compare data?

Yolanda is offered two summer jobs, as shown in the table.

Job	Hourly Rate	Bonus
A	\$10	\$50
B	\$15	\$0

a. Write an equation to represent the income from each job. Let y equal the salary and let x equal the number of hours worked. (*Hint*: Income = hourly rate · number of hours worked + bonus.)

b. Graph both equations on the same coordinate plane.

c. What are the coordinates of the point where the two lines meet? What does this point represent?

SOLVE SYSTEMS BY GRAPHING The equations $y = 10x + 50$ and $y = 15x$ together are called a **system of equations**. The solution of this system is the ordered pair that is a solution of both equations, (10, 150).

$$y = 10x + 50$$
$$150 \stackrel{?}{=} 10(10) + 50$$
$$150 = 150 \checkmark$$

Replace (x, y) with (10, 150).

$$y = 15x$$
$$150 \stackrel{?}{=} 15(10)$$
$$150 = 150 \checkmark$$

One method for solving a system of equations is to graph the equations on the same coordinate plane. The coordinates of the point where the graphs intersect is the solution of the system of equations.

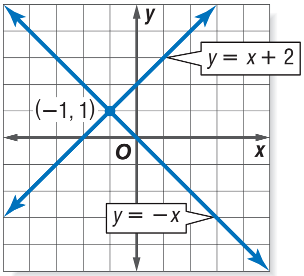
Example 1 Solve by Graphing

Solve the system of equations by graphing.

$y = -x$

$y = x + 2$

The graphs appear to intersect at (−1, 1). Check this estimate by substituting the coordinates into each equation.



CHECK

$y = -x$
$1 \stackrel{?}{=} -(-1)$
$1 = 1 \checkmark$

$y = x + 2$
$1 \stackrel{?}{=} -1 + 2$
$1 = 1 \checkmark$

The solution of the system of equations is (−1, 1).

Concept Check Is (0, 0) a solution of the system of equations in Example 1? Explain.

CONTENTS

Example 2 One Solution

ENTERTAINMENT Video Planet offers two rental plans.

Plan	Rental Fee Per Video	Annual Fee
A	\$4.00	\$0
B	\$1.50	\$20

a. How many videos would Walt need to rent in a year for the plans to cost the same?

Explore You know the rental fee per video and the annual fee.

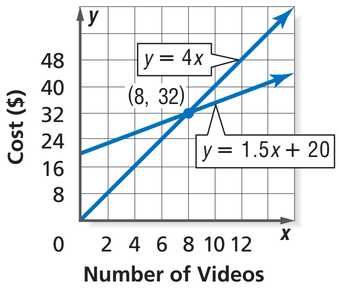
Plan Write an equation to represent each plan, and then graph the equations to find the solution.

Solve Let x = number of videos rented and let y = the total cost.

$$\text{Plan A} \quad \overbrace{y}^{\text{total cost}} = \overbrace{4x}^{\text{rental fee times number of videos}} + \overbrace{0}^{\text{annual fee}}$$

$$\text{Plan B} \quad y = 1.50x + 20$$

The graph of the system shows the solution is (8, 32). This means that if Walt rents 8 videos in a year, the plans cost the same, \$32.



Examine Check by substituting (8, 32) into both equations in the system.

b. Which plan would cost less if Walt rents 12 videos in a year?

For $x = 12$, the line representing Plan B has a smaller y value. So, Plan B would cost less.

Study Tip

Slopes/Intercepts

When the graphs of a system of equations have:

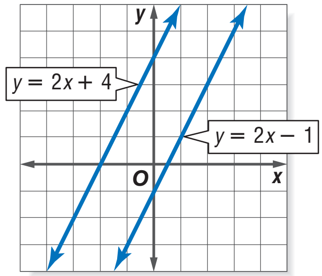
- different slopes, there is exactly one solution,
- the same slope and different y-intercepts, there is no solution,
- the same slope and the same y-intercept, there are infinitely many solutions.

Example 3 No Solution

Solve the system of equations by graphing.

$y = 2x + 4$

$y = 2x - 1$



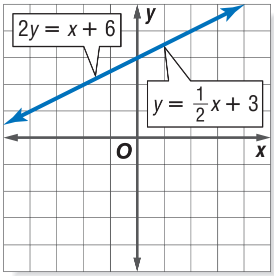
The graphs appear to be parallel lines. Since there is no coordinate pair that is a solution to both equations, there is no solution of this system of equations.

Example 4 Infinitely Many Solutions

Solve the system of equations by graphing.

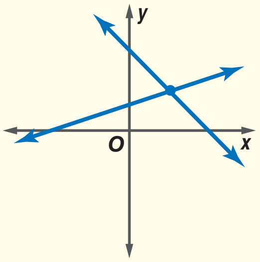
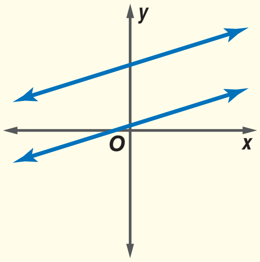
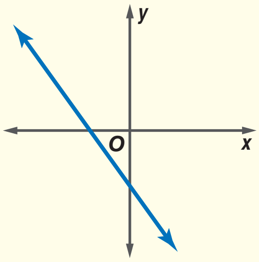
$2y = x + 6$

$y = \frac{1}{2}x + 3$



Both equations have the same graph. Any ordered pair on the graph will satisfy both equations. Therefore, there are infinitely many solutions of this system of equations.

Concept Summary — Solutions to Systems of Equations

One Solution	No Solution	Infinitely Many Solutions
		
Intersecting Lines	Parallel Lines	Same Line

SOLVE SYSTEMS BY SUBSTITUTION A more accurate way to solve a system of equations is by using a method called **substitution**.

