

Merrimack School District Mathematics Curriculum

Grade 3

Standards for Mathematical Practices

The College and Career Readiness Standards for Mathematical Practice are expected to be integrated into every mathematics lesson for all students Grades K-12. Below are a few examples of how these Practices may be integrated into tasks that students complete.

Mathematic Practices	Explanations and Examples
1. Make sense of problems and persevere in solving them.	In third grade, mathematically proficient students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Third grade students may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, “Does this make sense?” Students listen to other students’ strategies and are able to make connections between various methods for a given problem.
2. Reason abstractly and quantitatively.	Mathematically proficient third grade students should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities.
3. Construct viable arguments and critique the reasoning of others.	In third grade, mathematically proficient students may construct arguments using concrete referents, such as objects, pictures, and drawings. They refine their mathematical communication skills as they participate in mathematical discussions that the teacher facilitates by asking questions such as “How did you get that?” and “Why is that true?” They explain their thinking to others and respond to others’ thinking.
4. Model with mathematics.	Mathematically proficient students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart, list, or graph, creating equations, etc. Students require extensive opportunities to generate various mathematical representations and to both equations and story problems, and explain connections between representations as well as between representations and equations. Students should be able to use all of these representations as needed. They should evaluate their results in the context of the situation and reflect on whether the results make sense.

Mathematic Practices	Explanations and Examples
5. Use appropriate tools strategically.	Mathematically proficient third grader students consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper to find all the possible rectangles that have a given perimeter. They compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles.
6. Attend to precision.	Mathematically proficient third grader students develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the area of a rectangle they record their answers in square units.
7. Look for and make use of structure.	In third grade mathematically proficient students look closely to discover a pattern or structure. For instance, students use properties of operations as strategies to multiply and divide (commutative and distributive properties).
8. Look for and express regularity in repeated reasoning.	Mathematically proficient students in third grade should notice repetitive actions in computation and look for more shortcut methods. For example, students may use the distributive property as a strategy for using products they know to solve products that they don't know. For example, if students are asked to find the product of 7×8 , they might decompose 7 into 5 and 2 and then multiply 5×8 and 2×8 to arrive at $40 + 16$ or 56. In addition, third graders continually evaluate their work by asking themselves, "Does this make sense?"

Grade 3 Critical Areas

The Critical Areas are designed to bring focus to the standards at each grade by describing the big ideas that educators can use to build their curriculum and to guide instruction. The Critical Areas for third grade can be found in the *College and Career Readiness Standards for Mathematics*.

1. Developing understanding of multiplication and division and strategies for multiplication and division within 100.

Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

2. Developing understanding of fractions, especially unit fractions (fractions with numerator 1).

Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, $\frac{1}{2}$ of the paint in a small bucket could be less paint than $\frac{1}{3}$ of the paint in a larger bucket, but $\frac{1}{3}$ of a ribbon is longer than $\frac{1}{5}$ of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.

3. Developing understanding of the structure of rectangular arrays and of area.

Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.

4. Describing and analyzing two-dimensional shapes.

Students describe, analyze, and compare properties of two-dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

Grade 3 Overview

Operations and Algebraic Thinking

- Represent and solve problems involving multiplication and division.
- Understand properties of multiplication and the relationship between multiplication and division.
- Multiply and divide within 100.
- Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Number and Operations in Base Ten

- Use place value understanding and properties of operations to perform multi-digit arithmetic.

Number and Operations – Fractions

- Develop understanding of fractions as numbers.

Measurement and Data

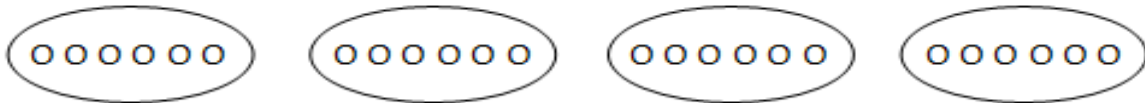
- Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.
- Present and interpret data.
- Geometric measurement: understand concepts of area and relate area to multiplication and to addition.
- Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

Geometry

- Reason with shapes and their attributes.

Operations and Algebraic Thinking	3.OA
College and Career Readiness Cluster	
Represent and solve problems involving multiplication and division.	
<p>Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size.</p> <p>Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: products, groups of, quotients, partitioned equally, multiplication, division, equal groups, group size, arrays, equations(number model), unknown, expression, whole number</p>	
<p>Enduring Understandings: Multiplication and division are used to solve problems and are inverse operations. Proficiency with basic facts aids estimation and computation of larger and smaller numbers.</p> <p>Essential Questions: What are the properties of multiplication and why are they important? What are different models of and for multiplication? What are different models of and for division? What are efficient methods for finding products and quotients? What are variables and how are they used?</p>	

College and Career Readiness Standards <i>Students are expected to:</i>	Mathematical Practices	Unpacking Explanations and Examples <i>What does this standard mean that a student will know and be able to do?</i>
3.OA.A.1 Interpret products of whole numbers , e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. <i>For example, describe a context in which a total number of objects can be expressed as 5×7.</i>	3.MP.1. Make sense of problems and persevere in solving them. 3.MP.4. Model with mathematics. 3.MP.7. Look for and make use of structure.	<p>This standard interprets products of whole numbers. Students recognize multiplication as a means to determine the total number of objects when there are a specific number of groups with the same number of objects in each group or of an equal amount of objects were added or collected numerous times. Multiplication requires students to think in terms of groups of things rather than individual things. Students learn that the multiplication symbol ‘x’ means “groups of” and problems such as 5×7 refer to 5 groups of 7.</p> <p>Example: Jim purchased 5 packages of muffins. Each package contained 3 muffins. How many muffins did Jim purchase? 5 groups of 3, $5 \times 3 = 15$. Describe another situation where there would be 5 groups of 3 or 5×3.</p> <p>Sonya earns \$7 a week pulling weeds. After 5 weeks of work, how much has Sonya earned? Write an equation and find the answer. Describe another situation that would match 7×5.</p>
3.OA.A.2 Interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each.	3.MP.1. Make sense of problems and persevere in solving them. 3.MP.4. Model with mathematics. 3.MP.7. Look for and make use of structure.	<p>This standard focuses on two distinct models of division: partition models and measurement (repeated subtraction) models.</p> <p>Partition models provide students with a total number and the number of groups. These models focus on the question, “How many objects are in each group so that the groups are equal?” A context for partition models would be: There are 12 cookies on the counter. If you are sharing the cookies equally among three bags, how many cookies will go in each bag?</p> <p>Measurement (repeated subtraction) models provide students with a total number and the number of objects in each group. These models focus on the question, “How many equal groups can you make?” A context for measurement models would be: "There are 12 cookies on the counter. If you put 3 cookies in each bag, how many bags will you fill?"</p> <div style="text-align: center;"> <div style="border: 1px solid black; padding: 2px 10px; display: inline-block;">○ ○ ○</div> <div style="border: 1px solid black; padding: 2px 10px; display: inline-block; margin: 0 10px;">○ ○</div> <div style="border: 1px solid black; padding: 2px 10px; display: inline-block; margin: 0 10px;">○ ○ ○</div> <div style="border: 1px solid black; padding: 2px 10px; display: inline-block;">○ ○</div> </div>

<i>For example, describe a context in which a number of shares or a number of groups can be expressed as $56 \div 8$.</i>																										
3.OA.A.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.	<p>3.MP.1. Make sense of problems and persevere in solving them.</p> <p>3.MP.4. Model with mathematics.</p> <p>3.MP.7. Look for and make use of structure.</p>	<p>This standard references various problem solving context and strategies that students are expected to use while solving word problems involving multiplication & division. Students should use a variety of representations for creating and solving one-step word problems, such as: If you divide 4 packs of 9 brownies among 6 people, how many brownies does each person receive? ($4 \times 9 = 36$, $36 \div 6 = 6$).</p> <p>Students should be given ample experiences to explore all of the different problem structures.</p> <p><u>Examples of Multiplication:</u></p> <p>There are 24 desks in the classroom. If the teacher puts 6 desks in each row, how many rows are there?</p> <p>This task can be solved by drawing an array by putting 6 desks in each row. This is an array model.</p> <table border="1"><tr><td></td><td></td><td></td><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td><td></td><td></td><td></td></tr></table> <p>This task can also be solved by drawing pictures of equal groups.</p> <p>4 groups of 6 equals 24 objects</p> <div></div> <p>A student can also reason through the problem mentally or verbally, “I know 6 and 6 are 12. 12 and 12 are 24. Therefore, there are 4 groups of 6 giving a total of 24 desks in the classroom.”</p> <p>A number line could also be used to show equal jumps.</p>																								

Students in third grade should use a variety of pictures, such as stars, boxes, flowers to represent unknown numbers (variables). Letters are also introduced to represent unknowns in third grade.

Examples of Division:

There are some students at recess. The teacher divides the class into 4 lines with 6 students in each line. Write a division equation for this story and determine how many students are in the class ($\square \div 4 = 6$. There are 24 students in the class).

Determining the number of objects in each share (partition model of division, where the size of the groups is unknown):

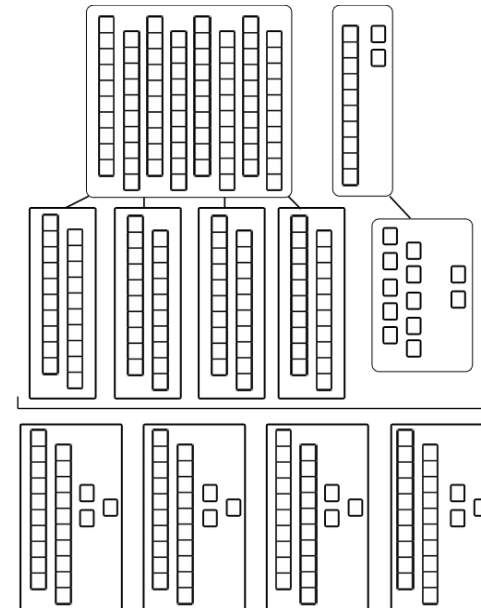
Example:

The bag has 92 hair clips, and Laura and her three friends want to share them equally. How many hair clips will each person receive?

