

Merrimack School District Mathematics Curriculum

Grade 6

Standards for Mathematical Practice

The College and Career Readiness Standards for Mathematical Practice are expected to be integrated into every mathematics lesson for all students Grades K-12. Below are a few examples of how these Practices may be integrated into tasks that students complete.

| Standards for Mathematical Practice | Explanations and Examples |
|---|--|
| 1. Make sense of problems and persevere in solving them. | In grade 6, students solve real world problems through the application of algebraic and geometric concepts. These problems involve ratio, rate, area and statistics. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, “What is the most efficient way to solve the problem?”, “Does this make sense?”, and “Can I solve the problem in a different way?”. Students can explain the relationships between equations, verbal descriptions, tables and graphs. Mathematically proficient students check answers to problems using a different method. |
| 2. Reason abstractly and quantitatively. | In grade 6, students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations. |
| 3. Construct viable arguments and critique the reasoning of others. | In grade 6, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like “How did you get that?”, “Why is that true?” “Does that always work?” They explain their thinking to others and respond to others’ thinking. |
| 4. Model with mathematics. | In grade 6, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students begin to explore covariance and represent two quantities simultaneously. Students use number lines to compare numbers and represent inequalities. They use measures of center and variability and data displays (i.e. box plots and histograms) to draw inferences about and make comparisons between data sets. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context. |

| Standards for Mathematical Practice | Explanations and Examples |
|---|--|
| 5. Use appropriate tools strategically. | Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 6 may decide to represent figures on the coordinate plane to calculate area. Number lines are used to understand division and to create dot plots, histograms and box plots to visually compare the center and variability of the data. Additionally, students might use physical objects or applets to construct nets and calculate the surface area of three-dimensional figures. |
| 6. Attend to precision. | In grade 6, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to rates, ratios, geometric figures, data displays, and components of expressions, equations or inequalities. |
| 7. Look for and make use of structure. | Students routinely seek patterns or structures to model and solve problems. For instance, students recognize patterns that exist in ratio tables recognizing both the additive and multiplicative properties. Students apply properties to generate equivalent expressions (i.e. $6 + 2x = 2(3 + x)$ by distributive property) and solve equations (i.e. $2c + 3 = 15$, $2c = 12$ by subtraction property of equality, $c=6$ by division property of equality). Students compose and decompose two- and three-dimensional figures to solve real world problems involving area and volume. |
| 8. Look for and express regularity in repeated reasoning. | In grade 6, students use repeated reasoning to understand algorithms and make generalizations about patterns. During multiple opportunities to solve and model problems, they may notice that $a/b \div c/d = ad/bc$ and construct other examples and models that confirm their generalization. Students connect place value and their prior work with operations to understand algorithms to fluently divide multi-digit numbers and perform all operations with multi-digit decimals. Students informally begin to make connections between covariance, rates, and representations showing the relationships between quantities. |

Grade 6 Critical Areas

The Critical Areas are designed to bring focus to the standards at each grade by describing the big ideas that educators can use to build their curriculum and to guide instruction.

1. Connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems.

Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

2. Completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers.

Students use the meaning of fractions, the meaning of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

3. Writing, interpreting, and using expressions and equations.

Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as $3x = y$) to describe relationships between quantities.

4. Developing understanding of statistical thinking.

Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected.

5. Reasoning about relationships among shapes to determine area, surface area, and volume.

Students in Grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.

Grade 6 Overview

Ratios and Proportional relationships

- Understand ratio concepts and use ratio reasoning to solve problems.

The Number System

- Apply and extend previous understanding of multiplication and division to divide fractions by fractions.
- Compute fluently with multi-digit numbers and find common factors and multiples.
- Apply and extend previous understandings of numbers to the system of rational numbers.

Expressions and Equations


- Apply and extend previous understandings of arithmetic to algebraic expressions.
- Reason about and solve one-variable equations and inequalities.
- Represent and analyze quantitative relationships between dependent and independent variables.

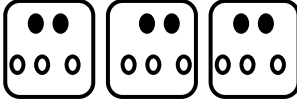
Geometry

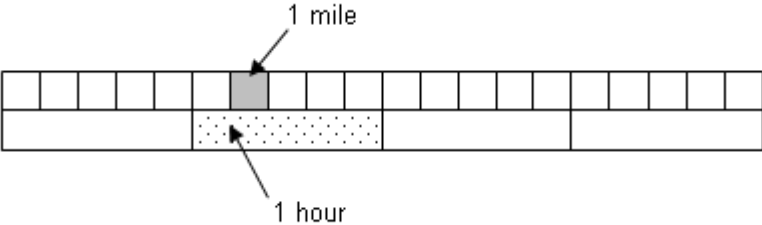
- Solve real-world and mathematical problems involving area, surface area, and volume.

Statistics and Probability

- Develop understanding of statistical variability.
- Summarize and describe distributions.

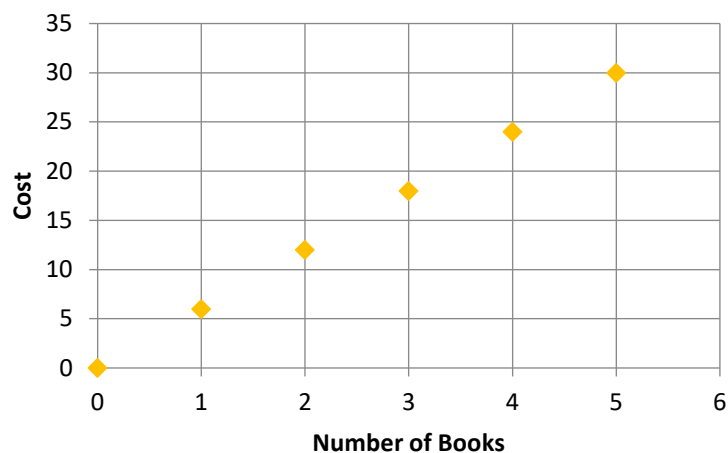
| Ratios and Proportional Relationships | | 6.RP |
|--|---|---|
| College and Career Readiness Cluster | | |
| Understand ratio concepts and use ratio reasoning to solve problems. | | |
| Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: ratio, equivalent ratios, tape diagram, unit rate, part-to-part, part-to-whole, percentage, proportional relationship | | |
| Enduring Understandings: Proportional relationships express how quantities change in relationship to each other. Essential Questions: What is a ratio? How do I use ratios to understand and describe the relationship between quantities? | | |
| College and Career Readiness Standards <i>Students are expected to:</i> | Mathematical Practices | Unpacking Explanations and Examples <i>What does this standard mean that a student will know and be able to do?</i> |
| 6.RP.A.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. <i>For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2</i> | 6.MP.2. Reason abstractly and quantitatively. 6.MP.6. Attend to precision. | A ratio is the comparison of two quantities or measures. The comparison can be part-to-whole (ratio of guppies to all fish in an aquarium) or part-to-part (ratio of guppies to goldfish). Example 1: A comparison of 6 guppies and 9 goldfish could be expressed in any of the following forms: $\frac{6}{9}$ If the number of guppies is represented by black circles and the number of goldfish is represented by white circles, this ratio could be a model. <div style="text-align: center;">  </div> |

| | | |
|--|--|---|
| <p>wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”</p> | | <p>These values can be regrouped into 2 black circles (guppies) to 3 white circles (goldfish), which would reduce the ratio to, $\frac{2}{3}$, 2 to 3 or 2:3.</p>  <p>Students should be able to identify and describe any ratio using “For every _____, there are _____” In the example above, the ratio could be expressed saying, “For every 2 guppies, there are 3 goldfish”.</p> <p>NOTE: Ratios are often expressed in fraction notation, although ratios and fractions do not have identical meaning. For example, ratios are often used to make “part-part” comparisons but fractions are not.</p> |
| <p>6.RP.A.2 Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $\frac{3}{4}$ cup of flour for each cup of sugar.” “We paid \$75 for 15 hamburgers, which</p> | <p>6.MP.2. Reason abstractly and quantitatively.</p> <p>6.MP.6. Attend to precision.</p> | <p>A unit rate compares a quantity in terms of one unit of another quantity. Students will often use unit rates to solve missing value problems. Cost per item or distance per time unit are common unit rates, however, students should be able to flexibly use unit rates to name the amount of either quantity in terms of the other quantity. Students will begin to notice that related unit rates are reciprocals as in the first example. It is not intended that this be taught as an algorithm or rule because at this level, students should primarily use reasoning to find these unit rates.</p> <p>In Grade 6, students are not expected to work with unit rates expressed as complex fractions. Both the numerator and denominator of the original ratio will be whole numbers.</p> <p>Examples:</p> <ul style="list-style-type: none"> On a bicycle you can travel 20 miles in 4 hours. What are the unit rates in this situation, (the distance you can travel in 1 hour and the amount of time required to travel 1 mile)? |

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| <p>is a rate of \$5 per hamburger.”¹</p> <p>¹ Expectations for unit rates in this grade are limited to non-complex fractions.</p> | | <p>Solution: You can travel 5 miles in 1 hour written as $\frac{5 \text{ mi}}{1 \text{ hr}}$ and it takes $\frac{1}{5}$ of an hour to travel each mile written as $\frac{1 \text{ hr}}{5 \text{ mi}}$. Students can represent the relationship between 20 miles and 4 hours.</p>  <ul style="list-style-type: none"> A simple modeling clay recipe calls for 1 cup corn starch, 2 cups salt, and 2 cups boiling water. How many cups of corn starch are needed to mix with each cup of salt? |
| <p>6.RP.A.3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.</p> <p>a. Make tables of equivalent ratios relating quantities</p> | <p>6.MP.1. Make sense of problems and persevere in solving them.</p> <p>6.MP.2. Reason abstractly and quantitatively.</p> <p>6.MP.4. Model with mathematics</p> <p>6.MP.5. Use appropriate</p> | <p>Ratios and rates can be used in ratio tables and graphs to solve problems. Previously, students have used additive reasoning in tables to solve problems. To begin the shift to proportional reasoning, students need to begin using multiplicative reasoning. Scaling up or down with multiplication maintains the equivalence. To aid in the development of proportional reasoning the cross-product algorithm is <i>not</i> expected at this level. When working with ratio tables and graphs, <i>whole number</i> measurements are the expectation for this standard.</p> <p>Example 1: At Books Unlimited, 3 paperback books cost \$18. What would 7 books cost? How many books could be purchased with \$54.</p> <p>Solution: To find the price of 1 book, divide \$18 by 3. One book costs \$6. To find the price of 7 books, multiply \$6 (the cost of one book times 7 to get \$42. To find the number of books that can be purchased with \$54, multiply \$6 times 9 to get \$54 and then multiply 1 book times 9 to get 9 books.</p> |

| with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. | tools strategically. 6.MP.7. Look for and make use of structure. | <p>Students use ratios, unit rates and multiplicative reasoning to solve problems in various contexts, including measurement, prices, and geometry. Notice in the table below, a multiplicative relationship exists between the numbers both horizontally (times 6) and vertically. (ie. $1 \cdot 7 = 7$; $6 \cdot 7 = 42$). Red numbers indicate solutions.</p> <table><tr><th>Number of Books (n)</th><th>Cost (C)</th></tr><tr><td>1</td><td>6</td></tr><tr><td>3</td><td>18</td></tr><tr><td></td><td></td></tr><tr><td>7</td><td>42</td></tr><tr><td>9</td><td>54</td></tr></table> <p>Students use tables to compare ratios. Another bookstore offers paperback books at the prices below. Which bookstore has the best buy? Explain your answer.</p> <table><tr><th>Number of Books (n)</th><th>Cost (C)</th></tr><tr><td></td><td></td></tr><tr><td>4</td><td>20</td></tr><tr><td></td><td></td></tr><tr><td>8</td><td>40</td></tr></table> <p>To help understand the multiplicative relationship between the number of books and cost, students write equations to express the cost of any number of books. Writing equations is foundational for work in 7th grade. For example, the equation for the first table would be $C = 6n$, while the equation for the second bookstore is $C = 5n$.</p> <p>The numbers in the table can be expressed as ordered pairs (number of books, cost) and plotted on a coordinate plane.</p> <p>Students are able to plot ratios as ordered pairs as seen in the graph below.</p> | Number of Books (n) | Cost (C) | 1 | 6 | 3 | 18 | | | 7 | 42 | 9 | 54 | Number of Books (n) | Cost (C) | | | 4 | 20 | | | 8 | 40 |
|--|---|--|---------------------|----------|---|---|---|----|--|--|---|----|---|----|---------------------|----------|--|--|---|----|--|--|---|----|
| Number of Books (n) | Cost (C) | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 6 | | | | | | | | | | | | | | | | | | | | | | | |
| 3 | 18 | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | |
| 7 | 42 | | | | | | | | | | | | | | | | | | | | | | | |
| 9 | 54 | | | | | | | | | | | | | | | | | | | | | | | |
| Number of Books (n) | Cost (C) | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | |
| 4 | 20 | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | |
| 8 | 40 | | | | | | | | | | | | | | | | | | | | | | | |

Books Unlimited



Example 2:

Ratios can also be used in problem solving by thinking about the total amount for each ratio unit. The ratio of cups of orange juice concentrate to cups of water in punch is 1: 3. If James made 32 cups of punch, how many cups of orange did he need?