Example 5 Solve by Substitution

Solve the system of equations by substitution.

$y = x + 5$

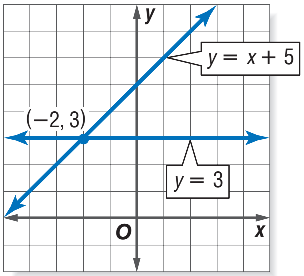
$y = 3$

Since y must have the same value in both equations, you can replace y with 3 in the first equation.

$y = x + 5$ Write the first equation.

$3 = x + 5$ Replace y with 3.

$-2 = x$ Solve for x.



The solution of this system of equations is $(-2, 3)$. You can check the solution by graphing. The graphs appear to intersect at $(-2, 3)$, so the solution is correct.

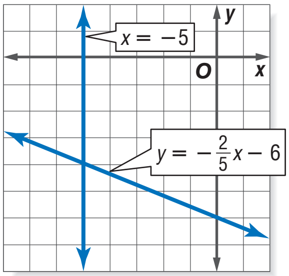
Check for Understanding

Concept Check

1. **Explain** what is meant by a system of equations and describe its solution.
2. **OPEN ENDED** Draw a graph of a system of equations that has one solution, a system that has no solution, and a system that has infinitely many solutions.

Guided Practice

3. State the solution of the system of equations graphed at the right.



Solve each system of equations by graphing.

4. $y = 2x + 1$
   $y = -x + 1$
5. $x + y = 4$
   $x + y = 2$

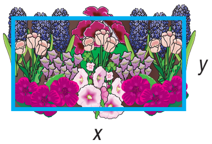
Solve each system of equations by substitution.

6. $y = 3x - 4$
   $x = 0$
7. $x + y = 8$
   $y = 6$

CONTENTS

Application

8. **GEOMETRY** The perimeter of a garden is 40 feet. If the width y equals 7 feet, write and solve a system of equations to find the length of the garden.



Practice and Apply

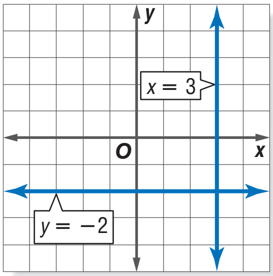
Homework Help

For Exercises	See Examples
9–17	1, 3, 4
18–23	5
24–26	2

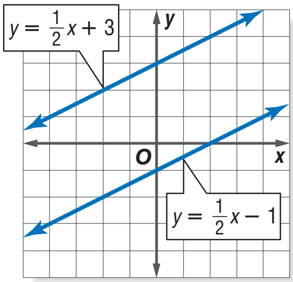
Extra Practice
See page 744.

State the solution of each system of equations.

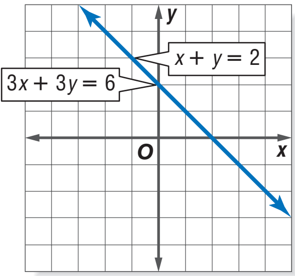
9.



10.



11.



Solve each system of equations by graphing.

12. $y = -x$
$y = x + 2$

13. $x + y = 6$
$y = x$

14. $2x + y = 1$
$y = -2x + 5$

15. $y = \frac{2}{3}x$
$2x - 3y = 0$

16. $x + y = -4$
$x - y = -4$

17. $y = -\frac{1}{2}x + 3$
$y = -2x$

Solve each system of equations by substitution.

18. $y = x + 1$
$x = 3$

19. $y = x + 2$
$y = 0$

20. $y = 2x + 7$
$y = -5$

21. $x + y = 6$
$x = -4$

22. $2x + 3y = 5$
$y = x$

23. $x + y = 9$
$y = 2x$

More About . . .

Snowboards

In 1998, snowboarding became an Olympic event in Nagano, Japan, with a giant slalom and halfpipe competition.

Source: www.snowboarding.about.com

SNOWBOARDS **For Exercises 24–26, use the following information and the table.** Two Internet sites sell a snowboard for the same price, but have different shipping charges.

Shipping Charges		
Internet Site	Base Fee	Charge per Pound
A	\$5.00	\$1.00
B	\$2.00	\$1.50

24. Write a system of equations that represents the shipping charges y for x pounds. (*Hint*: Shipping charge = base fee + charge per pound · number of pounds.)

25. Solve the system of equations. Explain what the solution means.

26. If the snowboard weighs 8 pounds, which Internet site would be less expensive? Explain.

27. **CRITICAL THINKING** Two runners A and B are 50 meters apart and running at the same rate along the same path.

a. If their rates continue, will the second runner ever catch up to the first? Draw a graph to explain why or why not.

b. Draw a graph that represents the second runner catching up to the first. What is different about the two graphs that you drew?

www.pre-alg.com/self_check_quiz

28. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can a system of equations be used to compare data?

Include the following in your answer:

- a situation that can be modeled by a system of equations, and
- an explanation of what a solution to such a system of equations means.