Step 1

Step 2

Step 3



		<p>Example:</p> <p>Max the monkey loves bananas. Molly, his trainer, has 24 bananas. If she gives Max 4 bananas each day, how many days will the bananas last?</p> <table><tr><th>Starting</th><th>Day 1</th><th>Day 2</th><th>Day 3</th><th>Day 4</th><th>Day 5</th><th>Day 6</th></tr><tr><td>24</td><td>$24 - 4 = 20$</td><td>$20 - 4 = 16$</td><td>$16 - 4 = 12$</td><td>$12 - 4 = 8$</td><td>$8 - 4 = 4$</td><td>$4 - 4 = 0$</td></tr></table> <p>Solution: The bananas will last for 6 days.</p>	Starting	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	24	$24 - 4 = 20$	$20 - 4 = 16$	$16 - 4 = 12$	$12 - 4 = 8$	$8 - 4 = 4$	$4 - 4 = 0$
Starting	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6										
24	$24 - 4 = 20$	$20 - 4 = 16$	$16 - 4 = 12$	$12 - 4 = 8$	$8 - 4 = 4$	$4 - 4 = 0$										
<p>3.OA.A.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. <i>For example, determine the unknown number that makes the equation true in each of the equations</i> $8 \times ? = 48$, $5 = _ \div 3$, $6 \times 6 = ?$</p>	<p>3.MP.1. Make sense of problems and persevere in solving them.</p> <p>3.MP.2. Reason abstractly and quantitatively.</p> <p>3.MP.6. Attend to precision.</p> <p>3.MP.7. Look for and make use of structure.</p>	<p>The focus of 3.OA.4 extends beyond the traditional notion of <i>fact families</i>, by having students explore the inverse relationship of multiplication and division.</p> <p>Students extend work from earlier grades with their understanding of the meaning of the equal sign as “the same amount as” to interpret an equation with an unknown. When given $4 \times ? = 40$, they might think:</p> <ul style="list-style-type: none">• 4 groups of some number is the same as 40• 4 times some number is the same as 40• I know that 4 groups of 10 is 40 so the unknown number is 10• The missing factor is 10 because 4 times 10 equals 40. <p>Equations in the form of $a \times b = c$ and $c = a \times b$ should be used interchangeably, with the unknown in different positions.</p> <p>Example: Solve the equations below: $24 = ? \times 6$ $72 \div \triangle = 9$ Rachel has 3 bags. There are 4 marbles in each bag. How many marbles does Rachel have altogether? $3 \times 4 = m$</p>														

<p>3.OA.B.5 Apply properties of operations as strategies to multiply and divide.²</p> <p><i>Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)</i></p> <p>² Students need not use formal terms for these properties.</p>	<p>3.MP.1. Make sense of problems and persevere in solving them.</p> <p>3.MP.4. Model with mathematics.</p> <p>3.MP.7. Look for and make use of structure.</p> <p>3.MP.8. Look for and express regularity in repeated reasoning.</p>	<p>This standard references properties (rules about how numbers work) of multiplication. This extends past previous expectations, in which students were asked to identify properties. While students DO NOT need to not use the formal terms of these properties, student must understand that properties are rules about how numbers work, and they need to be flexibly and fluently applying each of them in various situations. Students represent expressions using various objects, pictures, words and symbols in order to develop their understanding of properties. They multiply by 1 and 0 and divide by 1. They change the order of numbers to determine that the order of numbers does not make a difference in multiplication (but does make a difference in division). Given three factors, they investigate changing the order of how they multiply the numbers to determine that changing the order does not change the product. They also decompose numbers to build fluency with multiplication.</p> <p>The associative property states that the sum or product stays the same when the grouping of addends or factors is changed. For example, when a student multiplies $7 \times 5 \times 2$, a student could rearrange the numbers to first multiply $5 \times 2 = 10$ and then multiply $10 \times 7 = 70$.</p> <p>The commutative property states that the order of numbers does not matter when you are adding or multiplying numbers. For example, if a student knows that $5 \times 4 = 20$, then they also know that $4 \times 5 = 20$. The array below could be described as a 5×4 array for 5 columns and 4 rows, or a 4×5 array for 4 rows and 5 columns.</p> <p>Always write the dimensions of an array as rows x columns never columns x rows.</p> <p>Example:</p> <div><div>5×4<table><tr><td></td><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td><td></td></tr></table></div><div><table><tr><td></td><td></td><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td><td></td><td></td></tr></table>4×5</div></div> <p>Students are introduced to the distributive property of multiplication over addition as a strategy for using products they know to solve products they don't know. Students would be using mental math to determine a product. Here are ways that students could use the distributive property to determine</p>																																								

the product of 7×6 . Again, students should use the distributive property, but can refer to this in informal language such as “breaking numbers apart.”

Student 1

$$\begin{aligned} 7 \times 6 \\ 7 \times 5 = 35 \\ 7 \times 1 = 7 \\ 35 + 7 = 42 \end{aligned}$$

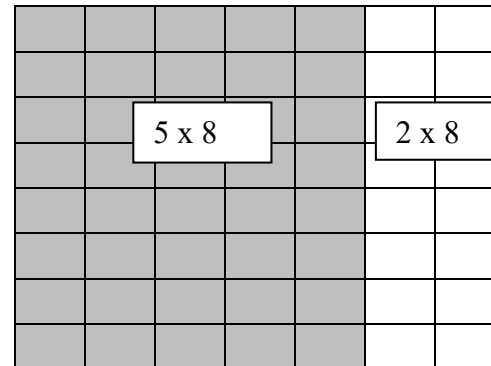
Student 2

$$\begin{aligned} 7 \times 6 \\ 7 \times 3 = 21 \\ 7 \times 3 = 21 \\ 21 + 21 = 42 \end{aligned}$$

Student 3

$$\begin{aligned} 7 \times 6 \\ 5 \times 6 = 30 \\ 2 \times 6 = 12 \\ 30 + 12 = 42 \end{aligned}$$

Another example of the distributive property helps students determine the products and factors of numbers by breaking them apart. For example, for the problem $7 \times 8 = ?$ students can decompose the 7 into a 5 and 2, and reach the answer by multiplying $5 \times 8 = 40$ and $2 \times 8 = 16$ and adding the two products ($40 + 16 = 56$).



To further develop understanding of properties related to multiplication and division, students use different representations and their understanding of the relationship between multiplication and division to determine if the following types of equations or inequalities are true or false.

- $0 \times 7 = 7 \times 0 = 0$ (Zero Property of Multiplication)
- $1 \times 9 = 9 \times 1 = 9$ (Multiplicative Identity Property of 1)
- $3 \times 6 = 6 \times 3$ (Commutative Property)
- $8 \div 2 = 2 \div 8$ (Students are only to determine that these are not equal)
- $2 \times 3 \times 5 = 6 \times 5$
- $10 \times 2 < 5 \times 2 \times 2$
- $2 \times 3 \times 5 = 10 \times 3$
- $0 \times 6 > 3 \times 0 \times 2$

<p>3.OA.B.6 Understand division as an unknown-factor problem. <i>For example, find $32 \div 8$ by finding the number that makes 32 when multiplied by 8.</i></p>	<p>3.MP.1. Make sense of problems and persevere in solving them. 3.MP.7. Look for and make use of structure.</p>	<p>Since multiplication and division are inverse operations, students are expected to solve problems and explain their processes of solving division problems that can also be represented as unknown factor multiplication problems.</p> <p><u>Example:</u> A student knows that $2 \times 9 = 18$. How can they use that fact to determine the answer to the following question: 18 people are divided into pairs in P.E. class? How many pairs are there? Write a division equation and explain your reasoning.</p> <p>Multiplication and division are inverse operations and that understanding can be used to find the unknown.</p> <p><u>Examples:</u></p> <ul style="list-style-type: none"> • $3 \times 5 = 15$ $5 \times 3 = 15$ • $15 \div 3 = 5$ $15 \div 5 = 3$ <p><u>Example:</u></p> <p>Sarah did not know the answer to 63 divided by 7. Are each of the following an appropriate way for Sarah to think about the problem? Explain why or why not with a picture or words for each one.</p> <ul style="list-style-type: none"> • “I know that $7 \times 9 = 63$, so 63 divided by 7 must be 9.” • “I know that $7 \times 10 = 70$. If I take away a group of 7 that means that I have $7 \times 9 = 63$. So 63 divided by 7 is 9.” • “I know that 7×5 is 35. 63 minus 35 is 28. I know that $7 \times 4 = 28$. So if I add 7×5 and 7×4 I get 63. That means that 7×9 is 63, or 63 divided by 7 is 9.”
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College and Career Readiness Cluster		
Multiply and divide within 100.		
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: operation, multiply, divide, factor, product, quotient, unknown, strategies, reasonableness, mental computation, property		
College and Career Readiness Standards <i>Students are expected to:</i>	Mathematical Practices	Unpacking Explanations and Examples <i>What does this standard mean that a student will know and be able to do?</i>
3.OA.C.7 Fluently multiply and divide within 100 , using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. *By the end of Grade 3, know from memory all products of two one-digit numbers.	<p>3.MP.2. Reason abstractly and quantitatively.</p> <p>3.MP.7. Look for and make use of structure.</p> <p>3.MP.8. Look for and express regularity in repeated reasoning.</p>	<p>This standard uses the word fluently, which means with accuracy, efficiency (using a reasonable amount of steps and time), and flexibility (using strategies such as the distributive property). “Know from memory” should not focus only on timed tests and repetitive practice, but ample experiences working with manipulatives, pictures, arrays, word problems, and numbers to internalize the basic facts.</p> <p>By studying patterns and relationships in multiplication facts and relating multiplication and division, students build a foundation for fluency with multiplication and division facts. Students demonstrate fluency with multiplication facts and the related division facts. Multiplying and dividing fluently refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently.</p> <p>Strategies students may use to attain fluency include:</p> <ul style="list-style-type: none"> • Multiplication by zeros and ones • Doubles (2s facts), Doubling twice (4s), Doubling three times (8s) • Tens facts (relating to place value, 5×10 is 5 tens or 50) • Five facts (half of tens) • Skip counting (counting groups of ___ and knowing how many groups have been counted) • Square numbers (ex: 3×3) • Nines (10 groups less one group, e.g., 9×3 is 10 groups of 3 minus one group of 3) • Decomposing into known facts (6×7 is 6×6 plus one more group of 6)