Solution: Students recognize that the total ratio would produce 4 cups of punch. To get 32 cups, the ratio would need to be duplicated 8 times, resulting in 8 cups of orange juice concentrate.

Example 3:

Using the information in the table, find the number of yards in 24 feet.

| | | | | | |
|-------|---|---|---|----|----|
| Feet | 3 | 6 | 9 | 15 | 24 |
| Yards | 1 | 2 | 3 | 5 | ? |

Solution:

There are several strategies that students could use to determine the solution to this problem:

- Add quantities from the table to total 24 feet (9 feet and 15 feet); therefore the number of yards in 24 feet must be 8 yards (3 yards and 5 yards).
- Use multiplication to find 24 feet: 1) $3 \text{ feet} \times 8 = 24 \text{ feet}$; therefore $1 \text{ yard} \times 8 = 8 \text{ yards}$, or 2) $6 \text{ feet} \times 4 = 24 \text{ feet}$; therefore $2 \text{ yards} \times 4 = 8 \text{ yards}$.

Example 4:

Compare the number of black circles to white circles. If the ratio remains the same, how many black circles will there be if there are 60 white circles?



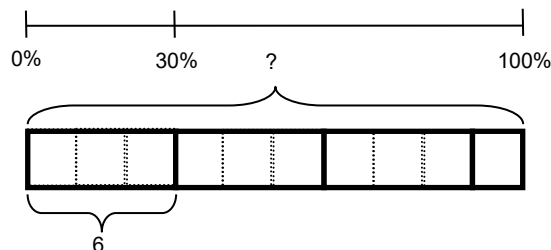
| | | | | | |
|-------|---|----|----|----|----|
| Black | 4 | 40 | 20 | 60 | ? |
| White | 3 | 30 | 15 | 45 | 60 |

Solution:

There are several strategies that students could use to determine the solution to this problem

- Add quantities from the table to total 60 white circles ($15 + 45$). Use the corresponding numbers to determine the number of black circles ($20 + 60$) to get 80 black circles.
- Use multiplication to find 60 white circles (one possibility 30×2). Use the corresponding numbers and operations to determine the number of black circles (40×2) to get 80 black circles.

| <p>b. Solve unit rate problems including those involving unit pricing and constant speed. <i>For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?</i></p> | <p>6.MP.1. Make sense of problems and persevere in solving them.</p> <p>6.MP.2. Reason abstractly and quantitatively.</p> <p>6.MP.4. Model with mathematics</p> <p>6.MP.5. Use appropriate tools strategically.</p> <p>6.MP.7. Look for and make use of structure.</p> | <p>Students recognize the use of ratios, unit rate and multiplication in solving problems, which could allow for the use of fractions and decimals.</p> <p>Example 1: In trail mix, the ratio of cups of peanuts to cups of chocolate candies is 3 to 2. How many cups of chocolate candies would be needed for 9 cups of peanuts?</p> <table><tr><th>Peanuts</th><th>Chocolate</th></tr><tr><td>3</td><td>2</td></tr><tr><td></td><td></td></tr></table> <p>Solution: One possible solution is for students to find the number of cups of chocolate candies for 1 cup of peanuts by dividing both sides of the table by 3, giving $\frac{2}{3}$ cup of chocolate for each cup of peanuts. To find the amount of chocolate needed for 9 cups of peanuts, students multiply the unit rate by nine ($9 \cdot \frac{2}{3}$), giving 6 cups of chocolate.</p> <p>Example 2: If steak costs \$2.25 per pound, how much does 0.8 pounds of steak cost? Explain how you determined your answer.</p> <p>Solution: The unit rate is \$2.25 per pound so multiply \$2.25 x 0.8 to get \$1.80 per 0.8 lbs. of steak.</p> | Peanuts | Chocolate | 3 | 2 | | |
|--|--|---|---------|-----------|---|---|--|--|
| Peanuts | Chocolate | | | | | | | |
| 3 | 2 | | | | | | | |
| | | | | | | | | |
| <p>c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems</p> | <p>6.MP.1. Make sense of problems and persevere in solving them.</p> <p>6.MP.2. Reason abstractly and quantitatively.</p> | <p>This is the students’ first introduction to percentages. Percentages are a rate per 100. Models, such as percent bars or 10 x 10 grids should be used for modeling percentages.</p> <p>Students use ratios to identify percentages.</p> <p>Example 1: What percent is 12 out of 25?</p> | | | | | | |

| involving finding the whole, given a part and the percent. | 6.MP.4. Model with mathematics | <p><u>Solution:</u></p> <p>One possible solution method is to set up a ratio table:</p> <p>Multiply 25 by 4 to get 100. Multiplying 12 by 4 will give 48, meaning that 12 out of 25 is equivalent to 48 out of 100 or 48%.</p> <p>Students use percentages to find the part when given the percent, by recognizing that the whole is being divided into 100 parts and then taking a part of them (the percent).</p> <p><u>Example 2:</u></p> <p>What is 40% of 30?</p> <p><u>Solution:</u></p> <p>There are several methods to solve this problem. One possible solution using rates is to use a 10 x 10 grid to represent the whole amount (or 30). If the 30 is divided into 100 parts, the rate for one block is 0.3. Forty percent would be 40 of the blocks, or 40 x 0.3, which equals 12.</p> <p>Students also determine the whole amount, given a part and the percent.</p> <p><u>Example 3:</u></p> <p>If 30% of the students in Mrs. Rutherford’s class like chocolate ice cream, then how many students are in Mrs. Rutherford’s class if 6 like chocolate ice cream</p> <div></div> <p><u>Solution:</u> 20 students</p> | <table><tr><th>Part</th><th>Whole</th></tr><tr><td>12</td><td>25</td></tr><tr><td>?</td><td>100</td></tr></table> | Part | Whole | 12 | 25 | ? | 100 |
|--|--------------------------------|---|---|------|-------|----|----|---|-----|
| | Part | | Whole | | | | | | |
| | 12 | | 25 | | | | | | |
| | ? | | 100 | | | | | | |
| 6.MP.5. Use appropriate tools strategically. | | | | | | | | | |
| 6.MP.7. Look for and make use of structure. | | | | | | | | | |

| | | | | | | | | | | | | | | |
|--|--|---|---------|-------|-------|-------|-------|-------|----------|--------|--------|------|------|---|
| | | <p>Example 4: A credit card company charges 17% interest fee on any charges not paid at the end of the month. Make a ratio table to show how much the interest would be for several amounts. If the bill totals \$450 for this month, how much interest would you have to be paid on the balance?</p> <p>Solution:</p> <table><tr><td>Charges</td><td>\$1</td><td>\$50</td><td>\$100</td><td>\$200</td><td>\$450</td></tr><tr><td>Interest</td><td>\$0.17</td><td>\$8.50</td><td>\$17</td><td>\$34</td><td>?</td></tr></table> <p>One possible solution is to multiply 1 by 450 to get 450 and then multiply 0.17 by 450 to get \$76.50</p> | Charges | \$1 | \$50 | \$100 | \$200 | \$450 | Interest | \$0.17 | \$8.50 | \$17 | \$34 | ? |
| Charges | \$1 | \$50 | \$100 | \$200 | \$450 | | | | | | | | | |
| Interest | \$0.17 | \$8.50 | \$17 | \$34 | ? | | | | | | | | | |
| <p>d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.</p> | <p>6.MP.1. Make sense of problems and persevere in solving them.</p> <p>6.MP.2. Reason abstractly and quantitatively.</p> <p>6.MP.4. Model with mathematics</p> <p>6.MP.5. Use appropriate tools strategically.</p> <p>6.MP.7. Look for and make use of structure.</p> | <p>A ratio can be used to compare measures of two different types, such as inches per foot, milliliters per liter and centimeters per inch. Students recognize that a conversion factor is a fraction equal to 1 since the numerator and denominator describe the same quantity. For example, $\frac{12 \text{ inches}}{1 \text{ foot}}$ is a conversion factor.</p> <p>Since the ratio is equivalent to 1, the identity property of multiplication allows an amount to be multiplied by the ratio. Also, the value of the ratio can also be expressed as $\frac{1 \text{ foot}}{12 \text{ inches}}$ allowing for the conversion ratios to be expressed in a format so that units will “cancel”.</p> <p>Students use ratios as conversion factors and the identity property for multiplication to convert ratio units.</p> <p>Example 1: How many centimeters are in 7 feet, given that 1 inch ≈ 2.54 cm.</p> <p>Solution:</p> <p>7 feet x $\frac{12 \text{ inches}}{1 \text{ foot}}$ x $\frac{2.54 \text{ cm}}{1 \text{ inch}}$ = 213.36 cm.</p> <p>Note: Conversion factors will be given. Conversions can occur both between and across the metric and English systems. Estimates are not expected.</p> | | | | | | | | | | | | |

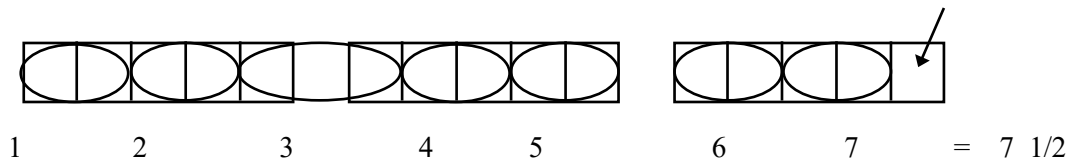
| The Number System | | 6.NS |
|--|--|--|
| College and Career Readiness Cluster | | |
| Apply and extend previous understandings of multiplication and division to divide fractions by fractions. | | |
| Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: reciprocal, multiplicative inverses, visual fraction model, greatest common factor, least common multiple, prime factorization, absolute value | | |
| Enduring Understandings: Number sense is developed through experience and can help determine effective and efficient strategies to solve problems/situations. Essential Questions: How do I determine the best numerical representation for a given problem/situation? What are standard procedures for estimating and finding quotients of fractions and mixed numbers? How do common factors and multiples help me compute fluently with multi-digit numbers? How can I express a number using its prime factorization? What makes a number a rational number? How does the use of a number line and a coordinate plane help in understanding rational numbers and their absolute value? How does understanding integers help me to write, interpret, and explain statements in real-world problems/situations? | | |
| College and Career Readiness Standards <i>Students are expected to:</i> | Mathematical Practices | Unpacking Explanations and Examples <i>What does this standard mean that a student will know and be able to do?</i> |
| 6.NS.A.1 Interpret and compute quotients of fractions, and solve word problems involving division of | <i>6.MP.1.</i> Make sense of problems and persevere in solving them. | In 5 th grade students divided whole numbers by unit fractions and divided unit fractions by whole numbers. Students continue to develop this concept by using visual models and equations to divide whole numbers by fractions and fractions by fractions to solve word problems. Students develop an understanding of the relationship between multiplication and division. |

fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for $(2/3) \div (3/4)$ and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that $(2/3) \div (3/4) = 8/9$ because $3/4$ of $8/9$ is $2/3$. (In general, $(a/b) \div (c/d) = ad/bc$.) How much chocolate will each person get if 3 people share $1/2$ lb of chocolate equally? How many $3/4$ -cup servings are in $2/3$ of a cup of yogurt? How wide is a rectangular strip of land with length

- 6.MP.2. Reason abstractly and quantitatively.
- 6.MP.3. Construct viable arguments and critique the reasoning of others.
- 6.MP.4. Model with mathematics.
- 6.MP.7. Look for and make use of structure.
- 6.MP.8. Look for and express regularity in repeated reasoning

Example 1:

Students understand that a division problem such as $3 \div \frac{2}{5}$ is asking, “how many $\frac{2}{5}$ are in 3?” One possible visual model would begin with three whole bars and divide each into fifths. There are 7 groups of two-fifths in the three wholes. However, one-fifth remains. Since one-fifth is half of a two-fifths group, there is a remainder of $\frac{1}{2}$. Therefore, $3 \div \frac{2}{5} = 7\frac{1}{2}$, meaning there are $7\frac{1}{2}$ groups of two-fifths. Students interpret the solution, explaining how division by fifths can result in an answer with halves. This section represents one-half of two-fifths.