Standardized Test Practice

29. Which system of equations represents the following verbal description? *The sum of two numbers is 6. The second number is three times greater than the first number.*

Ⓐ $x + y = 6$, $x = 3 + y$ Ⓑ $y = x + 6$, $y = x + 3$ Ⓒ $x + y = 6$, $y = 3x$ Ⓓ $x - y = 6$, $y = -3x$

30. Which equation, together with $x + y = 1$, forms a system that has a solution of $(-3, 4)$?

Ⓐ $y = x$ Ⓑ $x - y = 1$ Ⓒ $y = x + 7$ Ⓓ $-3x + 4y = 1$

Maintain Your Skills

Mixed Review

31. **SALES** Ice cream sales increase as the temperature outside increases. Describe the slope of a best-fit line that represents this situation. *(Lesson 8-8)*

Write an equation in slope-intercept form for the line passing through each pair of points. *(Lesson 8-7)*

32. (0, 1) and (3, 7)   **33.** (−2, 6) and (1, −3)   **34.** (8, 0) and (−8, −4)

Getting Ready for the Next Lesson

PREREQUISITE SKILL **State whether each number is a solution of the given inequality.** *(To review* ***inequalities****, see Lesson 7-3.)*

35. $1 < x + 3$; 0   **36.** $9 > t - 5$; 1   **37.** $2y \geq 2$; −1

38. $14 < 6 - n$; 0   **39.** $35 > 12 + k$; 12   **40.** $5n + 1 \leq 0$; −2

Practice Quiz 2

Lessons 8-6 through 8-9

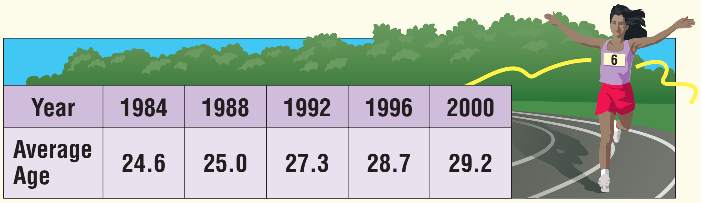
State the slope and the y-intercept for the graph of each equation. *(Lesson 8-6)*

1. $y = -x + 8$ **2.** $y = 3x - 5$ **3.** $x + 2y = 6$

Write an equation in slope-intercept form for each line. *(Lesson 8-7)*

4. slope = 6, y-intercept = −7   **5.** slope = 0, y-intercept = 1   **6.** slope = 1, y-intercept = 0

7. **STATISTICS** The table shows the average age of the Women's U.S. Olympic track and field team. Make a scatter plot of the data and draw a best-fit line. *(Lesson 8-8)*



Year	1984	1988	1992	1996	2000
Average Age	24.6	25.0	27.3	28.7	29.2

Source: *Sports Illustrated*

Solve each system of equations by substitution. *(Lesson 8-9)*

8. $y = x - 1$, $y = 2$ **9.** $y = x + 5$, $x = 0$ **10.** $y = 2x + 4$, $y = -4$

CONTENTS

8-10 Graphing Inequalities

What You'll Learn

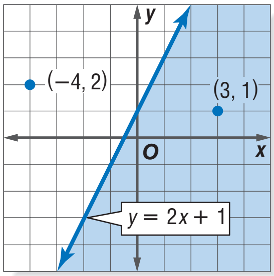
- Graph linear inequalities.
- Describe solutions of linear inequalities.

Vocabulary

- boundary
- half plane

How can shaded regions on a graph model inequalities?

Refer to the graph at the right.

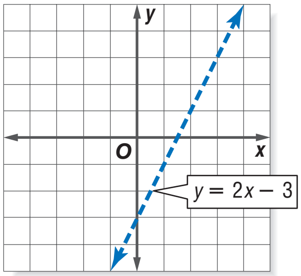


a. Substitute $(-4, 2)$ and $(3, 1)$ in $y > 2x + 1$. Which ordered pair makes the inequality true?

b. Substitute $(-4, 2)$ and $(3, 1)$ in $y < 2x + 1$. Which ordered pair makes the inequality true?

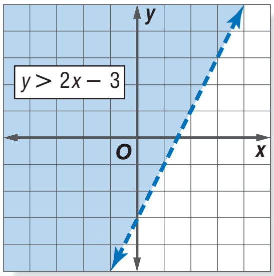
c. Which area represents the solution of $y < 2x + 1$?

GRAPH INEQUALITIES To graph an inequality such as $y > 2x - 3$, first graph the related equation $y = 2x - 3$. This is the **boundary**.



- If the inequality contains the symbol $\leq$ or $\geq$, then use a solid line to indicate that the boundary is included in the graph.
- If the inequality contains the symbol $<$ or $>$, then use a dashed line to indicate that the boundary is not included in the graph.

Next, test any point above or below the line to determine which region is the solution. It is easy to test (0, 0).



$y > 2x - 3$ Write the inequality.

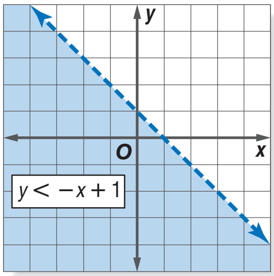
$0 \overset{?}{>} 2(0) - 3$ Replace *x* with 0 and *y* with 0.

$0 > -3$ ✓ Simplify.

Since $0 > -3$ is true, (0, 0) is a solution of $y > 2x - 3$. Shade the region that contains the solution. This region is called a **half plane**. All points in this region are solutions of the inequality.

Example 1 Graph Inequalities

a. Graph $y < -x + 1$.



Graph $y = -x + 1$. Draw a dashed line since the boundary is not part of the graph.

Test (0, 0): $y < -x + 1$

$0 \overset{?}{<} -0 + 1$ Replace (x, y) with $(0, 0)$.

$0 < 1$ ✓

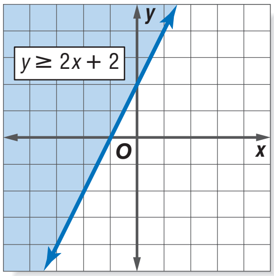
Thus, the graph is all points in the region below the boundary.

www.pre-alg.com/extra_examples

CONTENTS

b. Graph $y \geq 2x + 2$.

Graph $y = 2x + 2$. Draw a solid line since the boundary is part of the graph.



Test (0, 0): $y \geq 2x + 2$

$0 \overset{?}{\geq} 2(0) + 2$ Replace (x, y) with (0, 0).