		<ul style="list-style-type: none"> • Turn-around facts (Commutative Property) • Fact families (Ex: $6 \times 4 = 24$; $24 \div 6 = 4$; $24 \div 4 = 6$; $4 \times 6 = 24$) • Missing factors <p>Students should have exposure to multiplication and division problems presented in both vertical and horizontal forms.</p> <p>Note that mastering this material, and reaching fluency in single-digit multiplications and related divisions with understanding, may be quite time consuming because there are no general strategies for multiplying or dividing all single-digit numbers as there are for addition and subtraction. Instead, there are many patterns and strategies dependent upon specific numbers. So it is imperative that extra time and support be provided if needed.</p> <p>To achieve this by the end of Grade 3, students must begin working toward fluency for the easy numbers as early as possible. Because an unknown factor (a division) can be found from the related multiplication, the emphasis at the end of the year is on knowing from memory all products of two one-digit numbers. As should be clear from the foregoing, this isn't a matter of instilling facts divorced from their meanings, but rather the outcome of a carefully designed learning process that heavily involves the interplay of practice and reasoning.</p> <p>All of the work on how different numbers fit with the base-ten numbers culminates in these "just know" products and is necessary for learning products. Fluent dividing for all single-digit numbers, which will combine just knows, knowing from a multiplication, patterns, and best strategy, is also part of this vital standard.</p>
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College and Career Readiness Cluster		
Solve problems involving the four operations, and identify and explain patterns in arithmetic.		
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: operation, multiply, divide, factor, product, quotient, subtract, add, addend, sum, difference, equation, expression, unknown, strategies, reasonableness, mental computation, estimation, rounding, patterns, (properties)-rules about how numbers work, input and output table		
College and Career Readiness Standards <i>Students are expected to:</i>	Mathematical Practices	Unpacking Explanations and Examples <i>What does this standard mean that a student will know and be able to do?</i>
<p>3.OA.D.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.</p> <p>This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to</p>	<p>3.MP.1. Make sense of problems and persevere in solving them.</p> <p>3.MP.2. Reason abstractly and quantitatively.</p> <p>3.MP.4. Model with mathematics.</p> <p>3.MP.5. Use appropriate tools strategically.</p>	<p>Students in third grade begin the step to formal algebraic language by using a letter for the unknown quantity in expressions or equations for one and two-step problems. But the symbols of arithmetic, \times or \bullet or $*$ for multiplication and \div or $/$ for division, continue to be used in Grades 3, 4, and 5.</p> <p>This standard refers to two-step word problems using the four operations. The size of the numbers should be limited to related 3rd grade standards (e.g., 3.OA.7 and 3.NBT.2). Adding and subtracting numbers should include numbers within 1,000, and multiplying and dividing numbers should include factors up to 12.</p> <p>This standard calls for students to represent problems using equations with a letter to represent unknown quantities.</p> <p>Example:</p> <p>Mike runs 2 miles a day. His goal is to run 25 miles. After 5 days, how many miles does Mike have left to run in order to meet his goal? Write an equation and find the solution ($2 \times 5 + m = 25$).</p> <p>This standard refers to estimation strategies, including using compatible numbers (numbers whose sums are multiples of 10) or rounding. The focus in this standard is to have students use and discuss various strategies. Students should estimate during problem solving, and then revisit their estimate to check for reasonableness.</p>

<p>perform operations in the conventional order when there are no parentheses to specify a particular order</p>	<p><u>Example:</u></p> <p>Here are some typical estimation strategies for the problem: On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. How many total miles did they travel?</p> <div data-bbox="777 360 1148 695"> <p>Student 1 I first thought about 267 and 34. I noticed that their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I get 500.</p> </div> <div data-bbox="1178 360 1600 695"> <p>Student 2 I first thought about 194. It is really close to 200. I also have 2 hundreds in 267. That gives me a total of 4 hundreds. Then I have 67 in 267 and the 34. When I put 67 and 34 together that is really close to 100. When I add that hundred to the 4 hundreds that I already had, I end up with 500.</p> </div> <div data-bbox="1629 360 1955 695"> <p>Student 3 I rounded 267 to 300. I rounded 194 to 200. I rounded 34 to 30. When I added 300, 200 and 30, I know my answer will be about 530.</p> </div> <p>The assessment of estimation strategies should only have one reasonable answer (500 or 530), or a range (between 500 and 550). Problems should be structured so that all acceptable estimation strategies will arrive at a reasonable answer.</p>
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A two-step problem with diagram showing problem situation and equations showing the two parts

Carla has 4 packages of silly bands. Each package has 8 silly bands in it. Agustin is supposed to get 15 fewer silly bands than Carla. How many silly bands should Agustin get?

Carla: 

Agustin: 

C = number of Carla's silly bands
 A = number of Agustin's silly bands

$$C = 4 \times 8 = 32$$

$$A + 15 = C$$

$$A + 15 = 32$$

$$A = 17$$

Students may be able to solve this problem without writing such equations.

In the diagram above, Carla's bands are shown using 4 equal-sized bars that represent 4×8 or 32 bands. Agustin's bands are directly below showing that the number that Augustin has plus 15 = 32. The diagram can also be drawn like this:

8	8	8	8
15		?	

<p>3.OA.D.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.</p> <p><i>For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.</i></p>	<p>3.MP.1. Make sense of problems and persevere in solving them.</p> <p>3.MP.2. Reason abstractly and quantitatively.</p> <p>3.MP.3. Construct viable arguments and critique the reasoning of others.</p> <p>3.MP.6. Attend to precision.</p> <p>3.MP.7. Look for and make use of structure.</p>	<p>This standard calls for students to examine arithmetic patterns involving both addition and multiplication. Arithmetic patterns are patterns that change by the same rate, such as adding the same number. For example, the series 2, 4, 6, 8, 10 is an arithmetic pattern that increases by 2 between each term.</p> <p>This standard also mentions identifying patterns related to the properties of operations.</p> <p>Examples:</p> <ul style="list-style-type: none">• Even numbers are always divisible by 2. Even numbers can always be decomposed into 2 equal addends ($14 = 7 + 7$).• Multiples of even numbers (2, 4, 6, and 8) are always even numbers.• On a multiplication chart, the products in each row and column increase by the same amount (skip counting).• On an addition chart, the sums in each row and column increase by the same amount. <p>What do you notice about the numbers highlighted in pink in the multiplication table? Explain a pattern using properties of operations. <i>When (commutative property) one changes the order of the factors they will still get the same product, example $6 \times 5 = 30$ and $5 \times 6 = 30$.</i></p> <table><tr><th>x</th><th>0</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th><th>9</th><th>10</th></tr><tr><th>0</th><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><th>1</th><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr><tr><th>2</th><td>0</td><td>2</td><td>4</td><td>6</td><td>8</td><td>10</td><td>12</td><td>14</td><td>16</td><td>18</td><td>20</td></tr><tr><th>3</th><td>0</td><td>3</td><td>6</td><td>9</td><td>12</td><td>15</td><td>18</td><td>21</td><td>24</td><td>27</td><td>30</td></tr><tr><th>4</th><td>0</td><td>4</td><td>8</td><td>12</td><td>16</td><td>20</td><td>24</td><td>28</td><td>32</td><td>36</td><td>40</td></tr><tr><th>5</th><td>0</td><td>5</td><td>10</td><td>15</td><td>20</td><td>25</td><td>30</td><td>35</td><td>40</td><td>45</td><td>50</td></tr><tr><th>6</th><td>0</td><td>6</td><td>12</td><td>18</td><td>24</td><td>30</td><td>36</td><td>42</td><td>48</td><td>54</td><td>60</td></tr><tr><th>7</th><td>0</td><td>7</td><td>14</td><td>21</td><td>28</td><td>35</td><td>42</td><td>49</td><td>56</td><td>63</td><td>70</td></tr><tr><th>8</th><td>0</td><td>8</td><td>16</td><td>24</td><td>32</td><td>40</td><td>48</td><td>56</td><td>64</td><td>72</td><td>80</td></tr><tr><th>9</th><td>0</td><td>9</td><td>18</td><td>27</td><td>36</td><td>45</td><td>54</td><td>63</td><td>72</td><td>81</td><td>90</td></tr><tr><th>10</th><td>0</td><td>10</td><td>20</td><td>30</td><td>40</td><td>50</td><td>60</td><td>70</td><td>80</td><td>90</td><td>100</td></tr></table>	x	0	1	2	3	4	5	6	7	8	9	10	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	2	3	4	5	6	7	8	9	10	2	0	2	4	6	8	10	12	14	16	18	20	3	0	3	6	9	12	15	18	21	24	27	30	4	0	4	8	12	16	20	24	28	32	36	40	5	0	5	10	15	20	25	30	35	40	45	50	6	0	6	12	18	24	30	36	42	48	54	60	7	0	7	14	21	28	35	42	49	56	63	70	8	0	8	16	24	32	40	48	56	64	72	80	9	0	9	18	27	36	45	54	63	72	81	90	10	0	10	20	30	40	50	60	70	80	90	100
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Teacher: What pattern do you notice when 2, 4, 6, 8, or 10 are multiplied by any number (even or odd)?

Student: The product will always be an even number.

Teacher: Why?

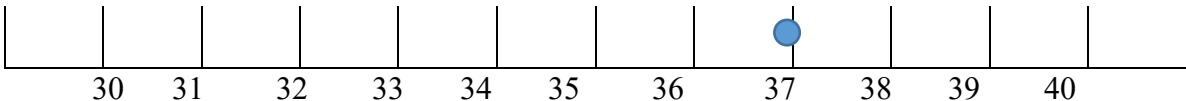
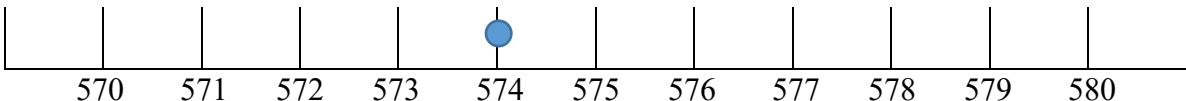
x	0	1	2	3	4	5	6	7	8	9	10
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1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

What patterns do you notice in this addition table? Explain why the pattern works this way?