Students also write contextual problems for fraction division problems. For example, the problem, $\frac{2}{3} \div \frac{1}{6}$ can be illustrated with the following word problem:

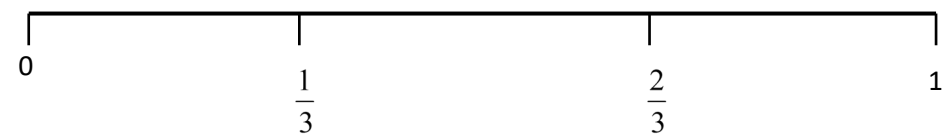
Example 2:

Susan has $\frac{2}{3}$ of an hour left to make cards. It takes her about $\frac{1}{6}$ of an hour to make each card.

About how many can she make?

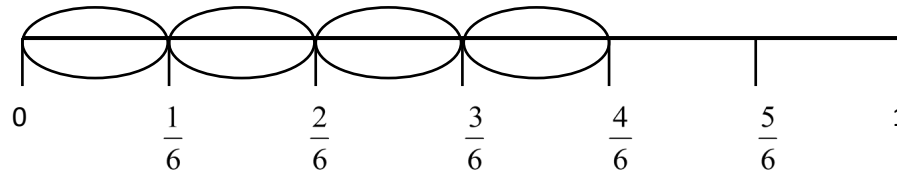
This problem can be modeled using a number line.

a. Start with a number line divided into thirds.



$\frac{3}{4}$ mi and area $\frac{1}{2}$ square mi?

b. The problem wants to know how many sixths are in two-thirds. Divide each third in half to create sixths.



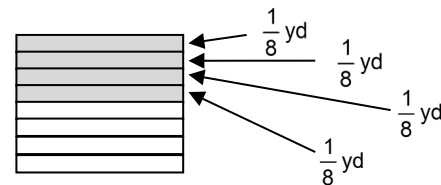
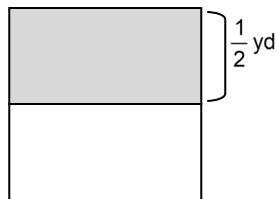
Each circled part represents $\frac{1}{6}$. There are four sixths in two-thirds; therefore, Susan can make 4 cards.

Example 3:

Michael has $\frac{1}{2}$ of a yard of fabric to make book covers. Each book cover is made from $\frac{1}{8}$ of a yard of fabric. How many book covers can Michael make?

Solution:

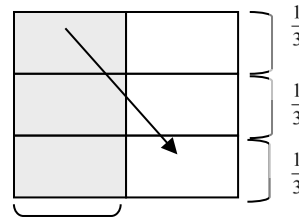
Michael can make 4 book covers.



Example 4:

Represent $\frac{1}{2} \div \frac{2}{3}$ in a problem context and draw a model to show your solution.

Context: A recipe requires $\frac{2}{3}$ of a cup of yogurt. Rachel has $\frac{1}{2}$ of a cup of yogurt from a snack pack. How much of the recipe can Rachel make?



Explanation of Model:

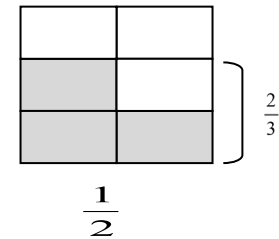
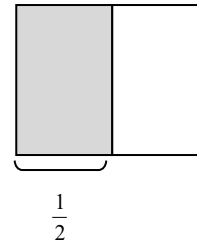
The first model shows $\frac{1}{2}$ cup. The shaded squares in all three models show the $\frac{1}{2}$ cup.

The second model shows $\frac{1}{2}$ cup and also shows $\frac{1}{3}$ cups horizontally.

The third model shows $\frac{1}{2}$ cup moved to fit in only the area shown by $\frac{2}{3}$ of the model.

$\frac{2}{3}$ is the new referent unit (whole).

3 out of the 4 squares in the $\frac{2}{3}$ portion are shaded. A $\frac{1}{2}$ cup is only $\frac{3}{4}$ of a $\frac{2}{3}$ cup portion, so only $\frac{3}{4}$ of the recipe can be made.



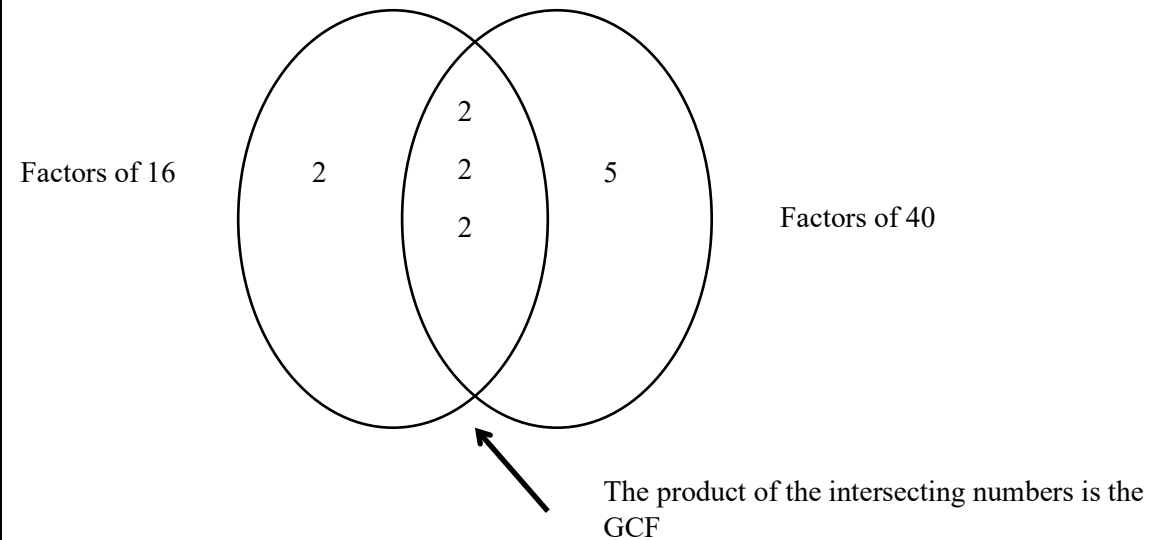
| The Number System | | 6.NS |
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| College and Career Readiness Cluster | | |
| Compute fluently with multi-digit numbers and find common factors and multiples. | | |
| Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: multi-digit, Distributive Property | | |
| College and Career Readiness Standards <i>Students are expected to:</i> | Mathematical Practices | Unpacking Explanations and Examples <i>What does this standard mean that a student will know and be able to do?</i> |
| 6.NS.B.2 Fluently divide multi-digit numbers using the standard algorithm . | <i>6.MP.2.</i> Reason abstractly and quantitatively. <i>6.MP.7.</i> Look for and make use of structure. <i>6.MP.8.</i> Look for and express regularity in repeated reasoning. | <p>In the elementary grades, students were introduced to division through concrete models and various strategies to develop an understanding of this mathematical operation (limited to 4-digit numbers divided by 2-digit numbers). In 6th grade, students become fluent in the use of the standard division algorithm, continuing to use their understanding of place value to describe what they are doing. Place value has been a major emphasis in the elementary standards. This standard is the end of this progression to address students’ understanding of place value.</p> <p>Students are expected to fluently and accurately divide multi-digit whole numbers. Divisors can be any number of digits at this grade level.</p> <p>As students divide they should continue to use their understanding of place value to describe what they are doing. When using the standard algorithm, students’ language should reference place value. For example, when dividing 32 into 8456, as they write a 2 in the quotient they should say, “there are 200 thirty-twos in 8456,” and could write 6400 beneath the 8456 rather than only writing 64.</p> |
| | | <table><tr><td><div><div>2</div><div>32</div><div>8456</div></div></td><td>There are 200 thirty twos in 8456.</td></tr></table> |
| <div><div>2</div><div>32</div><div>8456</div></div> | There are 200 thirty twos in 8456. | |

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| | | $\begin{array}{r} 2 \\ 32 \overline{)8456} \\ \underline{-6400} \\ 2056 \end{array}$ | <p>200 times 32 is 6400.</p> <p>8456 minus 6400 is 2056.</p> | |
| | | $\begin{array}{r} 26 \\ 32 \overline{)8456} \\ \underline{-6400} \\ 2056 \end{array}$ | There are 60 thirty twos in 2056. | |
| | | $\begin{array}{r} 264 \\ 32 \overline{)8456} \\ \underline{-6400} \\ 2056 \\ \underline{-1920} \\ 136 \\ \underline{-128} \end{array}$ | <p>There are 4 thirty twos in 136.</p> <p>4 times 32 is equal to 128.</p> | |
| | | $\begin{array}{r} 264 \\ 32 \overline{)8456} \\ \underline{-6400} \\ 2056 \\ \underline{-1920} \\ 136 \\ \underline{-128} \\ 8 \end{array}$ <p>The remainder is 8. There is not a full thirty two in 8; there is only part of a thirty two in 8.</p> <p>This can also be written as $\frac{8}{32}$ or $\frac{1}{4}$. There is $\frac{1}{4}$ of a thirty two in 8.</p> <p>$8456 = 264 * 32 + 8$</p> | | |

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| <p>6.NS.B.3 Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.</p> | <p>6.MP.2. Reason abstractly and quantitatively.</p> <p>6.MP.7. Look for and make use of structure.</p> <p>6.MP.8. Look for and express regularity in repeated reasoning.</p> | <p>Procedural fluency is defined by the College and Career Readiness as “skill in carrying out procedures flexibly, accurately, efficiently and appropriately”. In 4th and 5th grades, students added and subtracted decimals. Multiplication and division of decimals were introduced in 5th grade (decimals to the hundredth place). At the elementary level, these operations were based on concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. In 6th grade, students become fluent in the use of the standard algorithms of each of these operations. The use of estimation strategies supports student understanding of decimal operations.</p> <p>Example 1: First estimate the sum of 12.3 and 9.75.</p> <p>Solution: An estimate of the sum would be 12 + 10 or 22. Student could also state if their estimate is high or low. Answers of 230.5 or 2.305 indicate that students are not considering place value when adding.</p> |
| <p>6.NS.B.4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common</p> | <p>6.MP.7. Look for and make use of structure.</p> | <p>In elementary school, students identified primes, composites and factor pairs (4.OA.4). In 6th grade students will find the greatest common factor of two whole numbers less than or equal to 100. For example, the greatest common factor of 40 and 16 can be found by</p> <ol style="list-style-type: none"> 1) listing the factors of 40 (1, 2, 4, 5, 8, 10, 20, 40) and 16 (1, 2, 4, 8, 16), then taking the greatest common factor (8). Eight (8) is also the largest number such that the other factors are relatively prime (two numbers with no common factors other than one). For example, 8 would be multiplied by 5 to get 40; 8 would be multiplied by 2 to get 16. Since the 5 and 2 are relatively prime, then 8 is the greatest common factor. If students think 4 is the greatest, then show that 4 would be multiplied by 10 to get 40, while 16 would be 4 times 4. Since the 10 and 4 are not relatively prime (have 2 in common), the 4 cannot be the greatest common factor. 2) listing the prime factors of 40 ($2 \cdot 2 \cdot 2 \cdot 5$) and 16 ($2 \cdot 2 \cdot 2 \cdot 2$) and then multiplying the common factors ($2 \cdot 2 \cdot 2 = 8$). |

factor as a multiple of a sum of two whole numbers with no common factor. For example, express $36 + 8$ as $4(9 + 2)$.

Students also understand that the greatest common factor of two prime numbers is 1.



Example 1:

What is the greatest common factor (GCF) of 18 and 24?

Solution:

$2 * 3^2 = 18$ and $2^3 * 3 = 24$. Students should be able to explain that both 18 and 24 will have at least one factor of 2 and at least one factor of 3 in common, making $2 * 3$ or 6 the GCF.

Given various pairs of addends using whole numbers from 1-100, students should be able to identify if the two numbers have a common factor. If they do, they identify the common factor and use the distributive property to rewrite the expression. They prove that they are correct by simplifying both expressions.