$0 \geq 2$ not true

(0, 0) is not a solution, so shade the other half plane.

CHECK Test an ordered pair in the other half plane.

Concept Check

Is (0, 2) a solution of $y \geq 2x + 2$? Explain.

FIND SOLUTIONS You can write and graph inequalities to solve real-world problems. In some cases, you may have to solve the inequality for y first and then graph the inequality.

Example 2 Write and Graph an Inequality to Solve a Problem

SCHOOL **Nathan has at most 30 minutes to complete his math and science homework. How much time can he spend on each?**

Step 1 Write an inequality.

Let x represent the time spent doing math homework and let y represent the time spent doing science homework.

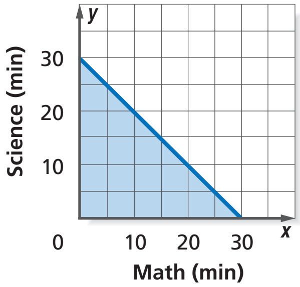
$$\underbrace{\text{Minutes doing math}}_{x}\quad\underbrace{\text{plus}}_{+}\quad\underbrace{\text{minutes doing science}}_{y}\quad\underbrace{\text{is at most}}_{\leq}\quad\underbrace{\text{30 minutes.}}_{30}$$

Step 2 Graph the inequality.

To graph the inequality, first solve for y.

$x + y \leq 30$ Write the inequality.

$y \leq -x + 30$ Subtract x from each side.



Graph $y \leq -x + 30$ as a solid line since the boundary is part of the graph. The origin is part of the graph since $0 \leq -0 + 30$. Thus, the coordinates of all points in the shaded region are possible solutions.

(10, 20) = 10 minutes on math, 20 minutes on science

(15, 15) = 15 minutes on math, 15 minutes on science

(10, 15) = 10 minutes on math, 15 minutes on science

(30, 0) = 30 minutes on math, 0 minutes on science

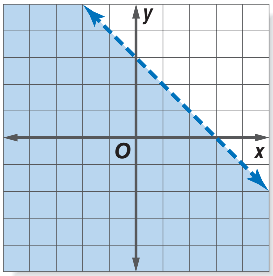
Note that the solutions are only in the first quadrant because negative values of time do not make sense.

Study Tip

Common Misconception
You may think that only whole number solutions are possible. Points in the shaded region such as (10.5, 18.5) are also solutions.

Check for Understanding

Concept Check



1. **Write** an inequality that describes the graph at the right.
2. **Explain** how to determine which side of the boundary line to shade when graphing an inequality.
3. **List** three solutions of each inequality.

   a. $y < x$ **b.** $y \geq x - 3$
4. **OPEN ENDED** Write an inequality that has (1, 4) as a solution.

Guided Practice **Graph each inequality.**

5. $y > x - 1$
6. $y \leq 2$
7. $y \geq -3x + 2$

Application **ENTERTAINMENT** **For Exercises 8 and 9, use the following information.** Adult passes for World Waterpark are $25, and children's passes are $15. A company is buying tickets for its employees and wants to spend no more than $630.

8. Write an inequality to represent this situation.
9. Graph the inequality and use the graph to determine three possible combinations of tickets that the company could buy.

Practice and Apply

Homework Help

For Exercises	See Examples
10–21	1
22–32	2

Extra Practice
See page 744.

Graph each inequality.

10. $y \geq x$
11. $y < x$
12. $y < 1$
13. $y \geq -3$
14. $y > x - 1$
15. $y \leq x + 4$
16. $y \leq -x$
17. $y \geq 0$
18. $y > 2x + 3$
19. $y < 2x - 4$
20. $y < \frac{1}{2}x + 2$
21. $y \geq -\frac{1}{3}x - 1$

BUSINESS **For Exercises 22–26, use the following information.** For a certain business to be successful, its monthly sales y must be at least $3000 greater than its monthly costs x.

22. Write an inequality to represent this situation.
23. Graph the inequality.
24. Do points above or below the boundary line indicate a successful business? Explain how you know.
25. Would negative numbers make sense in this problem? Explain.
26. List two solutions.

CRAFTS **For Exercises 27–29, use the following information.** Celia can make a small basket in 10 minutes and a large basket in 25 minutes. This month, she has no more than 24 hours to make these baskets for an upcoming craft fair.

27. Write an inequality to represent this situation.
28. Graph the inequality.
29. Use the graph to determine how many of each type of basket that she could make this month. List three possibilities.

Study Tip

Solutions

In Exercise 29, only whole-number solutions make sense since there cannot be parts of baskets.

www.pre-alg.com/self_check_quiz

More About. . .

Rafting

More than 25 whitewater rafting companies are located on the American River in California. A typical fee is $125 per person per day.

Source: www.rafting-whitewater.com

RAFTING **For Exercises 30–32, use the following information.**
Collin's Rent-a-Raft rents Super Rafts for $100 per day and Econo Rafts for $40 per day. He wants to receive at least $1500 per day renting out the rafts.

30. Write an inequality to represent this situation.

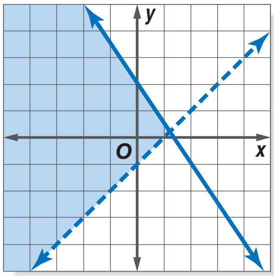
31. Graph the inequality.

32. Determine how many of each type of raft that Collin could rent out each day in order to receive at least $1500. List three possibilities.

33. **CRITICAL THINKING** The solution of a *system of inequalities* is the set of all ordered pairs that satisfies *both* inequalities.

a. Write a system of inequalities for the graph at the right.

b. List three solutions of the system.



34. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How can shaded regions on a graph model inequalities?

Include the following in your answer:

- a description of which points in the graph are solutions of the inequality.

