Merrimack School District
Mathematics Curriculum

Algebra Two

Standards for Mathematical Practices

The College and Career ReadinessState Standards for Mathematical Practice are expected to be integrated into every mathematics lesson for all students Grades K-12. Below are a few examples of how these Practices may be integrated into tasks that students complete.

Mathematic Practices	Explanations and Examples
1. Make sense of problems and persevere in solving them.	Mathematically proficient students in Algebra 2 should solve problems by applying their understanding of operations with whole numbers, decimals, and fractions including mixed numbers. They solve problems related to volume and measurement conversions. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?"
2. Reason abstractly and quantitatively.	Mathematically proficient students in Algebra 2 should recognize that a number represents a specific quantity. They connect quantities to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions that record calculations with numbers and represent or round numbers using place value concepts.
3. Construct viable arguments and critique the reasoning of others.	In Algebra 2 mathematical proficient students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations based upon models and properties of operations and rules that generate patterns. They demonstrate and explain the relationship between volume and multiplication. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.
4. Model with mathematics.	Mathematically proficient students in Algebra 2 experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Students should evaluate their results in the context of the situation and whether the results make sense. They also evaluate the utility of models to determine which models are most useful and efficient to solve problems.

5. Use appropriate tools strategically.	Mathematically proficient students consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use unit cubes to fill a rectangular prism and then use a ruler to measure the dimensions. They use graph paper to accurately create graphs and solve problems or make predictions from real world data.
6. Attend to precision.	Mathematically proficient students in Algebra 2 continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to expressions, fractions, geometric figures, and coordinate grids. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the volume of a rectangular prism they record their answers in cubic units.
7. Look for and make use of structure.	In Algebra 2 mathematically proficient students look closely to discover a pattern or structure. For instance, students use properties of operations as strategies to add, subtract, multiply and divide with whole numbers, fractions, and decimals. They examine numerical patterns and relate them to a rule or a graphical representation.
8. Look for and express regularity in repeated reasoning.	Mathematically proficient students use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and their prior work with operations to understand algorithms to fluently multiply multi-digit numbers and perform all operations with decimals to hundredths. Students explore operations with fractions with visual models and begin to formulate generalizations.

Probability and Statistics: Interpreting Categorical and Quantitative Data

College and Career Readiness Cluster

Summarize, represent, and interpret data on two categorical and quantitative variables.

Mathematically proficient students can produce functions to model data sets. The terms students should learn to use with increasing precision with this cluster are: **regression, scatter plot, dependent variable, independent variable,**

Enduring Understandings:

Functions can be used to model the relationships between sets of data.

Essential Questions:

How can we use equations to represent relationships?

How do we determine which algebraic model fits the given data?

Why is it necessary to distinguish between a dependent variable and an independent variable?

College and Career Readiness Standards *Students are expected to:*	Mathematical Practices	**Unpacking Explanations and Examples** *What does this standard mean that a student will know and be able to do?*
S.ID.B.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the	MP.2 Reason abstractly and quantitatively. MP.3 Construct viable arguments and critique the reasoning of others. MP.4 Model with mathematics. MP.5 Use appropriate tools strategically.	**Example**: In an experiment, 300 pennies were shaken in a cup and poured onto a table. Any penny 'heads up' was removed. The remaining pennies were returned to the cup and the process was repeated. The results of the experiment are shown below. # Of rolls 0 1 2 3 4 5 # Pennies 300 164 100 46 20 8 Write the function suggested by the context and determine how well it fits the data. Explore the two allowance options below and complete the tasks that follow. Option 1 – you earn a dollar amount equivalent to the number of the day. (Fro example: on August 1st you earn \$1, on August 20th you earn \$20.) Option 2 – you earn one penny on August 1st and double that amount each day - \$0.01 - \$0.02 - \$0.04 and so on.

data. Use given functions or choose a function suggested by the context. ==Emphasize== <mark style="background-color:cyan">linear, quadratic, and exponential models</mark>. b. Informally ==assess== the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association.	MP.7 Look for and make use of structure. MP.8 Look for and express regularity in repeated reasoning.	**Example:** Determine which model results in the greatest overall income for the month of August. **Example:** Complete a regression model for each option and write the regression equation. **Example:** Calculate the total allowance earned for the month of August.

Probability and Statistics: Interpreting Categorical and Quantitative Data

College and Career Readiness Cluster

Interpret Linear Models

Mathematically proficient students can glean information from linear models.