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| | | <p><u>Example 2:</u> Use the greatest common factor and the distributive property to find the sum of 36 and 8. $36 + 8 = 4(9) + 4(2)$ $44 = 4(9 + 2)$ $44 = 4(11)$ $44 = 44✓$</p> <p><u>Example 3:</u> Ms. Spain and Mr. France have donated a total of 90 hot dogs and 72 bags of chips for the class picnic. Each student will receive the same amount of refreshments. All refreshments must be used.</p> <ol style="list-style-type: none"> What is the greatest number of students that can attend the picnic? How many bags of chips will each student receive? How many hotdogs will each student receive? <p><i>Solution:</i></p> <ol style="list-style-type: none"> Eighteen (18) is the greatest number of students that can attend the picnic (GCF). Each student would receive 4 bags of chips. Each student would receive 5 hot dogs. <p>Students find the least common multiple of two whole numbers less than or equal to twelve. For example, the least common multiple of 6 and 8 can be found by</p> <ol style="list-style-type: none"> listing the multiplies of 6 (6, 12, 18, 24, 30, ...) and 8 (8, 26, 24, 32, 40...), then taking the least in common from the list (24); or using the prime factorization. <p>Step 1: find the prime factors of 6 and 8.</p> $6 = 2 \cdot 3$ $8 = 2 \cdot 2 \cdot 2$ <p>Step 2: Find the common factors between 6 and 8. In this example, the common factor is 2.</p> <p>Step 3: Multiply the common factors and any extra factors: $2 \cdot 2 \cdot 2 \cdot 3$ or 24 (one of the twos is in common; the other twos and the three are the extra factors.)</p> |
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Example 4:

The elementary school lunch menu repeats every 20 days; the middle school lunch menu repeats every 15 days. Both schools are serving pizza today. In how many days will both schools serve pizza again?

Solution:

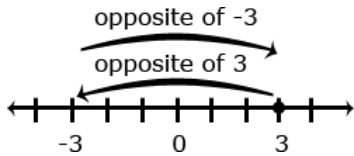
The solution to this problem will be the least common multiple (LCM) of 15 and 20. Students should be able to explain that the least common multiple is the smallest number that is a multiple of 15 and a multiple of 20.

One way to find the least common multiple is to find the prime factorization of each number:

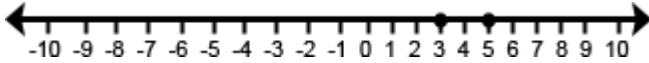
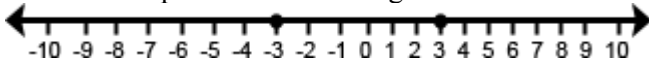
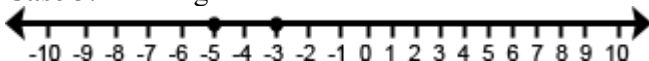
$2^2 * 5 = 20$ and $3 * 5 = 15$. To be a multiple of 20, a number must have 2 factors of 2 and one factor of 5: $(2 * 2 * 5)$.

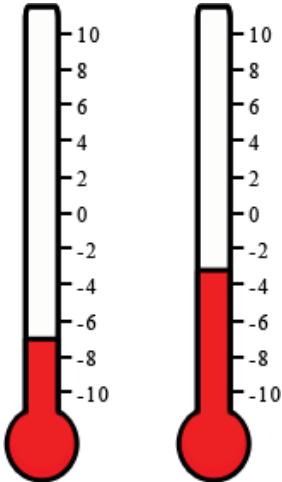
To be a multiple of 15, a number must have factors of 3 and 5. The least common multiple of 20 and 15 must have 2 factors of 2, one factor of 3 and one factor of 5: $(2 * 2 * 3 * 5)$ or 60.

| The Number System | | 6.NS |
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| College and Career Readiness Cluster | | |
| Apply and extend previous understandings of numbers to the system of rational numbers. | | |
| Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: rational numbers, opposites, absolute value, greater than, >, less than, <, greater than or equal to, \geq, less than or equal to, \leq, origin, quadrants, coordinate plane, ordered pairs, x-axis, y-axis, coordinates, positive numbers, negative numbers, integer | | |
| College and Career Readiness Standards <i>Students are expected to:</i> | Mathematical Practices | Unpacking Explanations and Examples <i>What does this standard mean that a student will know and be able to do?</i> |
| 6.NS.C.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real- | <p>6.MP.1. Make sense of problems and persevere in solving them.</p> <p>6.MP.2. Reason abstractly and quantitatively.</p> <p>6.MP.4. Model with mathematics.</p> | <p>Students use rational numbers (fractions, decimals, and integers) to represent real-world contexts and understand the meaning of 0 in each situation.</p> <p>Example 1:</p> <ol style="list-style-type: none"> Use an integer to represent 25 feet below sea level. Use an integer to represent 25 feet above sea level. What would 0 (zero) represent in the scenario above? <p>Solution:</p> <ol style="list-style-type: none"> -25 +25 0 would represent sea level. |

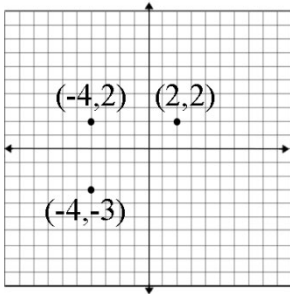
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| world contexts, explaining the meaning of 0 in each situation. | | |
| <p>6.NS.C.6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.</p> <p>a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that</p> | <p><i>6.MP.2.</i> Reason abstractly and quantitatively.</p> <p><i>6.MP.4.</i> Model with mathematics.</p> | <p>In elementary school, students worked with positive fractions, decimals and whole numbers on the number line and in quadrant 1 of the coordinate plane.</p> <p>In 6th grade, students extend the number line to represent all rational numbers and recognize that number lines may be either horizontal or vertical (i.e. thermometer) which facilitates the movement from number lines to coordinate grids. Students recognize that a number and its opposite are equidistance from zero (reflections about the zero). The opposite sign (–) shifts the number to the opposite side of 0.</p> <p>For example, – 4 could be read as “the opposite of 4” which would be negative 4. In the example, – (–6.4) would be read as “the opposite of the opposite of 6.4” which would be 6.4. Zero is its own opposite.</p> <div style="text-align: center;">  </div> <p><u>Example 1:</u></p> $\frac{1}{2}$ <p><i>Solution:</i></p> $\frac{1}{2}$ |

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| <p>0 is its own opposite.</p> <p>b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.</p> <p>c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.</p> | | <p>Example 2:</p> <p>Students place the following numbers on a number line: -4.5, 2, 3.2, $-3\frac{3}{5}$, 0.2, -2, $\frac{11}{2}$.</p> <p>Based on number line placement, numbers can be placed in order.</p> <p>Solution:</p> <p>The numbers in order from least to greatest are:</p> $-4.5, -3\frac{3}{5}, -2, 0.2, 2, 3.2, \frac{11}{2}$ <p>Students place each of these numbers on a number line to justify this order.</p> |
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| <p>6.NS.C.7 Understand ordering and absolute value of rational numbers.</p> <p>a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. <i>For example, interpret $-3 > -7$ as a statement that -3 is located to the right of -7 on a number line oriented from left to right.</i></p> <p>b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. <i>For example, write $-3^{\circ}\text{C} > -7^{\circ}\text{C}$ to express</i></p> | <p><i>6.MP.2. Reason abstractly and quantitatively.</i></p> <p><i>6.MP.4. Model with mathematics.</i></p> | <p>Common models to represent and compare integers include number line models, temperature models and the profit-loss model. On a number line model, the number is represented by an arrow drawn from zero to the location of the number on the number line; the absolute value is the length of this arrow. The number line can also be viewed as a thermometer where each point of on the number line is a specific temperature. In the profit-loss model, a positive number corresponds to profit and the negative number corresponds to a loss. Each of these models is useful for examining values but can also be used in later grades when students begin to perform operations on integers.</p> <p>In working with number line models, students internalize the order of the numbers; larger numbers on the right or top of the number line and smaller numbers to the left or bottom of the number line. They use the order to correctly locate integers and other rational numbers on the number line. By placing two numbers on the same number line, they are able to write inequalities and make statements about the relationships between the numbers.</p> <p>Case 1: Two positive numbers</p>  <p style="text-align: center;">$5 > 3$ 5 is greater than 3</p> <p>Case 2: One positive and one negative number</p>  <p style="text-align: center;">$3 > -3$ positive 3 is greater than negative 3 negative 3 is less than positive 3</p> <p>Case 3: Two negative numbers</p>  <p style="text-align: center;">$-3 > -5$ negative 3 is greater than negative 5 negative 5 is less than negative 3</p> |
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| <p><i>the fact that -3°C is warmer than -7°C.</i></p> <p>c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. <i>For example, for an account balance of -30 dollars, write $-30 = 30$ to describe the size of the debt in dollars.</i></p> <p>d. Distinguish comparisons of absolute value from statements about order. <i>For example, recognize that an account balance less than -30</i></p> | | <p>Comparative statements generate informal experience with operations and lay the foundation for formal work with operations on integers in Grade 7.</p> <p>Example: One of the thermometers shows -3°C and the other shows -7°C. Which thermometer shows which temperature? Which is the colder temperature? How much colder? Write an inequality to show the relationship between the temperatures and explain how the model shows this relationship.</p>  <p>Students recognize the distance from zero as the absolute value or magnitude of a rational number. Students need multiple experiences to understand the relationships between numbers, absolute value, and statements about order.</p> <p>Example: The Great Barrier Reef is the world's largest reef system and is located off the coast of Australia. It reaches from the surface of the ocean to a depth of 150 meters. Students could represent this value as less than 150 meters or a depth no greater than 150 meters below sea level.</p> |
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| <i>dollars represents a debt greater than 30 dollars.</i> | | |
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| <p>6.NS.C.8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.</p> | <p>6.MP.1. Make sense of problems and persevere in solving them.</p> <p>6.MP.2. Reason abstractly and quantitatively.</p> <p>6.MP.4. Model with mathematics.</p> <p>6.MP.5. Use appropriate tools strategically.</p> <p>6.MP.7. Look for and make use of structure.</p> | <p>Example:</p> <p>If the points on the coordinate plane below are the three vertices of a rectangle, what are the coordinates of the fourth vertex? How do you know? What are the length and width of the rectangle?</p>  <p>To determine the distance along the x-axis between the point $(-4, 2)$ and $(2, 2)$ a student must recognize that -4 is -4 or 4 units to the left of 0 and 2 is 2 or 2 units to the right of zero, so the two points are total of 6 units apart along the x-axis. Students should represent this on the coordinate grid and numerically with an absolute value expression, $-4 + 2$.</p> |
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| Expressions and Equations | | 6.EE |
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| College and Career Readiness Cluster | | |
| Apply and extend previous understandings of arithmetic to algebraic expressions. | | |
| Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: exponents, base, numerical expressions, algebraic expressions, evaluate, sum, term, product, factor, quantity, quotient, coefficient, constant, like terms, equivalent expressions, variables | | |
| Enduring Understandings: Doing things in the correct sequential order ensures the validity of your result. | | |
| Essential Questions: What are numerical and algebraic expressions and how can they be written and evaluated? How is an expression evaluated that includes parentheses, exponents, and multiple operations? How does understanding the question being asked help me to write an expression to solve for a logical answer? What information would you use to support, evaluate, and express the relationship between dependent and independent variables? How do you determine which of the arithmetic properties (commutative, associative, or distributive) to apply when evaluating an expression? How do we determine if two expressions are equivalent to each other? How do we determine if expressions are equalities or inequalities? | | |
| College and Career Readiness Standards <i>Students are expected to:</i> | Mathematical Practices | Unpacking Explanations and Examples <i>What does this standard mean that a student will know and be able to do?</i> |
| 6.EE.A.1. Write and evaluate numerical expressions involving whole-number exponents. | 6.MP.2. Reason abstractly and quantitatively. | Examples: Write the following as a numerical expressions using exponential notation. <ul style="list-style-type: none"> ○ The area of a square with a side length of 8 m (Solution: $8^2 m^2$) ○ The volume of a cube with a side length of 5 ft.: (Solution: $5^3 ft^3$) |

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| | | <ul style="list-style-type: none"> Yu-Lee has a pair of mice. The mice each have 2 babies. The babies grow up and have two babies of their own: (Solution: 2^3 mice) <p><u>Evaluate:</u></p> <ul style="list-style-type: none"> 4^3 (Solution: 64) $5 + 2^4 \bullet 6$ (Solution: 101) $7^2 - 24 \div 3 + 26$ (Solution: 67) |
| <p>6.EE.A.2. Write, read, and evaluate expressions in which letters stand for numbers.</p> <p>a. Write expressions that record operations with numbers and with letters standing for numbers. <i>For example, express the calculation “Subtract y from 5” as $5 - y$.</i></p> | <p><i>6.MP.1. Make sense of problems and persevere in solving them.</i></p> <p><i>6.MP.2. Reason abstractly and quantitatively.</i></p> <p><i>6.MP.3. Construct viable arguments and critique the reasoning of others.</i></p> <p><i>6.MP.4. Model with mathematics.</i></p> <p><i>6.MP.6. Attend to precision</i></p> | <p>It is important for students to read algebraic expressions in a manner that reinforces that the variable represents a number.</p> <ul style="list-style-type: none"> $r + 21$ as “some number plus 21 as well as “r plus 21” $n \bullet 6$ as “some number times 6 as well as “n times 6” $\frac{s}{6}$ and $s \div 6$ as “as some number divided by 6” as well as “s divided by 6” <p>Students should identify the parts of an algebraic expression including variables, coefficients, constants, and the names of operations (sum, difference, product, and quotient).</p> <p>Development of this common language helps students to understand the structure of expressions and explain their process for simplifying expressions.</p> |
| <p>b. Identify parts of an expression using mathematical</p> | | <p>Terms are the parts of a sum. When the term is an explicit number, it is called a constant. When the term is a product of a number and a variable, the number is called the coefficient of the variable.</p> |