Standardized Test Practice

35. Determine which ordered pair is a solution of $y - 8 > x$.

Ⓐ (0, 2) Ⓑ (2, 5) Ⓒ (−9, 0) Ⓓ (1, 4)

36. Which inequality does *not* have the boundary included in its graph?

Ⓐ $y > x$ Ⓑ $y \leq 1$ Ⓒ $y \geq x + 5$ Ⓓ $y \geq 0$

Maintain Your Skills

Mixed Review

Solve each system of equations by graphing. *(Lesson 8-9)*

37. $y = x + 3$
$x = 0$

38. $y = -x + 2$
$y = x + 4$

39. $y = -2x - 1$
$y = -x - 3$

40. Explain how you can make predictions from a set of ordered-pair data. *(Lesson 8-8)*

Write each fraction or mixed number as a decimal. Use a bar to show a repeating decimal. *(Lesson 5-1)*

41. $\frac{2}{5}$

42. $3\frac{7}{10}$

43. $-\frac{5}{9}$

WebQuest Internet Project

Just for Fun

It is time to complete your project. Use the information and data you have gathered about recreational activities to prepare a Web page or poster. Be sure to include a scatter plot and a prediction for each activity.

www.pre-alg.com/webquest

Graphing Calculator Investigation

A Follow-Up of Lesson 8-10

Graphing Inequalities

You can use a TI-83 Plus graphing calculator to investigate the graphs of inequalities. Since the graphing calculator only shades between two functions, enter a lower boundary as well as an upper boundary for each inequality.

Graph two different inequalities on your graphing calculator.

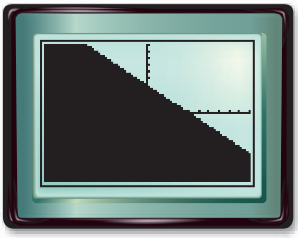
Step 1 *Graph $y \le -x + 4$.*

- Clear all functions from the Y= list.

  KEYSTROKES: Y= CLEAR

- Graph $y \le -x + 4$ in the standard window.

  KEYSTROKES: 2nd [DRAW] 7 (–) 10 ,
  (–) X,T,θ,n + 4)
  ENTER



Ymin or −10 is used as the lower boundary and $y = -x + 4$ as the upper boundary. All ordered pairs in the shaded region satisfy the inequality $y \le -x + 4$.

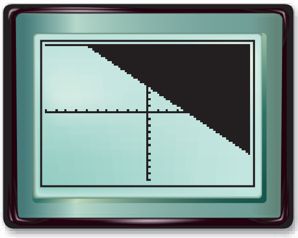
Step 2 *Graph $y \ge -x + 4$.*

- Clear the current drawing displayed.

  KEYSTROKES: 2nd [DRAW] ENTER

- Graph $y \ge -x + 4$ in the standard window.

  KEYSTROKES: 2nd [DRAW] 7 (–)
  X,T,θ,n + 4 , 10)
  ENTER



In this case, the lower boundary is $y = -x + 4$. The upper boundary is Ymax or 10. All ordered pairs in the shaded region satisfy the inequality $y \ge -x + 4$.

Exercises

1. Compare and contrast the two graphs shown above.

2. a. Graph $y \ge -2x - 6$ in the standard viewing window. Draw the graph on grid paper.

b. What functions do you enter as the lower and upper boundaries?

c. Use the graph to name four solutions of the inequality.

Use a graphing calculator to graph each inequality. Draw each graph on grid paper.

3. $y \le x - 3$

4. $y \le -1$

5. $x + y \ge 6$

6. $y \ge 3x$

7. $y \le 0$

8. $y + 3 \le -x$

9. $x + y \le 5$

10. $2y - x \ge 2$

www.pre-alg.com/other_calculator_keystrokes

Chapter 8 Study Guide and Review

Vocabulary and Concept Check

best-fit line (p. 409)
boundary (p. 419)
constant of variation (p. 394)
direct variation (p. 394)
family of graphs (p. 402)
function (p. 369)
half plane (p. 419)
linear equation (p. 375)
rate of change (p. 393)
slope (p. 387)
slope-intercept form (p. 398)
substitution (p. 416)
system of equations (p. 414)
vertical line test (p. 370)
x-intercept (p. 381)
y-intercept (p. 381)

Choose the letter of the term that best matches each statement or phrase.

1. a relation in which each member of the domain is paired with exactly one member of the range
2. in a graph of an inequality, the line of the related equation
3. a value that describes the steepness of a line
4. a group of two or more equations
5. can be drawn through data points to approximate a linear relationship
6. a change in one quantity with respect to another quantity
7. a graph of this is a straight line
8. one type of method for solving systems of equations
9. a linear equation that describes rate of change
10. in the graph of an inequality, the region that contains all solutions

a. best-fit line
b. boundary
c. direct variation
d. function
e. half plane
f. linear equation
g. rate of change
h. slope
i. substitution
j. system of equations

Lesson-by-Lesson Review

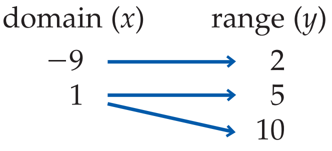
8-1 Functions

See pages 369–373.

Concept Summary

- In a function, each member in the domain is paired with exactly one member in the range.

Example **Determine whether {(−9, 2), (1, 5), (1, 10)} is a function. Explain.**



This relation is not a function because 1 in the domain is paired with two range values, 5 and 10.

Exercises **Determine whether each relation is a function. Explain.**
See Example 1 on page 369.