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
2	2	3	4	5	6	7	8	9	10	11	12
3	3	4	5	6	7	8	9	10	11	12	13
4	4	5	6	7	8	9	10	11	12	13	14
5	5	6	7	8	9	10	11	12	13	14	15
6	6	7	8	9	10	11	12	13	14	15	16
7	7	8	9	10	11	12	13	14	15	16	17
8	8	9	10	11	12	13	14	15	16	17	18
9	9	10	11	12	13	14	15	16	17	18	19
10	19	11	12	13	14	15	16	17	18	19	20

		<p>Students need ample opportunities to observe and identify important numerical patterns related to operations. They should build on their previous experiences with properties related to addition and subtraction. Students investigate addition and multiplication tables in search of patterns and explain why these patterns make sense mathematically.</p> <p><u>Example:</u></p> <ul style="list-style-type: none"> • The sum of any two even numbers is even. • The sum of any two odd numbers is even. • The sum of any even number and any odd number is odd. • The multiples of 4, 6, 8, and 10 are all even because they can all be decomposed into two equal groups. • The doubles (2 addends the same) in an addition table fall on a diagonal while the doubles (multiples of 2) in a multiplication table fall on horizontal and vertical lines. • The multiples of any number fall on a horizontal and a vertical line due to the commutative property. • All the multiples of 5 end in a 0 or 5 while all the multiples of 10 end with 0. Every other multiple of 5 is a multiple of 10. <p>Students also investigate a hundreds chart in search of addition and subtraction patterns. They record and organize all the different possible sums of a number and explain why the pattern makes sense.</p> <table border="1"> <thead> <tr> <th>addend</th><th>addend</th><th>sum</th></tr> </thead> <tbody> <tr><td>0</td><td>20</td><td>20</td></tr> <tr><td>1</td><td>19</td><td>20</td></tr> <tr><td>2</td><td>18</td><td>20</td></tr> <tr><td>3</td><td>17</td><td>20</td></tr> <tr><td>4</td><td>16</td><td>20</td></tr> <tr><td>□</td><td>□</td><td>□</td></tr> <tr><td>□</td><td>□</td><td>□</td></tr> <tr><td>□</td><td>□</td><td>□</td></tr> <tr><td>20</td><td>0</td><td>20</td></tr> </tbody> </table>	addend	addend	sum	0	20	20	1	19	20	2	18	20	3	17	20	4	16	20	□	□	□	□	□	□	□	□	□	20	0	20
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Numbers and Operations in Base Ten		3.NBT
College and Career Readiness Cluster		
Use place value understanding and properties of operations to perform multi-digit arithmetic.		
<p>A range of algorithms may be used.</p> <p>Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: place value, round, addition, add, addend, sum, subtraction, subtract, <u>difference, strategies, (properties)-rules about how numbers work</u></p>		
<p>Enduring Understandings: Place value is based on groups of ten. Estimation is a way to get an approximate answer and to know if an answer makes sense. Computation involves taking apart and combining numbers using a variety of strategies.</p> <p>Essential Questions: How do I know when to round numbers and how does it help me to solve problems? What are efficient methods for finding sums and differences? How can I use my knowledge of place value to multiply?</p>		
College and Career Readiness Standards <i>Students are expected to:</i>	Mathematical Practices	Unpacking Explanations and Examples <i>What does this standard mean that a student will know and be able to do?</i>
3.NBT.A.1 Use place value understanding to round whole numbers to the nearest 10 or 100.	<p>3.MP.5. Use appropriate tools strategically.</p> <p>3.MP.7. Look for and make use of structure.</p> <p>3.MP.8. Look for and express regularity in repeated reasoning.</p>	<p>This standard refers to place value understanding, which extends beyond an algorithm or memorized procedure for rounding. The expectation is that students have a deep understanding of place value and number sense and can explain and reason about the answers they get when they round. Students should have numerous experiences using a number line and a hundreds chart as tools to support their work with rounding.</p> <p>Example: Mrs. Rutherford drives 158 miles on Saturday and 171 miles on Sunday. When she told her husband, she estimated how many miles to the nearest 10 before adding the total. When she told her sister, she estimated to the nearest 100 before adding the total. Which method provided a closer estimate?</p>

		<p>The number line is a tool that can be used to support students' development related to rounding numbers. For example, round 37 to the nearest ten.</p> <p>Example: Teacher: Between which two tens does the number 37 fall? Student: 37 falls between 30 and 40. Teacher: Let's make a number line. Teacher: Where would 37 be on the number line? Students mark 37. Teacher: Is 37 closer to 30 or 40? Student: 40</p>  <p>With larger numbers a similar approach could be used. Teacher: We want to round 574 to the nearest ten. Between which two tens does 574 fall? Student: Between 570 and 580. Teacher: Let's make a number line. Teacher: Where would 574 be on the number line? Students mark 574. Teacher: Is 574 closer to 570 or 580?</p> 
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<p>3.NBT.A.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.</p> <p>A range of algorithms may be used</p>	<p>3.MP.2. Reason abstractly and quantitatively.</p> <p>3.MP.7. Look for and make use of structure.</p> <p>3.MP.8. Look for and express regularity in repeated reasoning.</p>	<p>This standard refers to fluency, which means accuracy, efficiency (using a reasonable amount of steps and time), and flexibility (using strategies such as the distributive property). The word algorithm refers to a procedure or a series of steps. There are other algorithms other than the standard algorithm. Third grade students should have experiences beyond the standard algorithm.</p> <p>Problems should include both vertical and horizontal forms, including opportunities for students to apply the commutative and associative properties. Students explain their thinking, show their work by using strategies and algorithms, and verify that their answer is reasonable.</p> <p>Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly.</p> <p>Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another.</p> <p>Example: There are 178 fourth graders and 225 fifth graders on the playground. What is the total number of students on the playground?</p> <table border="1" data-bbox="926 958 1782 1320"> <tr> <td data-bbox="926 958 1150 1295"> <p>Student 1</p> $100 + 200 = 300$ $70 + 20 = 90$ $8 + 5 = 13$ $300 + 90 + 13 = 403$ students </td><td data-bbox="1150 958 1419 1295"> <p>Student 2</p> <p>I added 2 to 178 to get 180. I added 220 to get 400. I added the 3 left over to get 403.</p> </td><td data-bbox="1419 958 1782 1295"> <p>Student 3</p> <p>I know the 75 plus 25 equals 100. I then added 1 hundred from 178 and 2 hundreds from 225. I had a total of 4 hundreds and I had 3 more left to add. So I have 4 hundreds plus 3 more which is 403.</p> </td></tr> <tr> <td data-bbox="926 1295 1150 1320"></td><td data-bbox="1150 1295 1419 1320"></td><td data-bbox="1419 1295 1782 1320"></td></tr> </table>	<p>Student 1</p> $100 + 200 = 300$ $70 + 20 = 90$ $8 + 5 = 13$ $300 + 90 + 13 = 403$ students	<p>Student 2</p> <p>I added 2 to 178 to get 180. I added 220 to get 400. I added the 3 left over to get 403.</p>	<p>Student 3</p> <p>I know the 75 plus 25 equals 100. I then added 1 hundred from 178 and 2 hundreds from 225. I had a total of 4 hundreds and I had 3 more left to add. So I have 4 hundreds plus 3 more which is 403.</p>			
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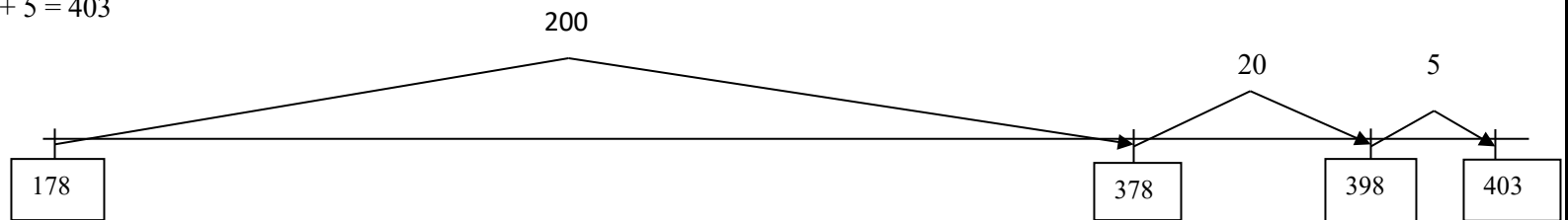
Student 4

$$178 + 225 = ?$$

$$178 + 200 = 378$$

$$378 + 20 = 398$$

$$398 + 5 = 403$$



3.NBT.A.3 Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9×80 , 5×60) using strategies based on place value and properties of operations.

3.MP.2. Reason abstractly and quantitatively.

3.MP.7. Look for and make use of structure.

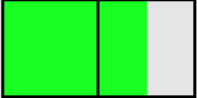
3.MP.8. Look for and express regularity in repeated reasoning.

This standard extends students' work in multiplication by having them apply their understanding of place value. This standard expects that students go beyond tricks that hinder understanding such as "just adding zeros" and explain and reason about their products. For example, for the problem 50×4 , students should think of this as 4 groups of 5 tens or 20 tens, and twenty tens equals 200.

The special role of 10 in the base-ten system is important in understanding multiplication of one-digit numbers with multiples of 10. For example, the product 3×50 can be represented as 3 groups of 5 tens, which is 15 tens, which is 150. This reasoning relies on the associative property of multiplication: $3 \times 50 = 3 \times (5 \times 10) = (3 \times 5) \times 10 = 15 \times 10 = 150$. It is an example of how to explain an instance of a calculation pattern for these products: calculate the product of the non-zero digits, and then shift the product one place to the left to make the result ten times as large.

		<p>● Grade 3 explanations for “15 tens is 150”</p> <ul style="list-style-type: none"> ● Skip-counting by 50. 5 tens is 50, 100, 150. ● Counting on by 5 tens. 5 tens is 50, 5 more tens is 100, 5 more tens is 150. ● Decomposing 15 tens. 15 tens is 10 tens and 5 tens. 10 tens is 100. 5 tens is 50. So 15 tens is 100 and 50, or 150. ● Decomposing 15. $ \begin{aligned} 15 \times 10 &= (10 + 5) \times 10 \\ &= (10 \times 10) + (5 \times 10) \\ &= 100 + 50 \\ &= 150 \end{aligned} $ <p><i>All of these explanations are correct. However, skip-counting and counting on become more difficult to use accurately as numbers become larger, e.g., in computing 5×90 or explaining why 45 tens is 450, and needs modification for products such as 4×90. The first does not indicate any place value understanding.</i></p>
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Number and Operation – Fractions		3.NF
College and Career Readiness Cluster		
Develop understanding of fractions as numbers.		
<p>Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, 8.</p> <p>Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, $\frac{1}{2}$ of the paint in a small bucket could be less paint than $\frac{1}{3}$ of the paint in a larger bucket, but $\frac{1}{3}$ of a ribbon is longer than $\frac{1}{5}$ of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.</p> <p>Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: partition(ed), equal parts, fraction, equal distance (intervals), equivalent, equivalence, reasonable, denominator, numerator, comparison, compare, $<$, $>$, $=$, justify, inequality</p>		
<p>Enduring Understandings: A fraction represents the relationship between the part and the whole.</p> <p>Essential Questions: How do I use fractions to represent part of a whole? Why is it important to know the size of the whole? How can I use my knowledge of number lines to help me understand fractions? What are some ways to name the same part of a whole? How can I compare fractions?</p>		
College and Career Readiness Standards <i>Students are expected to:</i>	Mathematical Practices	Unpacking Explanations and Examples <i>What does this standard mean that a student will know and be able to do?</i>
3.NF.A.1 Understand a fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is	3.MP.1. Make sense of problems	This standard refers to the sharing of a whole being partitioned. Fraction models in third grade include only area (parts of a whole) models (circles, rectangles, squares) and number lines. Set models (parts of a group) are not addressed in third grade.