Enduring Understandings:

Functions can be used to model linear relationships between sets of data.

Essential Questions:

How can we use the linear equations to represent relationships?

What do the slope and the *y*-intercept represent in relation to the data?

Why is it important to confirm the correlation coefficient by analyzing the data?

How do we predict future events based on a set of data?

College and Career Readiness Standards *Students are expected to:*	Mathematical Practices	Unpacking Explanations and Examples *What does this standard mean that a student will know and be able to do?*
S.ID.C.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	MP.2 Reason abstractly and quantitatively. MP.3 Construct viable arguments and critique the reasoning of others. MP.4 Model with mathematics. MP.5 Use appropriate tools strategically. MP.7 Look for and make use of structure.	**Example**: Data was collected of the weight of a male white laboratory rat for the first 25 weeks after its birth. A scatterplot of the rat's weight (in grams) and the time since birth (in weeks) shows a fairly strong, positive linear relationship. The linear regression equation $w = 100 + 40t$ (where w = weight in grams and t = number of weeks since birth) models the data fairly well. a. What is the slope of the linear regression equation? Explain what it means in context. b. What is the y-intercept of the linear regression equation? Explain what it means in context.

	MP.8 Look for and express regularity in repeated reasoning.	
S.ID.C.8 <mark>Compute</mark> (using technology) and interpret the <mark>correlation coefficient</mark> of a linear fit	MP.2 Reason abstractly and quantitatively. MP.3 Construct viable arguments and critique the reasoning of others. MP.4 Model with mathematics. MP.5 Use appropriate tools strategically. MP.7 Look for and make use of structure. MP.8 Look for and express regularity in repeated reasoning.	**Example**: The correlation coefficient of a given data set is 0.97. List three specific things this tells you about the data. **Example**: Given several scatter plots, match the appropriate correlation coefficient to each scatter plot.

Geometry: Expressing Geometric Properties with Equations

College and Career Readiness Cluster

Translate between the geometric description and the equation for a conic section.

Mathematically proficient students can write equations for conic sections.

The terms students should learn to use with increasing precision in this cluster are: **focus, directrix, ellipse, hyperbola, parabola, completing the square, major axis, minor axis, and conic section**

Enduring Understandings:

Geometric figures can be modeled with algebraic equations.

Essential Questions:

How are algebraic expressions used to model geometric firgures?

What does completing the square reveal about a conic section?

College and Career Readiness Standards *Students are expected to:*	Mathematical Practices	Unpacking Explanations and Examples *What does this standard mean that a student will know and be able to do?*
G.GPE.A.1 <mark>Derive</mark> the equation of a <mark>circle</mark> of given center and radius using the <mark>Pythagorean Theorem</mark>; complete the square to find the center and radius of a circle given by an equation.	MP.1 Make sense of problems and persevere in solving them. MP.2 Reason abstractly and quantitatively. MP.4 Model with mathematics. MP.7 Look for and make use of structure.	**Example**: Given the equation for a circle, rewrite into standard form, identify the center and radius, and graph. $$x^2 + y^2 - 8x - 18y + 48 = 0$$ $$(x - 4)^2 + (y - 9)^2 = 49$$ Center (4, 9) Radius 7 **Example**: Determine whether a given point is on, in, or outside a given circle.

G.GPE.A.2 Derive the equation of a parabola given a focus and directrix.	MP.1 Make sense of problems and persevere in solving them. MP.2 Reason abstractly and quantitatively. MP.4 Model with mathematics. MP.7 Look for and make use of structure.	**Example**: Write and graph an equation for a parabola whose focus is at (2, 3) and with a directrix at $y = 1$. **Example**: A parabola has focus (–2, 1) and directrix $y = –3$. Determine whether or not the point (2, 1) is part of the parabola. Justify your answer. **Example**: Given the equation $20(y - 5) = (x + 3)^2$, find the focus, vertex and directrix. **Example**: Determine whether a given point is on the parabola.
G.GPE.A.3 (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.	MP.1 Make sense of problems and persevere in solving them. MP.2 Reason abstractly and quantitatively. MP.4 Model with mathematics. MP.7 Look for and make use of structure.	**Example**: Given the key features of a graph, determine the type of conic section and its equation. **Example**: Determine whether a given point is on, in or outside the ellipse. **Example**: Determine whether a given point is on the hyperbola.