11. {(1, 12), (−4, 3), (6, 36), (10, 6)}
12. {(11.8, −9), (10.4, −2), (11.8, 3.8)}
13. {(0, 0), (2, 2), (3, 3), (4, 4)}
14. {(−0.5, 1.2), (3, 1.2), (2, 36)}

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8-2 Linear Equations in Two Variables

See pages 375–379.

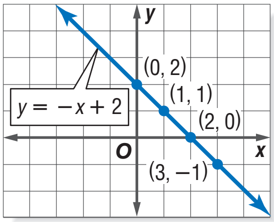
Concept Summary

- A solution of a linear equation is an ordered pair that makes the equation true.
- To graph a linear equation, plot points and draw a line through them.

Example **Graph $y = -x + 2$ by plotting ordered pairs.**

Find ordered pair solutions. Then plot and connect the points.

x	$-x + 2$	y	(x, y)
0	$-0 + 2$	2	(0, 2)
1	$-1 + 2$	1	(1, 1)
2	$-2 + 2$	0	(2, 0)
3	$-3 + 2$	-1	(3, -1)



Exercises **Graph each equation by plotting ordered pairs.**

See Example 3 on page 377.

15. $y = x + 4$ **16.** $y = x - 2$ **17.** $y = -x$ **18.** $y = 2x$

19. $y = 3x + 2$ **20.** $y = -2x - 4$ **21.** $x + y = 4$ **22.** $x - y = -3$

8-3 Graphing Linear Equations Using Intercepts

See pages 381–385.

Concept Summary

- To graph a linear equation, you can find and plot the points where the graph crosses the x-axis and the y-axis. Then connect the points.

Example **Graph $-3x + y = 3$ using the x- and y-intercepts.**

$-3x + y = 3$

$-3x + 0 = 3$

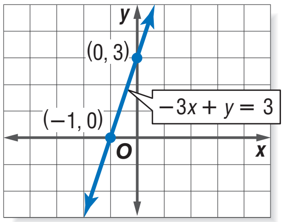
$x = -1$ The x-intercept is -1.

$-3x + y = 3$

$-3(0) + y = 3$

$y = 3$ The y-intercept is 3.

Graph the points at $(-1, 0)$ and $(0, 3)$ and draw a line through them.



Exercises **Graph each equation using the x- and y-intercepts.**

See Example 3 on page 382.

23. $y = x + 2$ **24.** $y = x + 6$ **25.** $y = -x - 3$ **26.** $y = x - 1$

27. $x = -4$ **28.** $y = 5$ **29.** $x + y = -2$ **30.** $3x + y = 6$

8-4 Slope

See pages 387–391.

Concept Summary

- Slope is the ratio of the *rise*, or the vertical change, to the *run*, or the horizontal change.

Example **Find the slope of the line that passes through $A(0, 6)$, $B(4, -2)$.**

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Definition of slope

$m = \frac{-2 - 6}{4 - 0}$ $(x_1, y_1) = (0, 6)$, $(x_2, y_2) = (4, -2)$

$m = \frac{-8}{4}$ or -2 The slope is -2.

Exercises **Find the slope of the line that passes through each pair of points.** *See Examples 2–5 on pages 388 and 389.*

31. $J(3, 4)$, $K(4, 5)$

32. $C(2, 8)$, $D(6, 7)$

33. $R(7, 3)$, $B(-1, -4)$

34. $Q(2, 10)$, $B(4, 6)$

35. $X(-1, 5)$, $Y(-1, 9)$

36. $S(0, 8)$, $T(-3, 8)$

8-5 Rate of Change

See pages 393–397.

Concept Summary

- A change in one quantity with respect to another quantity is called the rate of change.
- Slope can be used to describe rates of change.

Example **Find the rate of change in population from 1990 to 2000 for Oakland, California, using the graph.**

Year	Population (1000s)
x	y
1990	372
2000	400

rate of change $= \frac{y_2 - y_1}{x_2 - x_1}$ Definition of slope

$= \frac{400 - 372}{2000 - 1990}$ ← change in population ← change in time

$= 2.8$ Simplify.

So, the rate of change in population was an increase of about 2.8 thousand, or 2800 people per year.

Exercises **Find the rate of change for the linear function represented in each table.** *See Example 1 on page 393.*

37.

Time (s)	Distance (m)
x	y
0	0
1	8
2	16

38.

Time (h)	Temperature (°F)
x	y
1	45
2	43
3	41

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8-6 Slope-Intercept Form

See pages 398–401.

Concept Summary

- In the slope-intercept form $y = mx + b$, m is the slope and b is the y-intercept.

Example **State the slope and the y-intercept of the graph of $y = -2x + 3$.**

The slope of the graph is -2, and the y-intercept is 3.

Exercises **Graph each equation using the slope and y-intercept.**
See Example 3 on page 399.

39. $y = x + 4$ **40.** $y = -2x + 1$ **41.** $y = \frac{1}{3}x - 2$ **42.** $x + y = -5$

8-7 Writing Linear Equations

See pages 404–408.

Concept Summary

- You can write a linear equation by using the slope and y-intercept, two points on a line, a graph, a table, or a verbal description.

Example **Write an equation in slope-intercept form for the line having slope 4 and y-intercept -2.**

$y = mx + b$ Slope-intercept form
$y = 4x + (-2)$ Replace m with 4 and b with -2.
$y = 4x - 2$ Simplify.

Exercises **Write an equation in slope-intercept form for each line.**
See Example 1 on page 404.

43. slope $= -1$, y-intercept $= 3$ **44.** slope $= 6$, y-intercept $= -3$

8-8 Best-Fit Lines

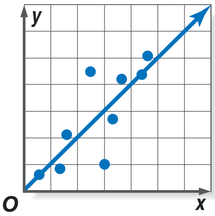
See pages 409–413.

Concept Summary

- A best-fit line can be used to approximate data.