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| <p>terms (sum, term, product, factor, quotient, and coefficient); view one or more parts of an expression as a single entity. <i>For example, describe the expression $2(8+7)$ as a product of two factors; view $(8+7)$ as both a single entity and a sum of two terms.</i></p> <p>c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic</p> | | <p>Variables are letters that represent numbers. There are various possibilities for the numbers they can represent; students can substitute these possible numbers for the letters in the expression for various different purposes.</p> <p>Consider the following expression:</p> $x^2 + 5y + 3x + 6$ <p>The variables are x and y.</p> <p>There are 4 terms, x^2, $5y$, $3x$, and 6.</p> <p>There are 3 variable terms, x^2, $5y$, $3x$. They have coefficients of 1, 5, and 3 respectively.</p> <p>The coefficient of x^2 is 1, since $x^2 = 1x^2$. The term $5y$ represent 5 y's or $5 * y$.</p> <p>There is one constant term, 6.</p> <p>The expression shows a sum of all four terms.</p> <p>Examples:</p> <ul style="list-style-type: none"> • 7 more than 3 times a number (Solution: $3x + 7$) • 3 times the sum of a number and 5 (Solution: $3(x + 5)$) • 7 less than the product of 2 and a number (Solution: $2x - 7$) • Twice the difference between a number and 5 (Solution: $2(z - 5)$) • Evaluate $5(n + 3) - 7n$, when $n = \frac{1}{2}$. • The expression $c + 0.07c$ can be used to find the total cost of an item with 7% sales tax, where c is the pre-tax cost of the item. Use the expression to find the total cost of an item that cost \$25. <p>The perimeter of a parallelogram is found using the formula $p = 2l + 2w$. What is the perimeter of a rectangular picture frame with dimensions of 8.5 inches by 11 inches.</p> |
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| <p>operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). <i>For example, use the formulas $V=s^3$ and $A=6s^2$ to find the volume and surface area of a cube with sides of length $s=1/2$.</i></p> | | |
| <p>6.EE.A.3. Apply the properties of operations to generate equivalent expressions. <i>For example, apply the distributive property to the expression</i></p> | <p>6.MP.2. Reason abstractly and quantitatively.</p> <p>6.MP.3. Construct viable arguments and critique the reasoning of others.</p> <p>6.MP.4. Model with mathematics.</p> | <p>Students use their understanding of multiplication to interpret $3(2 + x)$. <i>For example, 3 groups of $(2 + x)$. They use a model to represent x, and make an array to show the meaning of $3(2 + x)$. They can explain why it makes sense that $3(2 + x)$ is equal to $6 + 3x$.</i></p> <p>An array with 3 columns and $x + 2$ in each column:</p>  |

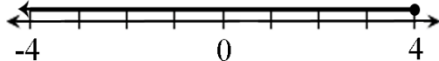
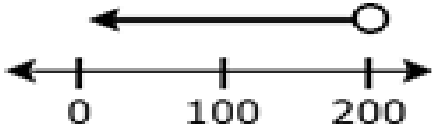
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| <p><i>3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.</i></p> | <p>6.MP.6. Attend to precision.</p> <p>6.MP.7. Look for and make use of structure.</p> | <p>Students interpret y as referring to one y. Thus, they can reason that one y plus one y plus one y must be $3y$. They also use the distributive property, the multiplicative identity property of 1, and the commutative property for multiplication to prove that</p> <p>$y + y + y = 3y$:</p> <p>$y + y + y = y \times 1 + y \times 1 + y \times 1 = y \times (1 + 1 + 1) = y \times 3 = 3y$</p> |
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| <p>6.EE.A.4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). <i>For example, the expressions $y + y + y$ and $3y$ are equivalent because they name the same number regardless of which number y stands for.</i></p> | <p>6.MP.2. Reason abstractly and quantitatively.</p> <p>6.MP.3. Construct viable arguments and critique the reasoning of others.</p> <p>6.MP.4. Model with mathematics.</p> <p>6.MP.6. Attend to precision.</p> <p>6.MP.7. Look for and make use of structure.</p> | <p>Students connect their experiences with finding and identifying equivalent forms of whole numbers and can write expressions in various forms. Students generate equivalent expressions using the associative, commutative, and distributive properties. They can prove that the expressions are equivalent by simplifying each expression into the same form.</p> <p>Example: Are the expressions equivalent? How do you know?</p> $4m + 8 \qquad 4(m+2) \qquad 3m + 8 + m \qquad 2 + 2m + m + 6 + m$ <p>Solution:</p> <table border="1"> <thead> <tr> <th>Expression</th><th>Simplifying the Expression</th><th>Explanation</th></tr> </thead> <tbody> <tr> <td>$4m + 8$</td><td>$4m + 8$</td><td>Already in simplest form</td></tr> <tr> <td>$4(m+2)$</td><td>$4(m+2)$ $4m + 8$</td><td>Distributive property</td></tr> <tr> <td>$3m + 8 + m$</td><td>$3m + 8 + m$ $3m + m + 8$ $(3m + m) + 8$ $4m + 8$</td><td>Combined like terms</td></tr> <tr> <td>$2 + 2m + m + 6 + m$</td><td>$2 + 2m + m + 6 + m$ $2 + 6 + 2m + m + m$ $(2 + 6) + (2m + m + m)$ $8 + 4m$ $4m + 8$</td><td>Combined like terms</td></tr> </tbody> </table> | Expression | Simplifying the Expression | Explanation | $4m + 8$ | $4m + 8$ | Already in simplest form | $4(m+2)$ | $4(m+2)$ $4m + 8$ | Distributive property | $3m + 8 + m$ | $3m + 8 + m$ $3m + m + 8$ $(3m + m) + 8$ $4m + 8$ | Combined like terms | $2 + 2m + m + 6 + m$ | $2 + 2m + m + 6 + m$ $2 + 6 + 2m + m + m$ $(2 + 6) + (2m + m + m)$ $8 + 4m$ $4m + 8$ | Combined like terms |
|--|--|---|------------|----------------------------|-------------|----------|----------|--------------------------|----------|----------------------|-----------------------|--------------|--|---------------------|----------------------|--|---------------------|
| Expression | Simplifying the Expression | Explanation | | | | | | | | | | | | | | | |
| $4m + 8$ | $4m + 8$ | Already in simplest form | | | | | | | | | | | | | | | |
| $4(m+2)$ | $4(m+2)$ $4m + 8$ | Distributive property | | | | | | | | | | | | | | | |
| $3m + 8 + m$ | $3m + 8 + m$ $3m + m + 8$ $(3m + m) + 8$ $4m + 8$ | Combined like terms | | | | | | | | | | | | | | | |
| $2 + 2m + m + 6 + m$ | $2 + 2m + m + 6 + m$ $2 + 6 + 2m + m + m$ $(2 + 6) + (2m + m + m)$ $8 + 4m$ $4m + 8$ | Combined like terms | | | | | | | | | | | | | | | |

| Expressions and Equations | | 6.EE |
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| College and Career Readiness Cluster | | |
| Reason about and solve one variable equations and inequalities. | | |
| Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: exponents, base, numerical expressions, algebraic expressions, evaluate, sum, term, product, factor, quantity, quotient, coefficient, constant, like terms, equivalent expressions, variables | | |
| College and Career Readiness Standards <i>Students are expected to:</i> | Mathematical Practices | Unpacking Explanations and Examples <i>What does this standard mean that a student will know and be able to do?</i> |
| 6.EE.B.5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an | <p>6.MP.1. Make sense of problems and persevere in solving them.</p> <p>6.MP.2. Reason abstractly and quantitatively.</p> <p>6.MP.4. Model with mathematics.</p> <p>6.MP.7. Look for and make use of structure.</p> | <p>Beginning experiences in solving equations should require students to understand the meaning of the equation as well as the question being asked. Solving equations using reasoning and prior knowledge should be required of students to allow them to develop effective strategies such as using reasoning, fact families, and inverse operations. Students may use balance models in representing and solving equations and inequalities.</p> <p>Consider the following situation: Joey had 26 papers in his desk. His teacher gave him some more and now he has 100. How many papers did his teacher give him?</p> <p>This situation can be represented by the equation $26 + n = 100$ where n is the number of papers the teacher gives to Joey. This equation can be stated as “some number was added to 26 and the result was 100.” Students ask themselves “What number was added to 26 to get 100?” to help them determine the value of the variable that makes the equation true. Students could use several different strategies to find a solution to the problem.</p> <ul style="list-style-type: none"> Reasoning: $26 + 70$ is 96. $96 + 4$ is 100, so the number added to 26 to get 100 is 74. Use knowledge of fact families to write related equations: $n + 26 = 100$, $100 - n = 26$, $100 - 26 = n$. Select the equation that helps you find n easily. Use knowledge of inverse operations: Since subtraction “undoes” addition then subtract 26 from 100 to get the numerical value of n |

| | | | | | | |
|---|--|--|-----|--|----|-----|
| equation or inequality true. | | <ul style="list-style-type: none">Scale Model: There are 26 blocks on the left side of the scale and 100 blocks on the right side of the scale. All the blocks are the same size. 74 blocks need to be added to the left side of the scale to make the scale balance.Bar Model: Each bar represents one of the values. Students use this visual representation to demonstrate that 26 and the unknown value together make 100. <div><table><tr><td colspan="2">100</td></tr><tr><td>26</td><td>n</td></tr></table></div> <p>Examples:</p> <p>The equation $0.44s = 11$ where s represents the number of stamps in a booklet. The booklet of stamps costs 11 dollars and each stamp costs 44 cents. How many stamps are in the booklet? Explain the strategies you used to determine your answer. Show that your solution is correct using substitution.</p> <p>Twelve is less than 3 times another number can be shown by the inequality $12 < 3n$. What numbers could possibly make this a true statement?</p> | 100 | | 26 | n |
| 100 | | | | | | |
| 26 | n | | | | | |
| <p>6.EE.B.6.</p> <p>Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the</p> | <p>6.MP.2. Reason abstractly and quantitatively.</p> <p>6.MP.4. Model with mathematics.</p> <p>6.MP.7. Look for and make use of structure.</p> | <p>Connecting writing expressions with story problems and/or drawing pictures will give students a context for this work. It is important for students to read algebraic expressions in a manner that reinforces that the variable represents a number.</p> <p>Examples:</p> <ul style="list-style-type: none">Maria has three more than twice as many crayons as Elizabeth. Write an algebraic expression to represent the number of crayons that Maria has. <p>Solution:</p> <p>$2c + 3$ where c represents the number of crayons that Elizabeth has.</p> <ul style="list-style-type: none">An amusement park charges \$28 to enter and \$0.35 per ticket. Write an algebraic expression to represent the total amount spent. <p>Solution:</p> <p>$28 + 0.35t$ where t represents the number of tickets purchased</p> | | | | |