Geometry: Expressing Geometric Properties with Equations
College and Career Readiness Cluster
Use coordinates to prove simple geometric theorems algebraically.
Mathematically proficient students understand the relationship between equations and their graphs
Enduring Understandings: Properties of graphed lines are related to the properties of the equations which generate them.   **Essential Questions:**   How do we use the slopes of two lines to determine the relationship between the two lines?

College and Career Readiness Standards *Students are expected to:*	Mathematical Practices	**Unpacking Explanations and Examples** *What does this standard mean that a student will know and be able to do?*

G.GPE.B.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	MP.1 Make sense of problems and persevere in solving them.
	
	 MP.2 Reason abstractly and quantitatively.
		Example:
		 Suppose a line k in a coordinate plane has slope c/d.
		
		    a. What is the slope of a line parallel to k? Why must this be the case?
		
		    b. What is the slope of a line perpendicular to k? Why does this seem reasonable?
		
		 Example:
		 Two points $A(0,-4)$, $B(2,-1)$ determine line AB.
		
		    a. What is the equation of line AB?
		
		    b. What is the equation of the line perpendicular to line AB passing through the point $(2,-1)$?

Functions: Interpreting Functions		
College and Career Readiness Cluster		
Understand the concept of a function and use function notation		
Mathematically proficient students communicate using precise vocabulary and proper notation.		
Enduring Understandings: Functions are relations which pair each member of the domain with exactly one member of the range. **Essential Questions:** How are patterns of change related to the behavior of functions? How do the constraints of a situation impact domain and range?		
College and Career Readiness Standards *Students are expected to:*	**Mathematical Practices**	**Unpacking Explanations and Examples** *What does this standard mean that a student will know and be able to do?*
F.IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of $f(x)$	MP.1 Make sense of problems and persevere in solving them. MP.2 Reason abstractly and quantitatively. MP.4 Model with mathematics.	**Example**: Determine which of the following tables represent a function and explain why. **Example**: A pack of pencils cost $0.75. If n number of packs are purchased then the total purchase price is represented by the function $t(n) = 0.75n$. a. Explain why t is a function. b. What is a reasonable domain and range for the function t?

corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.		
F.IF.A.2 <mark>Use function notation</mark>, evaluate functions for inputs in their domains, and <mark>interpret</mark> statements that use function notation in terms of a context.	MP.1 Make sense of problems and persevere in solving them. MP.4 Model with mathematics.	**Example**: Evaluate $f(2)$ for the function $f(x) = 5(x - 3) + 17$. **Example**: Evaluate $f(2)$ for the function $f(x) = 1200(1 + .04)^x$

F.IF.A.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n +1) = f(n) + f(n-1) for n ≥ 1.*	MP.1 Make sense of problems and persevere in solving them. MP.4 Model with mathematics. MP.7 Look for and make use of structure. MP.8 Look for and express regularity in repeated reasoning.	**Example**: Given the sequence 1, 1, 2, 3, 5, 8, 13, 21,… Explore the sequence, write a recursive function to define it, and determine each of the following: a. $f(0)$ b. $f(3)$ c. $f(10)$ **Example**: A theater has 60 seats in the first row, 68 seats in the second row, 76 seats in the third row, and so on in the same increasing pattern. a. If the theater has 20 rows of seats, how many seats are in the twentieth row? b. Explain why the sequence is considered a function. c. What is the domain of the sequence? Explain what the domain represents in context.

Functions: Interpreting Functions

College and Career Readiness Cluster

Interpret functions that arise in applications in terms of the context.

Mathematically proficient students pull relevant information from real life contexts to build mathematical models. The terms students should learn to use with increasing precision with this cluster are: **Intercept, interval, maximum, minimum, symmetry, end behavior, increasing, decreasing , zeros, average rate of change**

Enduring Understandings:
Functions can be used to model situations in the real world.

Essential Questions:
How can patterns, relations, and functions be used as tools to best describe and help explain real life situations?