Example **Draw a best-fit line through the scatter plot.**

Draw a line that is close to as many data points as possible.



Exercises **The table shows the attendance for an annual art festival.** *See Example 1 on page 409.*

45. Make a scatter plot and draw a best-fit line.

46. Use the best-fit line to predict art festival attendance in 2008.

Year	Attendance
2000	2500
2001	2650
2002	2910
2003	3050

Chapter 8 **For More ...**

- Extra Practice, see pages 741–744.
- Mixed Problem Solving, see page 765.

8-9 Solving Systems of Equations

See pages 414–418.

Concept Summary

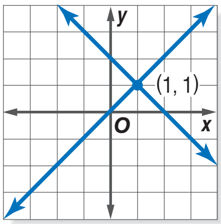
- The solution of a system of equations is the ordered pair that satisfies all equations in the system.

Examples

1 Solve the system of equations by graphing.

$y = x$

$y = -x + 2$



The graphs appear to intersect at (1, 1). The solution of the system of equations is (1, 1).

2 Solve the system $y = x - 1$ and $y = 3$ by substitution.

$y = x - 1$ Write the first equation.

$3 = x - 1$ Replace y with 3.

$4 = x$ Solve for x.

The solution of this system of equations is (4, 3). Check by graphing.

Exercises **Solve each system of equations by graphing.**

See Examples 1–4 on pages 414 and 415.

47. $y = x$, $y = 3$

48. $y = 2x + 4$, $x + y = -2$

49. $3x + y = 1$, $y = -3x + 5$

Solve each system of equations by substitution. *See Example 5 on page 416.*

50. $y = x + 6$, $y = -1$

51. $y = x$, $x = 4$

52. $y = 2x - 3$, $y = 0$

8-10 Graphing Inequalities

See pages 419–422.

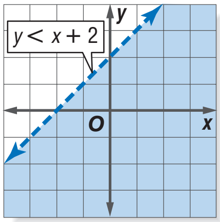
Concept Summary

- To graph an inequality, first graph the related equation, which is the boundary.
- All points in the shaded region are solutions of the inequality.

Example

Graph $y < x + 2$.

Graph $y = x + 2$. Draw a dashed line since the boundary is not part of the graph. Test a point in the original inequality and shade the appropriate region.



Exercises **Graph each inequality.** *See Example 1 on pages 419 and 420.*

53. $y \leq 3x - 1$ **54.** $y > \frac{1}{2}x - 3$ **55.** $y \geq -x + 4$ **56.** $y < -2x$

Chapter 8 Practice Test

Vocabulary and Concepts

1. **Describe** how to find the x-intercept and the y-intercept of a linear equation.
2. **State** how to choose which half plane to shade when graphing an inequality.
3. **OPEN ENDED** Write a system of equations and explain what the solution is.

Skills and Applications

Determine whether each relation is a function. Explain.

4. $\{(-3, 4), (2, 9), (4, -1), (-3, 6)\}$

5. $\{(1, 2), (4, -6), (-3, 5), (6, 2)\}$

Graph each equation by plotting ordered pairs.

6. $y = 2x + 1$

7. $3x + y = 4$

Find the x-intercept and y-intercept for the graph of each equation. Then, graph the equation using the x- and y-intercepts.

8. $y = x + 3$

9. $2x - y = 4$

Find the slope of the line that passes through each pair of points.

10. $A(2, 5), B(4, 11)$

11. $C(-4, 5), D(6, -3)$

12. Find the rate of change for the linear function represented in the table.

Hours Worked	1	2	3	4
Money Earned ($)	5.50	11.00	16.50	22.00

State the slope and y-intercept for the graph of each equation. Then, graph each equation using the slope and y-intercept.

13. $y = \frac{2}{3}x - 4$

14. $2x + 4y = 12$

15. Write an equation in slope-intercept form for the line with a slope of $\frac{3}{8}$ and y-intercept $= -2$.

16. Solve the system of equations $2x - y = 4$ and $4x + y = 2$.

17. Graph $y > 2x - 1$.

GARDENING **For Exercises 18 and 19, use the table at the right and the information below.**
The full-grown height of a tomato plant and the number of tomatoes it bears are recorded for five tomato plants.

Height (in.)	Number of Tomatoes
27	12
33	18
19	9
40	16
31	15

18. Make a scatter plot of the data and draw a best-fit line.

19. Use the best-fit line to predict the number of tomatoes a 43-inch tomato plant will bear.

20. **STANDARDIZED TEST PRACTICE** Which is a solution of $y \geq -2x + 5$?

Ⓐ $(1, -7)$　Ⓑ $(1, -3)$　Ⓒ $(-1, 3)$　Ⓓ $(-1, 7)$

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Chapter 8 Standardized Test Practice

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. A wooden rod is 5.5 feet long. If you cut the rod into 10 equal pieces, how many inches long will each piece be? (Prerequisite Skill, p. 715)

Ⓐ 0.55 in. Ⓑ 5.5 in.

Ⓒ 6.6 in. Ⓓ 66 in.