<p>partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.</p>	<p>and persevere in solving them.</p> <p>3.MP.4. Model with mathematics</p> <p>3.MP.7. Look for and make use of structure.</p>	<p>In 3.NF.1 students start with unit fractions (fractions with numerator 1), which are formed by partitioning a whole into equal parts and reasoning about one part of the whole, e.g., if a whole is partitioned into 4 equal parts then each part is $\frac{1}{4}$ of the whole, and 4 copies of that part make the whole. Next, students build fractions from unit fractions, seeing the numerator 3 of $\frac{3}{4}$ as saying that $\frac{3}{4}$ is the quantity you get by putting 3 of the $\frac{1}{4}$'s together. There is no need to introduce "improper fractions" initially.</p> <div data-bbox="1003 402 1757 740" data-label="Complex-Block"> <p style="text-align: center;">The importance of specifying the whole</p>  <p><i>Without specifying the whole it is not reasonable to ask what fraction is represented by the shaded area. If the left square is the whole, the shaded area represents the fraction $\frac{3}{2}$; if the entire rectangle is the whole, the shaded area represents $\frac{3}{4}$.</i></p> </div> <p>Some important concepts related to developing understanding of fractions include:</p> <ul style="list-style-type: none"> Understand fractional parts must have parts. <ul style="list-style-type: none"> Example Non-example <div data-bbox="970 889 1299 1049" data-label="Image"> </div> <div data-bbox="940 1060 1430 1089" data-label="Caption"> <p>These are thirds These are NOT thirds</p> </div> The number of equal parts tells how many make a whole. As the number of equal pieces in the whole increases, the size of the fractional pieces decreases. The size of the fractional part is relative to the whole. <ul style="list-style-type: none"> One-half of a small pizza is relatively smaller than one-half of a large pizza. When a whole is cut into equal parts, the denominator represents the number of equal parts. The numerator of a fraction is the count of the number of equal parts. <ul style="list-style-type: none"> $\frac{3}{4}$ means that there are 3 one-fourths.
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		<p>○ Students can count <i>one fourth, two fourths, three fourths</i>.</p> <p>Students express fractions as fair sharing or, parts of a whole. They use various contexts (candy bars, fruit, and cakes) and a variety of models (circles, squares, rectangles, fraction bars, and number lines) to develop understanding of fractions and represent fractions. Students need many opportunities to solve word problems that require them to create and reason about fair share.</p> <p>Initially, students can use an intuitive notion of “same size and same shape” (congruence) to explain why the parts are equal, e.g., when they divide a square into four equal squares or four equal rectangles. Students come to understand a more precise meaning for “equal parts” as “parts with equal measurements.” For example, when a ruler is partitioned into halves or quarters of an inch, they see that each subdivision has the same length. In area models they reason about the area of a shaded region to decide what fraction of the whole it represents.</p> <div data-bbox="1024 605 1734 984" data-label="Figure"> <p style="text-align: center;">Area representations of $\frac{1}{4}$</p> <p><i>In each representation the square is the whole. The two squares on the left are divided into four parts that have the same size and shape, and so the same area. In the three squares on the right, the shaded area is $\frac{1}{4}$ of the whole area, even though it is not easily seen as one part in a division of the square into four parts of the same shape and size.</i></p> </div>
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3.NF.A.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram.

a. **Represent** a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.

b. **Represent** a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.

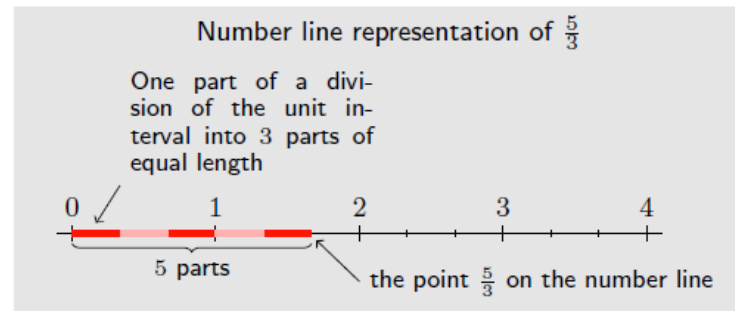
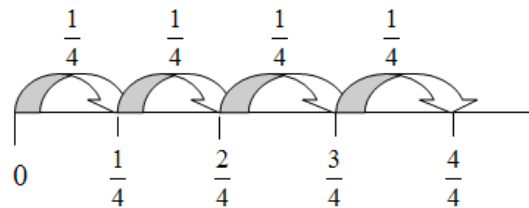
3.MP.1. Make sense of problems and persevere in solving them.

3.MP.4. Model with mathematics

3.MP.7. Look for and make use of structure.

The number line diagram is the first time students work with a number line for numbers that are between whole numbers (e.g., that $\frac{1}{2}$ is between 0 and 1). Students need ample experiences folding linear models (e.g., string, sentence strips) to help them reason about and justify the location of fractions, such that $\frac{1}{2}$ lies exactly halfway between 0 and 1.

In the number line diagram below, the space between 0 and 1 is divided (partitioned) into 4 equal regions. The distance from 0 to the first segment is 1 of the 4 segments from 0 to 1 or $\frac{1}{4}$ (**3.NF.2a**). Similarly, the distance from 0 to the third segment is 3 segments that are each one-fourth long. Therefore, the distance of 3 segments from 0 is the fraction $\frac{3}{4}$ (**3.NF.2b**).



<p>3.NF.A.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.</p> <p>a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.</p> <p>b. Recognize and generate simple equivalent fractions, e.g., $\frac{1}{2} = \frac{2}{4}$, $\frac{4}{6} = \frac{2}{3}$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.</p> <p>c. Express whole numbers as fractions, and recognize</p>	<p><i>3.MP.1.</i> Make sense of problems and persevere in solving them.</p> <p><i>3.MP.2.</i> Reason abstractly and quantitatively.</p> <p><i>3.MP.3.</i> Construct viable arguments and critique the reasoning of others.</p> <p><i>3.MP.4.</i> Model with mathematics.</p> <p><i>3.MP.6.</i> Attend to precision.</p> <p><i>3.MP.7.</i> Look for and make use of structure.</p> <p><i>3.MP.8.</i> Look for and express regularity in</p>	<p>An important concept when comparing fractions is to look at the size of the parts and the number of the parts.</p> <p>For example, $\frac{1}{8}$ is smaller than $\frac{1}{2}$ because when 1 whole is cut into 8 pieces, the pieces are much smaller than when 1 whole is cut into 2 pieces.</p> <p>3.NF.3a and 3.NF.3b These standards call for students to use visual fraction models (area models) and number lines to explore the idea of equivalent fractions. Students should only explore equivalent fractions using models, rather than using algorithms or procedures.</p> <p>This standard includes writing whole numbers as fractions. The concept relates to fractions as division problems, where the fraction $\frac{3}{1}$ is 3 wholes divided into one group. This standard is the building block for later work where students divide a set of objects into a specific number of groups. Students must understand the meaning of $\frac{a}{1}$.</p> <p>This standard involves comparing fractions with or without visual fraction models including number lines. Experiences should encourage students to reason about the size of pieces, the fact that $\frac{1}{3}$ of a cake is larger than $\frac{1}{4}$ of the same cake. Since the same cake (the whole) is split into equal pieces, thirds are larger than fourths.</p> <p>In this standard, students should also reason that comparisons are only valid if the wholes are identical. For example, $\frac{1}{2}$ of a large pizza is a different amount than $\frac{1}{2}$ of a small pizza. Students should be given opportunities to discuss and reason about which $\frac{1}{2}$ is larger.</p> <p>Previously, in second grade, students compared lengths using a standard measurement unit. In third grade they build on this idea to compare fractions with the same denominator. They see that for fractions that have the same denominator, the underlying unit fractions are the same size, so the fraction with the greater numerator is greater because it is made of more unit fractions.</p> <p>Students also see that for unit fractions, the one with the larger denominator is smaller, by reasoning, for example, that in order for more (identical) pieces to make the same whole, the pieces must be smaller. From this they reason that for fractions that have the same numerator, the fraction with the smaller denominator is greater.</p>
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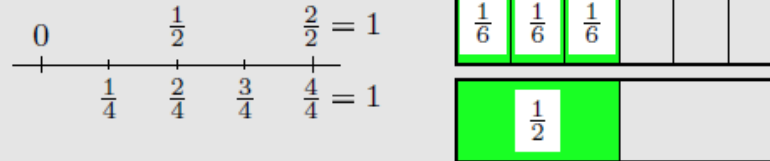
fractions that are equivalent to whole numbers.
Examples:
Express 3 in the form $3 = 3/1$; recognize that $6/1 = 6$; locate $4/4$ and 1 at the same point of a number line diagram.

d. **Compare** two **fractions** with the same numerator or the same denominator by reasoning about their size. **Recognize** that comparisons are valid only when the two **fractions** refer to the same whole. **Record** the results of comparisons with the symbols $>$, $=$, or $<$, and **justify** the conclusions, e.g., by using a visual fraction model

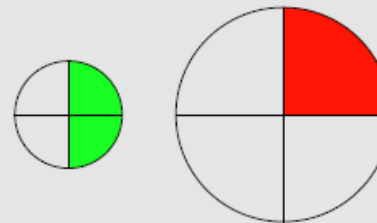
repeated reasoning.

As with equivalence of fractions, it is important in comparing fractions to make sure that each fraction refers to the same whole.

Using the number line and fraction strips to see fraction equivalence

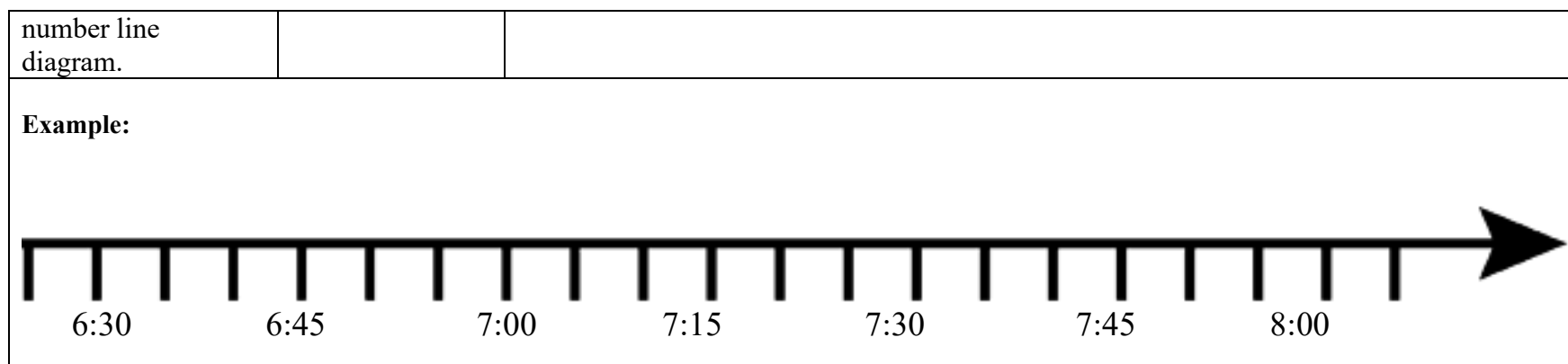


The importance of referring to the same whole when comparing fractions



A student might think that $\frac{1}{4} > \frac{1}{2}$, because a fourth of the pizza on the right is bigger than a half of the pizza on the left.