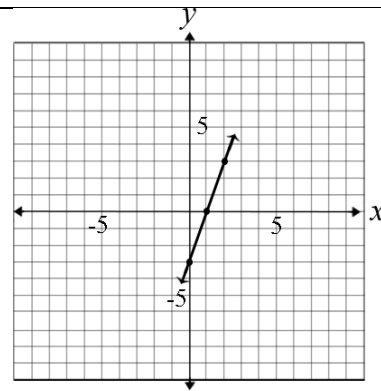
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| purpose at hand, any number in a specified set. | | <ul style="list-style-type: none"> Andrew has a summer job doing yard work. He is paid \$15 per hour and a \$20 bonus when he completes the yard. He was paid \$85 for completing one yard. Write an equation to represent the amount of money he earned. <p>Solution: $15h + 20 = 85$ where h is the number of hours worked</p> <ul style="list-style-type: none"> Describe a problem situation that can be solved using the equation $2c + 3 = 15$; where c represents the cost of an item. <p>Solution: Solutions may vary.</p> | | | | | | |
| <p>6.EE.B.7. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers</p> | <p>6.MP.1. Make sense of problems and persevere in solving them.</p> <p>6.MP.2. Reason abstractly and quantitatively.</p> <p>6.MP.3. Construct viable arguments and critique the reasoning of others.</p> <p>6.MP.4. Model with mathematics.</p> <p>6.MP.7. Look for and make use of structure.</p> | <p>Students create and solve equations that are based on real world situations. It may be beneficial for students to draw pictures that illustrate the equation in problem situations. Solving equations using reasoning and prior knowledge should be required of students to allow them to develop effective strategies.</p> <p>Examples:</p> <p>Meagan spent \$56.58 on three pairs of jeans. If each pair of jeans costs the same amount, write an algebraic equation that represents this situation and solve to determine how much one pair of jeans cost.</p> <div data-bbox="1192 857 1627 951" data-label="Table"> <table> <tr> <td colspan="3">\$56.58</td></tr> <tr> <td>J</td><td>J</td><td>J</td></tr> </table> </div> <p>Sample Solution:</p> <p>Students might say: “I created the bar model to show the cost of the three pairs of jeans. Each bar labeled J is the same size because each pair of jeans costs the same amount of money. The bar model represents the equation $3J = \\$56.58$. To solve the problem, I need to divide the total cost of 56.58 between the three pairs of jeans. I know that it will be more than \$10 each because 10×3 is only 30 but less than \$20 each because 20×3 is 60. If I start with \$15 each, I am up to \$45. I have \$11.58 left. I then give each pair of jeans \$3. That’s \$9 more dollars. I only have \$2.58 left. I continue until all the money is divided. I ended up giving each pair of jeans another \$0.86. Each pair of jeans costs \$18.86 ($15+3+0.86$). I double check that the jeans cost \$18.86 each because 18.86×3 is \$56.58.”</p> <p>Julio gets paid \$20 for babysitting. He spends \$1.99 on a package of trading cards and \$6.50 on lunch. Write and solve an equation to show how much money Julio has left.</p> | \$56.58 | | | J | J | J |
| \$56.58 | | | | | | | | |
| J | J | J | | | | | | |

| | | | | | | | | |
|--|---|---|----|--|--|------|------|---------------------|
| | | <p>Solution:</p> $20 = 1.99 + 6.50 + x, x = \11.51 <table border="1" data-bbox="1165 305 1852 399"> <tr> <td colspan="3">20</td></tr> <tr> <td>1.99</td><td>6.50</td><td>money left over (m)</td></tr> </table> | 20 | | | 1.99 | 6.50 | money left over (m) |
| 20 | | | | | | | | |
| 1.99 | 6.50 | money left over (m) | | | | | | |
| <p>6.EE.B.8. Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.</p> | <p>6.MP.1. Make sense of problems and persevere in solving them.</p> <p>6.MP.2. Reason abstractly and quantitatively.</p> <p>6.MP.3. Construct viable arguments and critique the reasoning of others.</p> <p>6.MP.4. Model with mathematics.</p> <p>6.MP.7. Look for and make use of structure.</p> | <p>Examples:</p> <p>Graph $x \leq 4$.</p>  <p>Jonas spent more than \$50 at an amusement park. Write an inequality to represent the amount of money Jonas spent. What are some possible amounts of money Jonas could have spent? Represent the situation on a number line</p> <p>Less than \$200.00 was spent by the Flores family on groceries last month. Write an inequality to represent this amount and graph this inequality on a number line.</p> <p>Solution:</p> $200 > x$  | | | | | | |

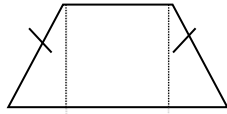
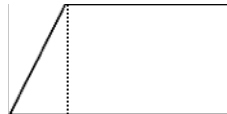
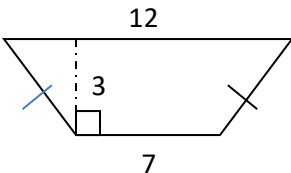
| Expressions and Equations | | 6.EE | | | | | | | | | | |
|---|---|--|-----|----|---|---|---|-----|-----|---|-----|----|
| College and Career Readiness Cluster | | | | | | | | | | | | |
| Represent and analyze quantitative relationships between dependent and independent variables. | | | | | | | | | | | | |
| Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: exponents, base, numerical expressions, algebraic expressions, evaluate, sum, term, product, factor, quantity, quotient, coefficient, constant, like terms, equivalent expressions, variable | | | | | | | | | | | | |
| College and Career Readiness Standards: <i>Students are expected to:</i> | Mathematical Practices | Unpacking Explanations and Examples <i>What does this standard mean that a student will know and be able to do?</i> | | | | | | | | | | |
| 6.EE.C.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity , thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze | <p>6.MP.1. Make sense of problems and persevere in solving them.</p> <p>6.MP.2. Reason abstractly and quantitatively.</p> <p>6.MP.3. Construct viable arguments and critique the reasoning of others.</p> <p>6.MP.4. Model with mathematics.</p> <p>6.MP.7. Look for and make use of structure.</p> | <p>Students can use many forms to represent relationships between quantities. Multiple representations include describing the relationship using language, a table, an equation, or a graph. Translating between multiple representations helps students understand that each form represents the same relationship and provides a different perspective on the function.</p> <p>Examples:</p> <ul style="list-style-type: none">What is the relationship between the two variables? Write an expression that illustrates the relationship. <table><tr><td>X</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>Y</td><td>2.5</td><td>5</td><td>7.5</td><td>10</td></tr></table> <ul style="list-style-type: none">Use the graph below to describe the change in y as x increases by 1. | X | 1 | 2 | 3 | 4 | Y | 2.5 | 5 | 7.5 | 10 |
| X | 1 | 2 | 3 | 4 | | | | | | | | |
| Y | 2.5 | 5 | 7.5 | 10 | | | | | | | | |

the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.*

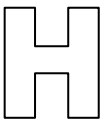
6.MP.8. Look for and express regularity in repeated reasoning

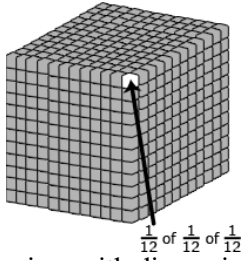
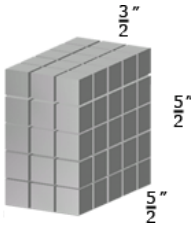


| Geometry | | 6.G |
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| College and Career Readiness Cluster | | |
| Solve real-world and mathematical problems involving area, surface area, and volume. | | |
| Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: area, surface area, volume, decomposing, edges, dimensions, net, vertices, face, base, height, trapezoid, isosceles, right triangle, quadrilateral, rectangles, squares, parallelograms, trapezoids, rhombi, kites, right rectangular prism, diagonal | | |
| Enduring Understandings: Everyday objects have a variety of attributes and can be measured, combined, decomposed, or constructed in many ways. | | |
| Essential Questions: What is the role of geometry in solving real-world problems involving area, surface area, and volume? How are composition and decomposition of polygons used to determine area, surface area, and volume of complex figures? | | |
| College and Career Readiness Standards <i>Students are expected to:</i> | Mathematical Practices | Unpacking Explanations and Examples <i>What does this standard mean that a student will know and be able to do?</i> |
| 6.G.A.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of | 6.MP.1. Make sense of problems and persevere in solving them. 6.MP.2. Reason abstractly and quantitatively. 6.MP.3. Construct viable arguments and critique the reasoning of others. 6.MP.4. Model with mathematics. | <p>Students continue to understand that area is the number of squares needed to cover a plane figure. Students should know the formulas for rectangles and triangles. “Knowing the formula” does not mean memorization of the formula. To “know” means to have an understanding of <i>why</i> the formula works and how the formula relates to the measure (area) and the figure. This understanding should be for <i>all</i> students.</p> <p>Finding the area of triangles is introduced in relationship to the area of rectangles – a rectangle can be decomposed into two congruent triangles. Therefore, the area of the triangle is $\frac{1}{2}$ the area of the rectangle. The area of a rectangle can be found by multiplying base x height; therefore, the area of the triangle is $\frac{1}{2}bh$ or $(b \times h)/2$.</p> <p>Students decompose shapes into rectangles and triangles to determine the area. For example, a trapezoid can be decomposed into triangles and rectangles (see figures below). Using the trapezoid’s dimensions, the area of the individual triangle(s) and rectangle can be found and then added together. Special quadrilaterals include rectangles, squares, parallelograms, trapezoids, rhombi, and kites.</p> |

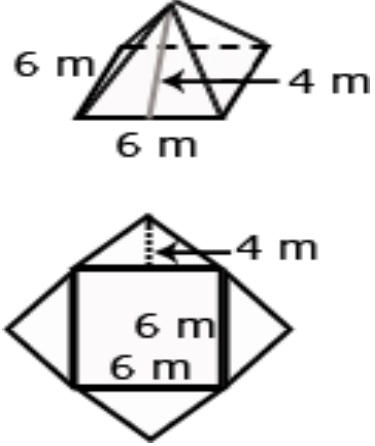
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| <p>solving real-world and mathematical problems.</p> | <p>6.MP.5. Use appropriate tools strategically.</p> <p>6.MP.6. Attend to precision.</p> <p>6.MP.7. Look for and make use of structure.</p> <p>6.MP.8. Look for and express regularity in repeated reasoning.</p> | <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">  <p>Isosceles trapezoid</p> </div> <div style="text-align: center;">  <p>Right trapezoid</p> </div> </div> <p>Note: Students recognize the marks on the isosceles trapezoid indicating the two sides have equal measure. This is the student's first exposure to the term diagonal.</p> <p>Example 1: Find the area of a right triangle with a base length of three units, a height of four units, and a hypotenuse of 5.</p> <p>Solution: Students understand that the hypotenuse is the longest side of a right triangle. The base and height would form the 90° angle and would be used to find the area using:</p> $A = \frac{1}{2}bh$ $A = \frac{1}{2}(3 \text{ units})(4 \text{ units})$ $A = \frac{1}{2}12 \text{ units}^2$ $A = 6 \text{ units}^2$ <p>Example 2: Find the area of the trapezoid shown below using the formulas for rectangles and triangles.</p> <div style="text-align: center;">  </div> <p>Solution: The trapezoid could be decomposed into a rectangle with a length of 7 units and a height of 3 units. The area of the rectangle would be 21 units².</p> |
|--|--|--|