2. Which of the following is a true statement? (Lesson 2-1)

Ⓐ $\frac{9}{3} < \frac{3}{9}$ Ⓑ $-\frac{3}{9} < -\frac{9}{3}$

Ⓒ $-\frac{9}{3} > \frac{3}{9}$ Ⓓ $-\frac{3}{9} > -\frac{9}{3}$

3. You roll a cube that has 2 blue sides, 2 yellow sides, and 2 red sides. What is the probability that a yellow side will face upwards when the cube stops rolling? (Lesson 6-9)

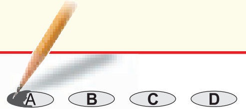
Ⓐ $\frac{1}{6}$ Ⓑ $\frac{1}{3}$

Ⓒ $\frac{1}{2}$ Ⓓ $\frac{2}{3}$

4. Luis used 3 quarts of paint to cover 175 square feet of wall. He now wants to paint 700 square feet in another room. Which proportion could he use to calculate how many quarts of paint he should buy? (Lesson 6-3)

Ⓐ $\frac{175}{700} = \frac{x}{3}$ Ⓑ $\frac{3}{700} = \frac{175}{x}$

Ⓒ $\frac{700}{3} = \frac{175}{x}$ Ⓓ $\frac{3}{175} = \frac{x}{700}$



Test-Taking Tip

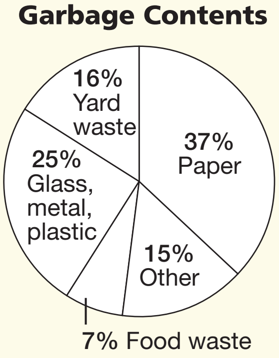
Question 1

Pay attention to units of measurement, such as inches, feet, grams, and kilograms. A problem may require you to convert units; for example, you may be told the length of an object in feet and need to find the length of that object in inches.

5. A shampoo maker offered a special bottle with 30% more shampoo than the original bottle. If the original bottle held 12 ounces of shampoo, how many ounces did the special bottle hold? (Lesson 6-7)

Ⓐ 3.6 Ⓑ 12.3 Ⓒ 15.6 Ⓓ 16

6. The graph shows the materials in a town's garbage collection. If the week's garbage collection totals 6000 pounds, how many pounds of garbage are *not* paper? (Lesson 6-7)



Ⓐ 2220 Ⓑ 3780 Ⓒ 5630 Ⓓ 5963

7. The sum of an integer and the next greater integer is more than 51. Which of these could be the integer? (Lesson 7-6)

Ⓐ 23 Ⓑ 24 Ⓒ 25 Ⓓ 26

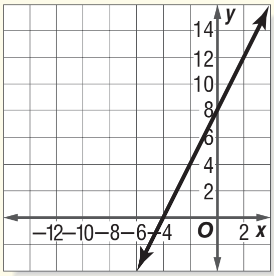
8. The table represents a function between x and y. What is the missing number in the table? (Lesson 8-1)

x	y
1	3
2	■
4	9
6	13

Ⓐ 4 Ⓑ 5

Ⓒ 6 Ⓓ 7

9. Which equation is represented by the graph? (Lesson 8-7)



Ⓐ $y = 2x - 4$

Ⓑ $y = 2x + 8$

Ⓒ $y = \frac{1}{2}x - 8$

Ⓓ $y = \frac{1}{2}x + 4$

10. Which ordered pair is the solution of this system of equations? (Lesson 8-9)

$$2x + 3y = 7$$
$$3x - 3y = 18$$

Ⓐ (25, 1) Ⓑ (2, 1)

Ⓒ (5, –1) Ⓓ (7, 1)

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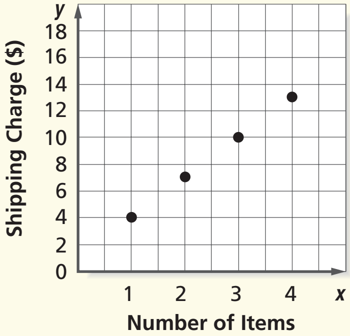
Preparing for Standardized Tests
For test-taking strategies and more practice, see pages 771–788.

Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

11. The graph shows the shipping charges per order, based on the number of items shipped in the order. What is the shipping charge for an order with 4 items? (Prerequisite Skill, pp. 722–723)



12. The length of a rectangle is 8 centimeters, and its perimeter is 24 centimeters. What is the area of the rectangle in square centimeters? (Lesson 3-5)

13. Write the statement *y is 5 more than one half the value of x* as an equation. (Lesson 3-6)

14. Write the number 0.09357 in scientific notation. Round your answer to two decimal places. (Lesson 4-8)

15. If $y = -\frac{3}{2}$, what is the value of x in $y = 4x - 3$? (Lesson 5-8)

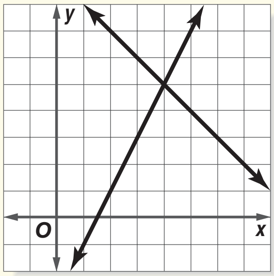
16. Write $\frac{3}{5}$ as a percent. (Lesson 6-4)

17. In a diving competition, the diver in first place has a total score of 345.4. Ming has scored 68.2, 68.9, 67.5, and 71.7 for her first four dives and has one more dive remaining. Write an inequality to show the score x that Ming must receive on her fifth dive in order to overtake the diver in first place. (Lesson 7-4)

18. Ms. Vang drove at 30 mph for 30 minutes and at 56 mph for one hour and fifteen minutes. How far did she travel? (Lesson 8-5)

19. What is the slope of the line that contains (0, 4) and (2, 5)? (Lesson 8-6)

20. Find the solution of the system of equations graphed below. (Lesson 8-9)



Part 3 Extended Response

Record your answers on a sheet of paper. Show your work.

21. Krishnan is considering three plans for cellular phone service. The plans each offer the same services for different monthly fees and different costs per minute. (Lesson 8-6)

Plan	Monthly Fee	Cost per Minute
X	$0	$0.24
Y	$15.95	$0.08
Z	$25.95	$0.04

a. For each plan, write an equation that shows the total monthly cost c for m minutes of calls.

b. What is the cost of each plan if Krishnan uses 100 minutes per month?

c. Which plan costs the least if Krishnan uses 100 minutes per month?

d. What is the cost of each plan if Krishnan uses 300 minutes per month?

e. Which plan costs the least if Krishnan uses 300 minutes per month?

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