Measurement and Data		3.MD
College and Career Readiness Cluster		
Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.		
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: estimate, time, time intervals, minute, hour, elapsed time, measure, liquid volume, mass, standard units, metric, gram (g), kilogram (kg), liter (L), milliliter (ML)		
Enduring Understandings: Standard units of measure convey understanding of data and allow for precise interpretation of data results. Essential Questions: How can we use measurement to describe the attributes of objects and intervals of time? Why is it important to use standard units of measure? How do you determine which tool to use when measuring? How do you solve problems involving intervals of time? How do you solve problems involving volume and mass? How can I use picture graphs, bar graphs, and line plots to help me solve problems? What is area and what are different ways to calculate it? What is perimeter and what are different ways to calculate it?		
College and Career Readiness Standards <i>Students are expected to:</i>	Mathematical Practices	Unpacking Explanations and Examples <i>What does this standard mean that a student will know and be able to do?</i>
3.MD.A.1 Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a	3.MP.1. Make sense of problems and persevere in solving them. 3.MP.4. Model with mathematics. 3.MP.6. Attend to precision.	This standard calls for students to calculate elapsed time, including word problems. Students could use clock models or number lines to solve. On the number line, students should be given the opportunities to determine the intervals and size of jumps on their number line. Students could use pre-determined number lines (intervals every 5 or 15 minutes) or open number lines (intervals determined by students). In third grade, students are expected to tell time within the hour, in fourth grade students are expected to tell time throughout the hour.



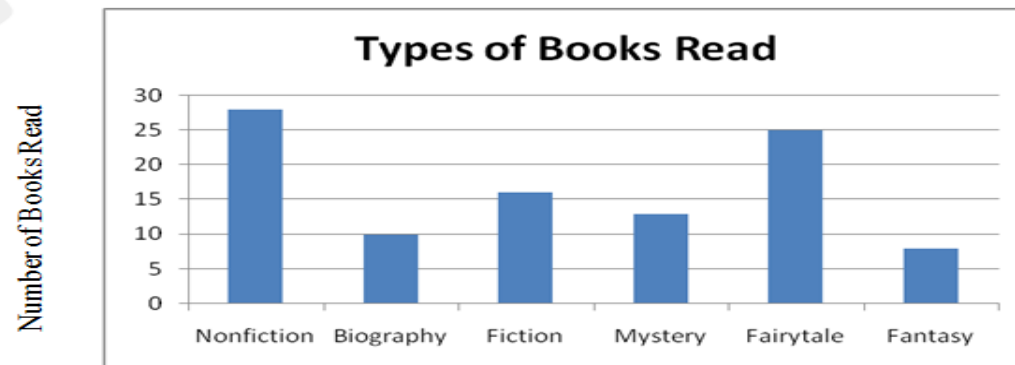
<p>3.MD.A.2 Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.</p>	<p><i>3.MP.1.</i> Make sense of problems and persevere in solving them.</p> <p><i>3.MP.2.</i> Reason abstractly and quantitatively,</p> <p><i>3.MP.4.</i> Model with mathematics.</p> <p><i>3.MP.5.</i> Use appropriate tools strategically.</p> <p><i>3.MP.6.</i> Attend to precision.</p>	<p>This standard asks for students to reason about the units of mass and volume using units g, kg, and L. Students need multiple opportunities weighing classroom objects and filling containers to help them develop a basic understanding of the size and weight of a liter, a gram, and a kilogram. Milliliters may also be used to show amounts that are less than a liter emphasizing the relationship between smaller units to larger units in the same system. Word problems should only be one-step and include the same units.</p> <p>Students are not expected to do conversions between units, but reason as they estimate, using benchmarks to measure weight and capacity.</p> <p>Example: Students identify 5 things that weigh about one gram. They record their findings with words and pictures. (Students can repeat this for 5 grams and 10 grams.) This activity helps develop gram benchmarks. One large paperclip weighs about one gram.</p> <p>Example: A paper clip weighs about a) a gram, b) 10 grams, c) 100 grams? Explain why.</p> <p>Foundational understandings to help with measure concepts: Understand that larger units can be subdivided into equivalent units (partition). Understand that the same unit can be repeated to determine the measure (iteration). Understand the relationship between the size of a unit and the number of units needed (compensatory principle).</p>
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<p>Excludes compound units such as cm³ and finding the geometric volume of a container.</p> <p>Excludes multiplicative comparison problems (problems involving notions of “times as much”; see Glossary, Table 2).</p> <p>(Table included at the end of this document for your convenience)</p>	<p>Before learning to measure attributes, children need to recognize them, distinguishing them from other attributes. That is, the attribute to be measured has to “stand out” for the student and be discriminated from the less sophisticated understanding that young children often have. Ex: "This cup holds a larger volume of water" versus "This cup is bigger."</p> <p>These standards do not differentiate between weight and mass. Technically, mass is the amount of matter in an object. Weight is the force exerted on the body by gravity. On the earth’s surface, the distinction is not important (on the moon, an object would have the same mass, but would weigh less due to the lower gravity).</p> <p>Much of the work involving measure supports the work that is emphasized in third on multiplication. Example:</p> <p style="text-align: center;">Table 1: Multiplication and division situations for measurement</p> <table><tr><th></th><th>Unknown Product $A \times B = \square$</th><th>Group Size Unknown $A \times \square = C$ and $C \div A = \square$</th><th>Number of Groups Unknown $\square \times B = C$ and $C \div B = \square$</th></tr><tr><td>Grouped Objects (Units of Units)</td><td>You need A lengths of string, each B inches long. How much string will you need altogether?</td><td>You have C inches of string, which you will cut into A equal pieces. How long will each piece of string be?</td><td>You have C inches of string, which you will cut into pieces that are B inches long. How many pieces of string will you have?</td></tr><tr><td>Arrays of Objects (Spatial Structuring)</td><td>What is the area of a A cm by B cm rectangle?</td><td>A rectangle has area C square centimeters. If one side is A cm long, how long is a side next to it?</td><td>A rectangle has area C square centimeters. If one side is B cm long, how long is a side next to it?</td></tr><tr><td>Compare</td><td>A rubber band is B cm long. How long will the rubber band be when it is stretched to be A times as long?</td><td>A rubber band is stretched to be C cm long and that is A times as long as it was at first. How long was the rubber band at first?</td><td>A rubber band was B cm long at first. Now it is stretched to be C cm long. How many times as long is the rubber band now as it was at first?</td></tr></table> <p>Adapted from box 2-4 of <i>Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity</i>, National Research Council, 2009, pp. 32–33. Note that Grade 3 work does not include Compare problems with “times as much,” see the Operations and Algebraic Thinking Progression, Table 3, also p. 29.</p>		Unknown Product $A \times B = \square$	Group Size Unknown $A \times \square = C$ and $C \div A = \square$	Number of Groups Unknown $\square \times B = C$ and $C \div B = \square$	Grouped Objects (Units of Units)	You need A lengths of string, each B inches long. How much string will you need altogether?	You have C inches of string, which you will cut into A equal pieces. How long will each piece of string be?	You have C inches of string, which you will cut into pieces that are B inches long. How many pieces of string will you have?	Arrays of Objects (Spatial Structuring)	What is the area of a A cm by B cm rectangle?	A rectangle has area C square centimeters. If one side is A cm long, how long is a side next to it?	A rectangle has area C square centimeters. If one side is B cm long, how long is a side next to it?	Compare	A rubber band is B cm long. How long will the rubber band be when it is stretched to be A times as long?	A rubber band is stretched to be C cm long and that is A times as long as it was at first. How long was the rubber band at first?	A rubber band was B cm long at first. Now it is stretched to be C cm long. How many times as long is the rubber band now as it was at first?
	Unknown Product $A \times B = \square$	Group Size Unknown $A \times \square = C$ and $C \div A = \square$	Number of Groups Unknown $\square \times B = C$ and $C \div B = \square$														
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College and Career Readiness Cluster		
Represent and interpret data.		
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: scale, scaled picture graph, scaled bar graph, line plot, data		
College and Career Readiness Standards <i>Students are expected to:</i>	Mathematical Practices	Unpacking Explanations and Examples <i>What does this standard mean that a student will know and be able to do?</i>
3.MD.B.3 Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs. <i>For example, draw a bar graph in which each square in the bar graph might represent 5 pets.</i>	3.MP.1. Make sense of problems and persevere in solving them. 3.MP.4. Model with mathematics. 3.MP.6. Attend to precision. 3.MP.7. Look for and make use of pattern.	<p>Students should have opportunities to read and solve problems using scaled graphs before being asked to draw one. Working with scaled graphs builds on students’ understandings of multiplication and division.</p> <p>The graphs below all use five as the scale interval, but students should experience different intervals to further develop their understanding of scale graphs and number facts.</p> <p>While exploring data concepts, students should Pose a question, Collect data, Analyze data, and Interpret data (PCAI). Students should be graphing sets of data that are relevant to their lives.</p> <p>Example: Pose a question: (Student should come up with a question.) What is the typical genre read in our class? Collect and organize data: student survey</p> <p>Pictographs: Scaled pictographs include symbols that represent multiple units. Below is an example of a pictograph with symbols that represent multiple units. Graphs should include a title, categories, category label, key, and data. How many more books did Juan read than Nancy?</p>

Number of Books Read	
Nancy	★ ★ ★ ★ ★
Juan	★ ★ ★ ★ ★ ★ ★
★ = 5 Books	

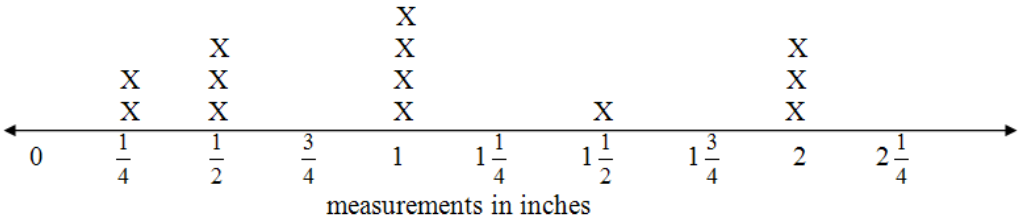
Single Bar Graphs: Students use both horizontal and vertical bar graphs. Bar graphs include a title, scale, scale label, categories, category label, and data.