| | | |
|--|--|--|
| | | <p>The triangles on each side would have the same area. The height of the triangles is 3 units. After taking away the middle rectangle's base length, there is a total of 5 units remaining for both of the side triangles. The base length of each triangle is half of 5. The base of each triangle is 2.5 units. The area of one triangle would be $\frac{1}{2} (2.5 \text{ units})(3 \text{ units})$ or 3.75 units^2.</p> <p>Using this information, the area of the trapezoid would be:</p> $\begin{array}{r} 21 \text{ units}^2 \\ 3.75 \text{ units}^2 \\ \hline +3.75 \text{ units}^2 \\ \hline 28.5 \text{ units}^2 \end{array}$ <p><u>Example 3:</u> A rectangle measures 3 inches by 4 inches. If the lengths of each side double, what is the effect on the area?</p> <p><u>Solution:</u> The new rectangle would have side lengths of 6 inches and 8 inches. The area of the original rectangle was 12 inches^2. The area of the new rectangle is 48 inches^2. The area increased 4 times (quadrupled). Students may also create a drawing to show this visually.</p> <p><u>Example 4:</u> The lengths of the sides of a bulletin board are 4 feet by 3 feet. How many index cards measuring 4 inches by 6 inches would be needed to cover the board?</p> <p><u>Solution:</u> Change the dimensions of the bulletin board to inches (4 feet = 48 inches; 3 feet = 36 inches). The area of the board would be 48 inches x 36 inches or 1728 inches^2. The area of one index card is 24 inches^2. Divide 1728 inches^2 by 24 inches^2 to get the number of index cards. 72 index cards would be needed.</p> <p><u>Example 5:</u> The sixth grade class at Hernandez School is building a giant wooden H for their school. The "H" will be 10 feet tall and 10 feet wide and the thickness of the block letter will be 2.5 feet.</p> <ol style="list-style-type: none"> How large will the H be if measured in square feet? |
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| | | <p>2. The truck that will be used to bring the wood from the lumberyard to the school can only hold a piece of wood that is 60 inches by 60 inches. What pieces of wood (how many and which dimensions) will need to be bought to complete the project?</p> <div style="text-align: center;">  </div> <p><i>Solution:</i></p> <p>1. One solution is to recognize that, if filled in, the area would be 10 feet tall and 10 feet wide or 100 ft². The size of one piece removed is 5 feet by 3.75 feet or 18.75 ft². There are two of these pieces. The area of the “H” would be 100 ft² – 18.75 ft² – 18.75 ft², which is 62.5ft². A second solution would be to decompose the “H” into two tall rectangles measuring 10 ft by 2.5 ft and one smaller rectangle measuring 2.5 ft by 5 ft. The area of each tall rectangle would be 25 ft² and the area of the smaller rectangle would be 12.5 ft². Therefore the area of the “H” would be 25 ft² + 25 ft² + 12.5 ft² or 62.5ft².</p> <p>2. Sixty inches is equal to 5 feet, so the dimensions of each piece of wood are 5ft by 5ft. Cut two pieces of wood in half to create four pieces 5 ft. by 2.5 ft. These pieces will make the two taller rectangles. A third piece would be cut to measure 5ft. by 2.5 ft. to create the middle piece.</p> <p><u>Example 6:</u> A border that is 2 ft wide surrounds a rectangular flowerbed 3 ft by 4 ft. What is the area of the border?</p> <p><i>Solution:</i> Two sides 4 ft. by 2 ft. would be 8ft² x 2 or 16 ft² Two sides 3 ft. by 2 ft. would be 6ft² x 2 or 12 ft² Four corners measuring 2 ft. by 2 ft. would be 4ft² x 4 or 16 ft²</p> <p>The total area of the border would be 16 ft² + 12 ft² + 16 ft² or 44ft²</p> |
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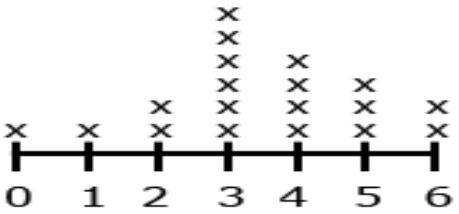
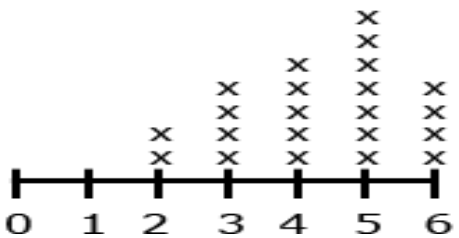
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| <p>6.G.A.2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.</p> | <p><i>6.MP.1.</i> Make sense of problems and persevere in solving them.</p> <p><i>6.MP.2.</i> Reason abstractly and quantitatively.</p> <p><i>6.MP.3.</i> Construct viable arguments and critique the reasoning of others.</p> <p><i>6.MP.4.</i> Model with mathematics.</p> <p><i>6.MP.5.</i> Use appropriate tools strategically.</p> <p><i>6.MP.6.</i> Attend to precision.</p> <p><i>6.MP.7.</i> Look for and make use of structure.</p> <p><i>6.MP.8.</i> Look for and express regularity in repeated reasoning.</p> | <p>Students need multiple opportunities to measure volume by filling rectangular prisms with blocks and looking at the relationship between the total volume and the area of the base. Through these experiences, students derive the volume formula (volume equals the area of the base times the height).</p> <p>In addition to filling boxes, students can draw diagrams to represent fractional side lengths, connecting with multiplication of fractions. This process is similar to composing and decomposing two dimensional shapes.</p> <p>Examples:</p> <ul style="list-style-type: none"> The model shows a cubic foot filled with cubic inches. The cubic inches can also be labeled as a fractional cubic unit with dimensions of $\frac{1}{12}$ ft³.  <ul style="list-style-type: none"> The models show a rectangular prism with dimensions $\frac{3}{2}$ inches, $\frac{5}{2}$ inches, and $\frac{5}{2}$ inches. Each of the cubic units in the model is $\frac{1}{8}$ in³. Students work with the model to illustrate $\frac{3}{2} \times \frac{5}{2} \times \frac{5}{2} = (3 \times 5 \times 5) \times \frac{1}{8}$. Students reason that a small cube has volume $\frac{1}{8}$ because 8 of them fit in a unit cube.  |
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| <p>6.G.A.3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.</p> | <p>6.MP.1. Make sense of problems and persevere in solving them.</p> <p>6.MP.2. Reason abstractly and quantitatively.</p> <p>6.MP.4. Model with mathematics.</p> <p>6.MP.5. Use appropriate tools strategically.</p> <p>6.MP.7. Look for and make use of structure.</p> | <p>Example:</p> <p>On a map, the library is located at $(-2, 2)$, the city hall building is located at $(0, 2)$, and the high school is located at $(0, 0)$. Represent the locations as points on a coordinate grid with a unit of 1 mile.</p> <ul style="list-style-type: none"> What is the distance from the library to the city hall building? The distance from the city hall building to the high school? How do you know? What shape is formed by connecting the three locations? The city council is planning to place a city park in this area. How large is the area of the planned park? |
| <p>6.G.A.4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques</p> | <p>6.MP.1. Make sense of problems and persevere in solving them.</p> <p>6.MP.2. Reason abstractly and quantitatively.</p> <p>6.MP.3. Construct viable arguments and critique the reasoning of others.</p> | <p>Students construct models and nets of three dimensional figures, describing them by the number of edges, vertices, and faces. Solids include rectangular and triangular prisms. Students are expected to use the net to calculate the surface area.</p> <p>Students also describe the types of faces needed to create a three-dimensional figure. Students make and test conjectures by determining what is needed to create a specific three-dimensional figure.</p> <p>Examples:</p> <ul style="list-style-type: none"> Describe the shapes of the faces needed to construct a rectangular pyramid. Cut out the shapes and create a model. Did your faces work? Why or why not? Create the net for a given prism or pyramid, and then use the net to calculate the surface area. |

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| <p>in the context of solving real-world and mathematical problems.</p> | <p>6.MP.4. Model with mathematics.</p> <p>6.MP.5. Use appropriate tools strategically.</p> <p>6.MP.6. Attend to precision.</p> <p>6.MP.7. Look for and make use of structure.</p> <p>6.MP.8. Look for and express regularity in repeated reasoning.</p> |  <p>The top diagram is a perspective view of a square pyramid. The base is a square with side length 6 m. A dashed line indicates the height of the pyramid. A slant height of 4 m is shown on one of the triangular faces.</p> <p>The bottom diagram is a net of the pyramid. It consists of a central square base with side length 6 m. Four triangular faces are attached to the sides of the base. Each triangle has a base of 6 m and a height of 4 m.</p> |
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| Statistics and Probability | | 6.SP |
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| College and Career Readiness Cluster | | |
| Develop understanding of statistical variability. | | |
| Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: statistics, data, variability, distribution, dot plot, histograms, box plots, median, mean | | |
| <p>Enduring Understanding:</p> <p>Being able to organize, read, and interpret data are critical elements needed to facilitate decision making, predicting and inferring in real-world situations.</p> <p>Essential Questions:</p> <p>What are the connections between data analysis, probability and discrete math that help solve real world problems?</p> <p>In what ways does the use of data analysis, probability, and discrete math help in identifying math processes?</p> <p>How are predictions and informed decisions influenced by probability?</p> <p>What is the role of distribution analysis in interpretation of data?</p> <p>Why and how can center measures (median & mean) and deviation from center be used to describe data sets?</p> | | |
| College and Career Readiness Standards <i>Students are expected to:</i> | Mathematical Practices | Unpacking Explanations and Examples <i>What does this standard mean that a student will know and be able to do?</i> |
| 6.SP.A.1 Recognize a statistical question as one that anticipates variability in the data related to | 6.MP.1. Make sense of problems and persevere in solving them. 6.MP.3. Construct viable | Statistics are numerical data relating to a group of individuals; statistics is also the name for the science of collecting, analyzing and interpreting such data. A statistical question anticipates an answer that varies from one individual to the next and is written to account for the variability in the data. Data are the numbers produced in response to a statistical question. Data are frequently collected from surveys or other sources (i.e. documents). |

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| <p>the question and accounts for it in the answers. <i>For example, “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages.</i></p> | <p>arguments and critique the reasoning of others.</p> <p>6.MP.6. Attend to precision.</p> | <p>Students differentiate between statistical questions and those that are not. A statistical question is one that collects information that addresses differences in a population. The question is framed so that the responses will allow for the differences. For example, the question, “How tall am I?” is not a statistical question because there is only one response; however, the question, “How tall are the students in my class?” is a statistical question since the responses anticipates variability by providing a variety of possible anticipated responses that have numerical answers. Questions can result in a narrow or wide range of numerical values.</p> <p>Students might want to know about the fitness of the students at their school. Specifically, they want to know about the exercise habits of the students. So rather than asking "Do you exercise?" they should ask about the amount of exercise the students at their school get per week. A statistical question for this study could be: “How many hours per week on average do students at Jefferson Middle School exercise?”</p> |
| <p>6.SP.A.2. Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.</p> | <p>6.MP.2. Reason abstractly and quantitatively.</p> <p>6.MP.4. Model with mathematics.</p> <p>6.MP.5. Use appropriate tools strategically.</p> <p>6.MP.6. Attend to precision.</p> <p>6.MP.7. Look for and make use of structure.</p> | <p>The two dot plots show the 6-trait writing scores for a group of students on two different traits, organization and ideas. The center, spread and overall shape can be used to compare the data sets. Students consider the context in which the data were collected and identify clusters, peaks, gaps, and symmetry. Showing the two graphs vertically rather than side by side helps students make comparisons. For example, students would be able to see from the display of the two graphs that the ideas scores are generally higher than the organization scores. One observation students might make is that the scores for organization are clustered around a score of 3 whereas the scores for ideas are clustered around a score of 5.</p> |

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| | | <p style="text-align: center;">6-Trait Writing Rubric Scores for Organization</p>  <p style="text-align: center;">6-Trait Writing Rubric Scores for Ideas</p>  |
| <p>6.SP.A.3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.</p> | <p>6.MP.2. Reason abstractly and quantitatively.</p> <p>6.MP.4. Model with mathematics.</p> <p>6.MP.5. Use appropriate tools strategically.</p> <p>6.MP.6. Attend to precision.</p> <p>6.MP.7. Look for and make use of structure.</p> | <p>When using measures of center (mean, median, and mode) and range, students are describing a data set in a single number. The range provides a single number that describes how the values vary across the data set. The range can also be expressed by stating the minimum and maximum values.</p> <p>Example:</p> <ul style="list-style-type: none"> • Consider the data shown in the dot plot of the six trait scores for organization for a group of students. <ul style="list-style-type: none"> ○ How many students are represented in the data set? ○ What are the mean, median, and mode of the data set? What do these values mean? How do they compare? ○ What is the range of the data? What does this value mean? |

| | | <p>6-Trait Writing Rubric Scores for Organization</p> <table><tr><th>Score</th><th>Frequency (Number of x's)</th></tr><tr><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td></tr><tr><td>2</td><td>1</td></tr><tr><td>3</td><td>3</td></tr><tr><td>4</td><td>4</td></tr><tr><td>5</td><td>3</td></tr><tr><td>6</td><td>1</td></tr></table> | Score | Frequency (Number of x's) | 0 | 1 | 1 | 1 | 2 | 1 | 3 | 3 | 4 | 4 | 5 | 3 | 6 | 1 |
|-------|---------------------------|---|-------|---------------------------|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Score | Frequency (Number of x's) | | | | | | | | | | | | | | | | | |
| 0 | 1 | | | | | | | | | | | | | | | | | |
| 1 | 1 | | | | | | | | | | | | | | | | | |
| 2 | 1 | | | | | | | | | | | | | | | | | |
| 3 | 3 | | | | | | | | | | | | | | | | | |
| 4 | 4 | | | | | | | | | | | | | | | | | |
| 5 | 3 | | | | | | | | | | | | | | | | | |
| 6 | 1 | | | | | | | | | | | | | | | | | |

| Statistics and Probability | | 6.SP |
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| Common Core Cluster | | |
| Summarize and describe distributions. | | |
| Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: statistics, data, variability, distribution, dot plot, histograms, box plots, median, mean | | |
| Common Core Standards <i>Students are expected to:</i> | Mathematical Practices | Unpacking Explanations and Examples <i>What does this standard mean that a student will know and be able to do?</i> |
| 6.SP.B.4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots. | <p>6.MP.2. Reason abstractly and quantitatively.</p> <p>6.MP.4. Model with mathematics.</p> <p>6.MP.5. Use appropriate tools strategically.</p> <p>6.MP.6. Attend to precision.</p> <p>6.MP.7. Look for and make use of structure.</p> | <p>In order to display numerical data in dot plots, histograms or box plots, students need to make decisions and perform calculations. Students are expected to display data graphically in a format appropriate for that data set as well as reading data from graphs generated by others students or contained in reference materials.</p> <p>Dot plots are simple plots on a number line where each dot represents a piece of data in the data set. Dot plots are suitable for small to moderate size data sets and are useful for highlighting the distribution of the data including clusters, gaps, and outliers.</p> <p>In most real data sets, there is a large amount of data and many numbers will be unique. A graph (such as a dot plot) that shows how many ones, how many twos, etc. would not be meaningful; however, a histogram can be used. Students organize the data into convenient ranges and use these intervals to generate a frequency table and histogram. Note that changing the size of the range changes the appearance of the graph and the conclusions you may draw from it.</p> <p>Box plots are another useful way to display data and are plotted horizontally or vertically on a number line. Box plots are generated from the five number summaries of a data set consisting of the minimum, maximum, median, and two quartile values. Students can readily compare two sets of data if they are displayed with side by side box plots on the same scale. Box plots display the degree of spread of the data and the skewness of the data.</p> |

Examples:

- Nineteen students completed a writing sample that was scored using the six traits rubric. The scores for the trait of organization were 0, 1, 2, 2, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 6, 6. Create a data display.