Analyze and Interpret data:

- How many more nonfiction books were read than fantasy books?
- Did more people read biography and mystery books or fiction and fantasy books?
- About how many books in all genres were read?
- Using the data from the graphs, what type of book was read more often than a mystery but less often than a fairytale?
- What interval was used for this scale?
- What can we say about types of books read? What is a typical type of book read? (beyond standard)

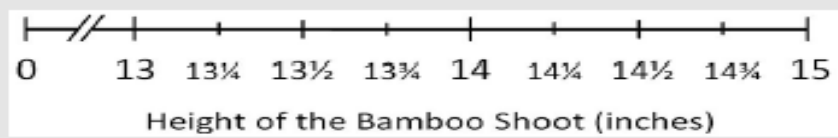
If you were to purchase a book for the class library which would be the best genre? Why? (beyond standard)

<p>3.MD.B.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.</p>	<p><i>3.MP.1.</i> Make sense of problems and persevere in solving them.</p> <p><i>3.MP.4.</i> Model with mathematics.</p> <p><i>3.MP.6.</i> Attend to precision.</p>	<p>Students in second grade measured length in whole units using both metric and U.S. customary systems. It's important to review with students how to read and use a standard ruler including details about halves and quarter marks on the ruler. Students should connect their understanding of fractions to measuring to one-half and one-quarter inch. Third graders need many opportunities measuring the length of various objects in their environment.</p> <p>This standard provides a context for students to work with fractions by measuring objects to a quarter of an inch.</p> <p>Example: Measure objects in your desk to the nearest $\frac{1}{2}$ or $\frac{1}{4}$ of an inch, display data collected on a line plot. How many objects measured $\frac{1}{4}$? $\frac{1}{2}$? etc.</p> <p style="text-align: center;">Objects in my Desk</p>  <p>In Grade 3, students are beginning to learn fraction concepts (3.NF). They understand fraction equivalence in simple cases, and they use visual fraction models to represent and order fractions. Grade 3 students also measure lengths using rulers marked with halves and fourths of an inch. They use their developing knowledge of fractions and number lines to extend their work from the previous grade by working with measurement data involving fractional measurement values.</p> <p>For example, every student in the class might measure the height of a bamboo shoot growing in the classroom, leading to the data set shown in the table. (Illustration below shows a larger data set than students would normally work with in elementary grades.)</p> <p>To make a line plot from the data in the table, the student can determine the greatest and least values in the data: $13\frac{1}{2}$ inches and $14\frac{3}{4}$ inches. The student can draw a segment of a number line diagram that includes these extremes, with tick marks indicating specific values on the</p>
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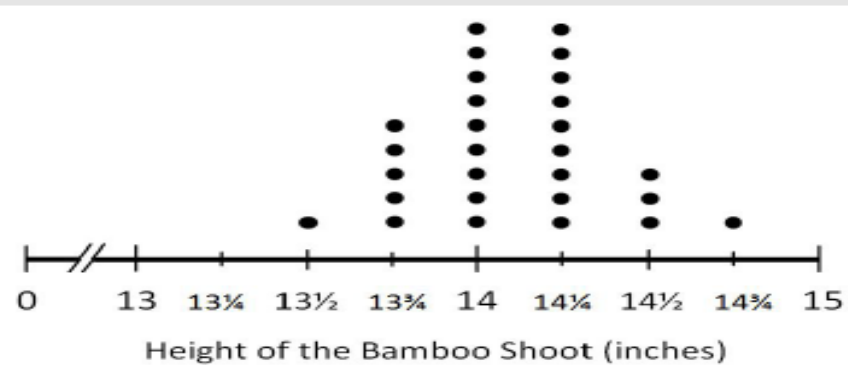
measurement scale. This is just like part of the scale on a ruler. Having drawn the number line diagram, the student can proceed through the data set recording each observation by drawing a symbol, such as a dot, above the proper tick mark. As with Grade 2 line plots, if a particular data value appears many times in the data set, dots will “pile up” above that value. There is no need to sort the observations, or to do any counting of them, before producing the line plot. Students can pose questions about data presented in line plots, such as how many students obtained measurements larger than $14 \frac{1}{4}$ inches.

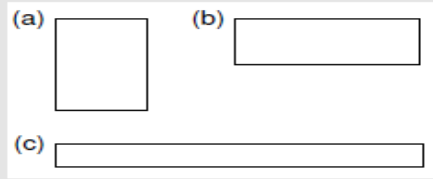
Students' measurements of a statue and of a bamboo shoot			
Statue measurements		Bamboo shoot measurements	
Student's initials	Student's measured value (inches)	Student's initials	Height value (inches)
W.B.	64	W.B.	$13 \frac{3}{4}$
D.W.	65	D.W.	$14 \frac{1}{2}$
H.D.	65	H.D.	$14 \frac{1}{4}$
G.W.	65	G.W.	$14 \frac{3}{4}$
V.Y.	67	V.Y.	$14 \frac{1}{4}$
T.T.	66	T.T.	$14 \frac{1}{2}$
D.F.	67	D.F.	14
B.H.	65	B.H.	$13 \frac{1}{2}$
H.H.	63	H.H.	$14 \frac{1}{4}$
V.H.	64	V.H.	$14 \frac{1}{4}$
I.O.	64	I.O.	$14 \frac{1}{4}$
W.N.	65	W.N.	14
B.P.	69	B.P.	$14 \frac{1}{2}$
V.A.	65	V.A.	$13 \frac{3}{4}$
H.L.	66	H.L.	14
O.M.	64	O.M.	$13 \frac{3}{4}$
L.E.	65	L.E.	$14 \frac{1}{4}$
M.J.	66	M.J.	$13 \frac{3}{4}$
T.D.	66	T.D.	$14 \frac{1}{4}$
K.P.	64	K.P.	14
H.N.	65	H.N.	14
W.M.	67	W.M.	14
C.Z.	64	C.Z.	$13 \frac{3}{4}$
J.I.	66	J.I.	14
M.S.	66	M.S.	$14 \frac{1}{4}$
T.C.	65	T.C.	14
G.V.	67	G.V.	14
O.F.	65	O.F.	$14 \frac{1}{4}$

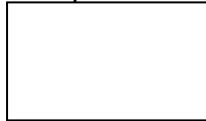
A scale for a line plot of the bamboo shoot data



A line plot of the bamboo shoot data



College and Career Readiness Cluster		
Geometric measurement: understand concepts of area and relate area to multiplication and to addition.		
<p>Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.</p> <p>Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: attribute, area, square unit, plane figure, gap, overlap, square cm, square m , square in., square ft, nonstandard units, tiling, side length, decomposing</p>		
College and Career Readiness Standards <i>Students are expected to:</i>	Mathematical Practices	Unpacking Explanations and Examples <i>What does this standard mean that a student will know and be able to do?</i>
<p>3.MD.C.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.</p> <p>a. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.</p>	<p>3.MP.2. Reason abstractly and quantitatively.</p> <p>3.MP.4. Model with mathematics.</p> <p>3.MP.5. Use appropriate tools strategically.</p> <p>3.MP.6. Attend to precision.</p>	<p>These standards call for students to explore the concept of covering a region with “unit squares,” which could include square tiles or shading on grid or graph paper. Based on students’ development, they should have ample experiences filling a region with square tiles before transitioning to pictorial representations on graph paper.</p> <p>Example:</p> <div data-bbox="966 974 1761 1396"> <p>Which rectangle covers the most area?</p>  <p>These rectangles are formed from unit squares (tiles students have used) although students are not informed of this or the rectangle’s dimensions: (a) 4 by 3, (b) 2 by 6, and (c) 1 row of 12. Activity from Lehrer, et al., 1998, “Developing understanding of geometry and space in the primary grades,” in R. Lehrer & D. Chazan (Eds.), <i>Designing Learning Environments for Developing Understanding of Geometry and Space</i>, Lawrence Erlbaum Associates.</p> </div>

b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square unit														
3.MD.C.6 Measure areas by counting unit squares (square cm, square m, square in, square ft., and improvised units).	<p>3.MP.5. Use appropriate tools strategically.</p> <p>3.MP.6. Attend to precision.</p>	<p>Students should be counting the square units to find the area. This could be done in metric, customary, or non-standard square units. Using different sized graph paper, students can explore the areas measured in square centimeters and square inches.</p> <p>The task shown above would provide a great experience for students to tile a region and count the number of square units.</p>												
<p>3.MD.C.7 Relate area to the operations of multiplication and addition.</p> <p>a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.</p> <p>b. Multiply side lengths to find</p>	<p>3.MP.1. Make sense of problems and persevere in solving them.</p> <p>3.MP.2. Reason abstractly and quantitatively.</p> <p>3.MP.4. Model with mathematics.</p> <p>3.MP.5. Use appropriate tools strategically.</p> <p>3.MP.6. Attend to precision.</p>	<p>Students should tile rectangle then multiply the side lengths to show it is the same.</p> <div><p>To find the area one could count the squares or multiply $3 \times 4 = 12$.</p><table><tr><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>5</td><td>6</td><td>7</td><td>8</td></tr><tr><td>9</td><td>10</td><td>11</td><td>12</td></tr></table></div> <p>Students should solve real world and mathematical problems</p> <p>Example: Drew wants to tile the bathroom floor using 1 foot tiles. How many square foot tiles will he need?</p> <div><p>8 square feet</p><div></div><p>6 square feet</p></div> <p>Students might solve problems such as finding all the rectangular regions with whole-number side lengths that have an area of 12 area-units, doing this for larger rectangles (e.g., enclosing 24, 48, 72 area-units), making</p>	1	2	3	4	5	6	7	8	9	10	11	12
1	2	3	4											
5	6	7	8											
9	10	11	12											