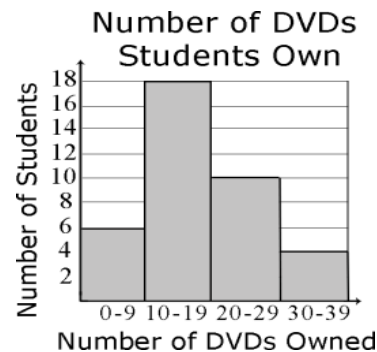
What are some observations that can be made from the data display?



- Grade 6 students were collecting data for a math class project. They decided they would survey the other two grade 6 classes to determine how many DVDs each student owns. A total of 48 students were surveyed. The data are shown in the table below in no specific order. Create a data display. What are some observations that can be made from the data display?

| | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|
| 11 | 21 | 5 | 12 | 10 | 31 | 19 | 13 | 23 | 33 |
| 10 | 11 | 25 | 14 | 34 | 15 | 14 | 29 | 8 | 5 |
| 22 | 26 | 23 | 12 | 27 | 4 | 25 | 15 | 7 | |
| 2 | 19 | 12 | 39 | 17 | 16 | 15 | 28 | 16 | |

A histogram using 5
organize the data is



ranges (0-9, 10-19, ...30-39) to
displayed below.

- Ms. Wheeler asked each student in her class to write their age in months on a sticky note. The 28 students in the class brought their sticky note to the front of the room and posted them in order on the white board. The data set is listed below in order from least to greatest. Create a data display. What are some observations that can be made from the data display?

| | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 130 | 130 | 131 | 131 | 132 | 132 | 132 | 133 | 134 | 136 |
| 137 | 137 | 138 | 139 | 139 | 139 | 140 | 141 | 142 | 142 |
| 142 | 143 | 143 | 144 | 145 | 147 | 149 | 150 | | |

Five number summary

Minimum – 130 months

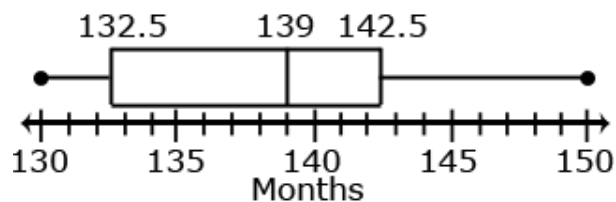
Quartile 1 (Q1) – $(132 + 133) \div 2 = 132.5$ months

Median (Q2) – 139 months

Quartile 3 (Q3) – $(142 + 143) \div 2 = 142.5$ months

Maximum – 150 months

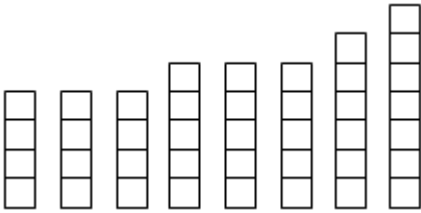
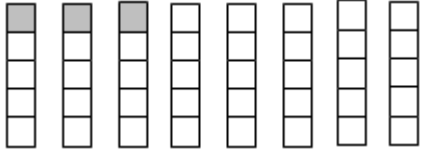
Ages in Months of a Class of 6th Grade Students

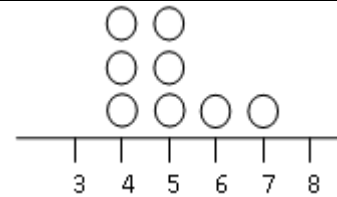


This box plot shows that:

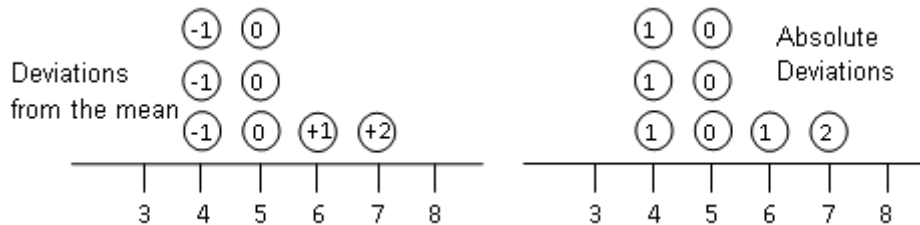
- $\frac{1}{4}$ of the students in the class are from 130 to 132.5 months old
 - $\frac{1}{4}$ of the students in the class are from 142.5 months to 150 months old
 - $\frac{1}{2}$ of the class are from 132.5 to 142.5 months old
- the median class age is 139 months.

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| <p>6.SP.B.5. Summarize numerical data sets in relation to their context, such as by:</p> <p>a. Reporting the number of observations.</p> <p>b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement</p> <p>c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean</p> | <p><i>6.MP.2.</i> Reason abstractly and quantitatively.</p> <p><i>6.MP.3.</i> Construct viable arguments and critique the reasoning of others.</p> <p><i>6.MP.4.</i> Model with mathematics.</p> <p><i>6.MP.5.</i> Use appropriate tools strategically.</p> <p><i>6.MP.6.</i> Attend to precision.</p> <p><i>6.MP.7.</i> Look for and make use of structure.</p> | <p>Students summarize numerical data by providing background information about the attribute being measured, methods and unit of measurement, the context of data collection activities, the number of observations, and summary statistics. Summary statistics include quantitative measures of center, spread, and variability including extreme values (minimum and maximum), mean, median, mode, range, quartiles, interquartile ranges, and mean absolute deviation.</p> <p>The measure of center that a student chooses to describe a data set will depend upon the shape of the data distribution and context of data collection. The mode is the value in the data set that occurs most frequently. The mode is the least frequently used as a measure of center because data sets may not have a mode, may have more than one mode, or the mode may not be descriptive of the data set. The mean is a very common measure of center computed by adding all the numbers in the set and dividing by the number of values. The mean can be affected greatly by a few data points that are very low or very high. In this case, the median or middle value of the data set might be more descriptive. In data sets that are symmetrically distributed, the mean and median will be very close to the same. In data sets that are skewed, the mean and median will be different, with the median frequently providing a better overall description of the data set.</p> <p><u>Understanding the Mean</u></p> <p>The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students develop understanding of what the mean represents by redistributing data sets to be level or fair. The leveling process can be connected to and used to develop understanding of the computation of the mean.</p> <p>For example, students could generate a data set by measuring the number of jumping jacks they can perform in 5 seconds, the length of their feet to the nearest inch, or the number of letters in their names. It is best if the data generated for this activity are 5 to 10 data points which are whole numbers between 1 and 10 that are easy to model with counters or stacking cube</p> <p>Students generate a data set by drawing eight student names at random from the popsicle stick cup. The number of letters in each of the names is used to create the data set. If the names drawn were Carol, Mike, Maria, Luis, Monique, Sierra, John, and Karen there would be 3 names with 4 letters each, 3 names with 5 letters each, 1 name with 6 letters and 1 name with 7 letters.</p> |
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| <p>absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.</p> <p>d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.</p> | <p>This data set could be represented with stacking cubes.</p>  <p>Students can model the mean by “leveling” the stacks or distributing the blocks so the stacks are “fair.” Students are seeking to answer the question “If all of the students had the same number of letters in their name, how many letters would each person have?”</p> <p>One block from the stack of six and two blocks from the stack of 7 can be moved down to the stacks of 4 and then all the stacks have five blocks. If all students had the same number of letters in their name, they would have five letters. The mean number of letters in a name in this data set is 5.</p>  <p>If it was not possible to make the stacks exactly even, students could begin to consider what part of the extra blocks each stack would have.</p> <p><u>Understanding Mean Absolute Deviation</u></p> <p>The use of mean absolute deviation in 6th grade is mainly exploratory. The intent is to build a deeper understanding of variability. Students would understand the mean distance between the pieces of data and the mean of the data set expresses the spread of the data set. Students can see that the larger the mean distance, the greater the variability. Comparisons can be made between different data sets.</p> <p>In the previous data set, the names drawn were Carol, Mike, Maria, Luis, Monique, Sierra, John, and Karen. There were 3 names with 4 letters each, 3 names with 5 letters each, 1 name with 6 letters and 1 name with 7 letters. This data can be represented on a dot plot. The mean of the data set is 5.</p> |
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To find the mean absolute deviation, students examine each of the data points and its difference from the mean. This analysis can be represented on the dot plot itself or in a table. Each of the names with 4 letters has one fewer letter than the mean, each of the names with 5 letters has zero difference in letters as compared to the mean, each of the names with 6 letters has one more letter than the mean, and each of the names with 7 letters has two more letters than the mean. The absolute deviations are the absolute value of each difference.



| Name | Number of letters in a name | Deviation from the Mean | Absolute Deviation from the Mean |
|--------|-----------------------------|-------------------------|----------------------------------|
| John | 4 | -1 | 1 |
| Luis | 4 | -1 | 1 |
| Mike | 4 | -1 | 1 |
| Carol | 5 | 0 | 0 |
| Maria | 5 | 0 | 0 |
| Karen | 5 | 0 | 0 |
| Sierra | 6 | +1 | 1 |

| | | | |
|--------------|----|----|---|
| Monique | 7 | +2 | 2 |
| Total | 40 | 0 | 6 |

The mean of the absolute deviations is found by summing the absolute deviations and dividing by the number of data points. In this case, the mean absolute deviation would be $6 \div 8$ or $\frac{3}{4}$ or 0.75. The mean absolute deviation is a small number, indicating that there is little variability in the data set.

Consider a different data set also containing 8 names. If the names were Sue, Joe, Jim, Amy, Sabrina, Monique, Timothy, and Adelita. Summarize the data set and its variability. How does this compare to the first data set?

The mean of this data set is still 5. $\frac{(3+3+3+3+7+7+7+7)}{8} = \frac{40}{8} = 5$

| Name | Number of letters in a name | Deviation from the Mean | Absolute Deviation from the Mean |
|--------------|-----------------------------|-------------------------|----------------------------------|
| Sue | 3 | -2 | 2 |
| Joe | 3 | -2 | 2 |
| Jim | 3 | -2 | 2 |
| Amy | 3 | -2 | 2 |
| Sabrina | 7 | +2 | 2 |
| Timothy | 7 | +2 | 2 |
| Adelita | 7 | +2 | 2 |
| Monique | 7 | +2 | 2 |
| Total | 40 | 0 | 16 |

The mean deviation of this data set is $16 \div 8$ or 2. Although the mean is the same, there is much more variability in this data set.

Understanding Medians and Quartiles

Students can also summarize and describe the center and variability in data sets using the median and a five number summary consisting of the minimum, quartiles, and maximum as seen in the box plot example in 6.SP.4. The median is the middle number of the data set with half the number below the median and half the numbers above the median. The quartiles partition the data set into four parts by dividing each of the halves of the data set into half again. Quartile 1 (Q1 or the lower quartile) is the middle value of the lower half of the data set and quartile 3 (Q3 or the upper quartile) is the middle value of the upper half of the data set. The median can also be referred to as quartile 2 (Q2). The range of the data is the difference between the minimum and maximum values. The interquartile range of the data is the difference between the lower and upper quartiles ($Q3 - Q1$). The interquartile range is a measure of the dispersion or spread of the data set: a small value indicates values that are clustered near the median whereas a larger value indicates values that are more distributed.

Consider the first data set again. Recall that the names drawn were Carol, Mike, Maria, Luis, Monique, Sierra, John, and Karen. The data set can be represented in a numerical list. To find the median and quartile, the values are placed in order from least to greatest.

5 4 5 4 7 6 4 5 → 4 4 4 5 5 5 6 7

The middle value in the ordered data set is the median. If there are even numbers of values, the median is the mean of the middle two values. In this case, the median would be 5 because 5 is the average of the 4th and 5th values which are both 5.

Students find quartile 1 (Q1) by examining the lower half of the data. Again there are 4 values which is an even number of values. Q1 would be the average of the 2nd and 3rd value in the data set or 4. Students find quartile 3 (Q3) by examining the upper half of the data. Q3 would be the average of the 6th and 7th value in the data set or 5.5. The mean of the data set was 5 and the median is also 5, showing that the values are probably clustered close to the mean. The interquartile range is 1.5 ($5.5 - 4$). The interquartile range is small, showing little variability in the data.

4 4 4 5 5 5 6 7
↑ ↑ ↑
Q1 = 4 Q3 = 5.5
Median = 5