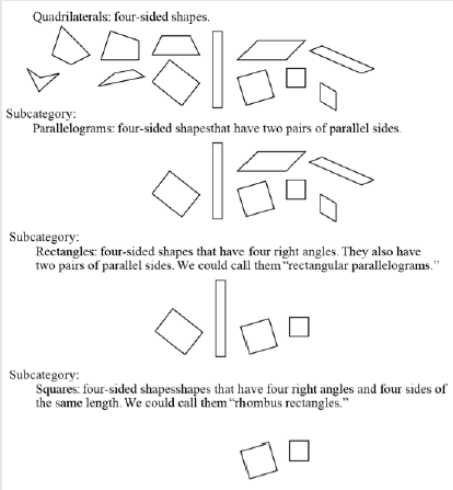
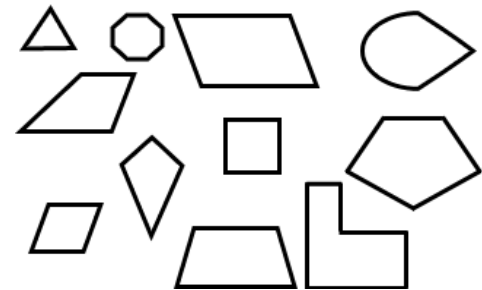
<p>areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.</p> <p>c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.</p> <p>d. Recognize area as additive. Find areas of rectilinear figures</p>		<p>sketches rather than drawing each square. Students learn to justify their belief they have found all possible solutions.</p> <p>Students tile areas of rectangles, determine the area, record the length and width of the rectangle, investigate the patterns in the numbers, and discover that the area is the length times the width.</p> <p>Joe and John made a poster that was 4' by 3'. Mary and Amir made a poster that was 4' by 2'. They placed their posters on the wall side-by-side so that there was no space between them. How much area will the two posters cover? Students use pictures, words, and numbers to explain their understanding of the distributive property in this context.</p> <div data-bbox="1050 487 1501 706" data-label="Diagram"> <p style="text-align: center;"> $4 \times 3 + 4 \times 2 = 20$ $4(3 + 2) = 20$ $4 \times 5 = 20$ </p> </div> <p>Students can decompose a rectilinear figure into different rectangles. They find the area of the figure by adding the areas of each of the rectangles together.</p> <div data-bbox="1050 852 1480 1144" data-label="Diagram"> <p style="text-align: center;">area is $12 \times 3 + 8 \times 7 = 92$ sq inches</p> </div>
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by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.		
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College and Career Readiness Cluster		
Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.		
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: attribute, perimeter, plane figure, linear, area, polygon, side length		
College and Career Readiness Standards <i>Students are expected to:</i>	Mathematical Practices	Unpacking Explanations and Examples <i>What does this standard mean that a student will know and be able to do?</i>
3.MD.D.8 Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths,	<p><i>3.MP.1.</i> Make sense of problems and persevere in solving them.</p> <p><i>3.MP.4.</i> Model with mathematics.</p>	Students develop an understanding of the concept of perimeter through various experiences, such as walking around the perimeter of a room, using rubber bands to represent the perimeter of a plane figure on a geoboard, or tracing around a shape on an interactive whiteboard. They find the perimeter of objects; use addition to find perimeters; and recognize the patterns that exist when finding the sum of the lengths and widths of rectangles. Students should also strategically use tools, such as geoboards, tiles, and graph paper to find all the possible rectangles that have a given perimeter (e.g., find the rectangles with a perimeter of 14 cm.) They record all the possibilities using dot or graph paper, compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles. Following this experience, students can reason about connections between their representations, side lengths, and the perimeter of the rectangles. Given a perimeter and a length or width, students use objects or pictures to find the missing length or width. They justify and communicate their solutions using words, diagrams, pictures, numbers, and an interactive whiteboard.

finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.	3.MP.7. Look for and make use of structure.	<p>Students use geoboards, tiles, graph paper, or technology to find all the possible rectangles with a given area (e.g. find the rectangles that have an area of 12 square units.) They record all the possibilities using dot or graph paper, compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles. Students then investigate the perimeter of the rectangles with an area of 12.</p> <table><tr><th>Area</th><th>Length</th><th>Width</th><th>Perimeter</th></tr><tr><td>12 sq. in.</td><td>1 in.</td><td>12 in.</td><td>26 in.</td></tr><tr><td>12 sq. in.</td><td>2 in.</td><td>6 in.</td><td>16 in.</td></tr><tr><td>12 sq. in.</td><td>3 in.</td><td>4 in.</td><td>14 in.</td></tr><tr><td>12 sq. in.</td><td>4 in.</td><td>3 in.</td><td>14 in.</td></tr><tr><td>12 sq. in.</td><td>6 in.</td><td>2 in.</td><td>16 in.</td></tr><tr><td>12 sq. in.</td><td>12 in.</td><td>1 in.</td><td>26 in.</td></tr></table> <p>The patterns in the chart allow the students to identify the factors of 12, connect the results to the commutative property, and discuss the differences in perimeter within the same area. This chart can also be used to investigate rectangles with the same perimeter. It is important to include squares in the investigation.</p>	Area	Length	Width	Perimeter	12 sq. in.	1 in.	12 in.	26 in.	12 sq. in.	2 in.	6 in.	16 in.	12 sq. in.	3 in.	4 in.	14 in.	12 sq. in.	4 in.	3 in.	14 in.	12 sq. in.	6 in.	2 in.	16 in.	12 sq. in.	12 in.	1 in.	26 in.
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Geometry	3.G
College and Career Readiness Cluster	
Reason with shapes and their attributes.	
<p>Students describe, analyze, and compare properties of two dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.</p> <p>Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: properties¹, attributes¹, features¹, quadrilateral, open figure, closed figure , three-sided, 2-dimensional, rectangles, and squares are subcategories of quadrilaterals, polygon, rhombus/rhombi, rectangle, square, partition, unit fraction, kite, parallelogram, examples, parallelogram, right angle, and non-examples</p> <p>From previous grades: triangle, quadrilateral, pentagon, hexagon, cube, trapezoid, half/quarter circle, circle, cone, cylinder, sphere, sides, vertices, corners</p> <p>The term “property” in these standards is reserved for those attributes that indicate a relationship between components of shapes. Thus, “having parallel sides” or “having all sides of equal lengths” are properties. “Attributes” and “features” are used interchangeably to indicate any characteristic of a shape, including properties, and other defining characteristics (e.g., straight sides) and non-defining characteristics (e.g., “right-side up”).</p>	
<p>Enduring Understandings: Objects can be identified, described, and compared using their geometric attributes in order to make sense of the physical environment.</p> <p>Essential Questions: How can I identify, describe, and classify triangles, quadrilaterals, and other geometric shapes? How can I divide two-dimensional shapes into equal parts and name the fractional parts?</p>	

College and Career Readiness Standards <i>Students are expected to:</i>	Mathematical Practices	Unpacking Explanations and Examples <i>What does this standard mean that a student will know and be able to do?</i>
<p>3.G.A.1 Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.</p>	<p>3.MP.5. Use appropriate tools strategically.</p> <p>3.MP.6. Attend to precision.</p> <p>3.MP.7. Look for and make use of structure.</p>	<p>In second grade, students identify and draw triangles, quadrilaterals, pentagons, and hexagons. Third Graders build on this experience and further investigate quadrilaterals. Students should be encouraged to provide details and use proper vocabulary when describing the properties of quadrilaterals. They sort geometric figures (see examples below) and identify squares, rectangles, and rhombuses as quadrilaterals.</p> <div data-bbox="800 524 1346 1182"> <p>Quadrilaterals and some special kinds of quadrilaterals</p>  <p>The representations above might be used by teachers in class. Note that the left-most four shapes in the first section at the top left have four sides but do not have properties that would place them in any of the other categories shown (parallelograms, rectangles, squares).</p> </div> <div data-bbox="1365 646 1843 933">  </div> <p>Students should classify shapes by attributes and drawing shapes that fit specific categories.</p> <p>Example: Students can form larger categories, such as the class of all shapes with four sides, or quadrilaterals, and recognize that it includes other categories, such as squares, rectangles, rhombuses, parallelograms, and trapezoids. They also recognize that there are quadrilaterals that are not in any of those subcategories.</p>

Quadrilaterals and some special kinds of quadrilaterals

Quadrilaterals: four-sided shapes.



Subcategory:
Parallelograms: four-sided shapes that have two pairs of parallel sides.



Subcategory:
Rectangles: four-sided shapes that have four right angles. They also have two pairs of parallel sides. We could call them "rectangular parallelograms."



Subcategory:
Squares: four-sided shapes that have four right angles and four sides of the same length. We could call them "rhombus rectangles."



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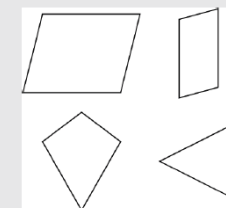
The representations above might be used by teachers in class. Note that the left-most four shapes in the first section at the top left have four sides but do not have properties that would place them in any of the other categories shown (parallelograms, rectangles, squares).

The standards do **not** require the above representation be constructed by students, but students should be able to draw examples of quadrilaterals that are not in the subcategories.

Example:

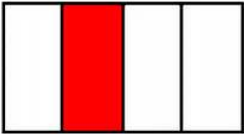
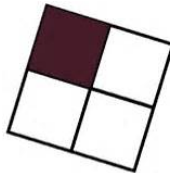
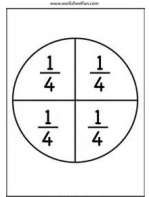
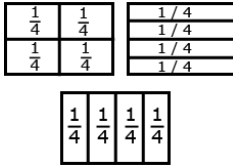
Parallelograms include: squares, rectangles, rhombuses, or other shapes that have two pairs of parallel sides. Also, the broad category quadrilaterals include all types of parallelograms, trapezoids and other four-sided figures.

Quadrilaterals that are not rectangles



These quadrilaterals have two pairs of sides of the same length but are not rectangles. A kite is on lower left and a deltoid is at lower right.

		<p>Example: Draw a picture of a quadrilateral. Draw a picture of a rhombus. How are they alike? How are they different? Is a quadrilateral a rhombus? Is a rhombus a quadrilateral? Justify your thinking.</p> <p>A kite is a quadrilateral whose four sides can be grouped into two pairs of equal-length sides that are beside each other.</p> <p>The notion of congruence (“same size and same shape”) may be part of classroom conversation but the concepts of congruence and similarity do not appear until middle school.</p>
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College and Career Readiness Standards <i>Students are expected to:</i>	Mathematical Practices	Unpacking Explanations and Examples <i>What does this standard mean that a student will know and be able to do?</i>
<p>3.G.A.2 Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as $\frac{1}{4}$ of the area of the shape.</p>	<p>3.MP.2. Reason abstractly and quantitatively. 3.MP.4. Model with mathematics. 3.MP.5. Use appropriate tools strategically.</p>	<p>In third grade students start to develop the idea of a fraction more formally, building on the idea of partitioning a whole into equal parts. The whole can be a shape such as a circle or rectangle. In Grade 4, this is extended to include wholes that are collections of objects. This standard also builds on students’ work with fractions and area. Students are responsible for partitioning shapes into halves, thirds, fourths, sixths and eighths. Example: This figure was partitioned/divided into four equal parts. Each part is $\frac{1}{4}$ of the total area of the figure.</p> <p>Given a shape, students partition it into equal parts, recognizing that these parts all have the same area. They identify the fractional name of each part and are able to partition a shape into parts with equal areas in several different ways.</p> <div style="text-align: center;">   </div> <div style="text-align: center;"> $\frac{1}{4}$ </div> <div style="text-align: center;">  </div> <div style="text-align: center;">  </div>