College and Career Readiness Standards *Students are expected to:*	Mathematical Practices	Unpacking Explanations and Examples *What does this standard mean that a student will know and be able to do?*
F.IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of	MP.1 Make sense of problems and persevere in solving them. MP.2 Reason abstractly and quantitatively. MP.7 Look for and make use of structure.	**Example**: Given a table which represents the relationship between daily profit for an amusement park and the number of paying visitors. a. What are the x-intercept(s) and *y*-intercept(s) and explain them in the context of the problem. b. Identify any maximums or minimums and explain their meaning in the context of the problem. c. Determine if the graph is symmetrical and identify which shape this pattern of change develops. d. Describe the intervals of increase and decrease and explain them in the context of the problem.

the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.		**Example**: Given a graph which represents the height (in feet) of a rocket as a function of the time (in seconds) since it was launched, use the graph to answer the following: a. What is the practical domain for t in this context? Why? b. What is the altitude of the rocket two seconds after it was launched? c. What is the maximum value of the function and what does it mean in context? d. When is the rocket 100 feet above the ground? e. When is the rocket 250 feet above the ground? f. Why are there two answers to part d but only one practical answer for part e? g. What are the intercepts of this function? What do they mean in the context of this problem? h. What are the intervals of increase and decrease on the practical domain? What do they mean in the context of the problem?

| F.IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* | MP.1 Make sense of problems and persevere in solving them.

MP.4 Model with mathematics. | **Example**:
An all-inclusive resort in Los Cabos, Mexico provides everything for their customers during their stay including food, lodging, and transportation. Use a provided graph to describe the domain of the total cost function.

Example:
Jennifer purchased a cell phone and the plan she decided upon charged her $50 for the phone and $0.10 for each minute she is on the phone. (The wireless carrier rounds up to the half minute.) She has budgeted $100 for her phone bill.

    a.   What would be the appropriate domain for the cost as a function of the total minutes she used the phone?

    b.   Describe what the point (10, 51) represents in the problem. |

F.IF.B.6 ==Calculate== and ==interpret== the average rate of change of a ==function== (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	MP.1 Make sense of problems and persevere in solving them. MP.6 Attend to precision.	**Example**: Using a graph, determine the average rate at which a bicycle rider traveled from four to ten minutes of her ride? **Example**: The plug is pulled from a small hot tub. Using a table which gives the volume of water in the tub from the moment the plug is pulled, until it is empty, find the average rate of change between: 60 seconds and 100 seconds? 0 seconds and 120 seconds? 70 seconds and 110 seconds?

Functions: Interpreting Functions

College and Career Readiness Cluster

Analyze functions using different representations.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **asymptotes, exponential function, logarithmic function, piecewise, roots, polynomial function, degree, leading coefficient, symmetry, maximum, minimum, growth, decay, absolute value functions**

Enduring Understandings:

Key features of a function's equation, graph, and/or table can be used to understand the relationship represented by the function.

Essential Questions:

How do the coefficients and exponents of the function relate to the key components of its graph?

How do you translate from one form of a function to another form to reveal the key features of the function?

College and Career Readiness Standards *Students are expected to:*	Mathematical Practices	Unpacking Explanations and Examples *What does this standard mean that a student will know and be able to do?*
F.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases	MP.1 Make sense of problems and persevere in solving them. MP.2 Reason abstractly and quantitatively. MP.3 Construct viable arguments and critique the reasoning of others. MP.4 Model with mathematics.	The following examples should be used with each type of function, using graphing technology where appropriate. **Example**: Describe the key features of the graph $f(x) = -23x + 8$ and use the key features to create a sketch of the function. **Example**: Without using the graphing capabilities of a calculator, sketch the graph of $f(x) = x^2 + 7x + 10$ and identify the x-intercepts, y-intercept, and the maximum or minimum point. **Example**: Given a set of functions and a set of graphs match the appropriate function to its graph.

F.IF.C.7a Graph linear and quadratic functions and show <mark>intercepts</mark>, <mark>maxima</mark>, and <mark>minima</mark>. **F.IF.C.7b** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. **F.IF.C.7c** Graph polynomial functions, <mark>identify</mark>ing <mark>zeros</mark> when suitable factorizations are available, and showing end behavior.	MP.6 Attend to precision. MP.7 Look for and make use of structure.	

F.IF.C.7d Graph rational functions, identifying zeros and <mark>asymptotes</mark> when suitable factorizations are available, and showing end behavior. **F.IF.C.7e** Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing <mark>period, midline, and amplitude</mark>.		

F.IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
 	
F.IF.C.8a Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
 	
F.IF.C.8b Use the properties of exponents to interpret expressions for exponential functions. *For*
	MP.1 Make sense of problems and persevere in solving them.
	
	MP.4 Model with mathematics.
		Example:
		Given a standard form quadratic equation, rewrite into vertex form. Interpret the key features of the graph from the rewritten equation.
		
		Example:
		The projected population of Merrimack is given by the function $p(t) = 1500(1.08)^{2t}$ where t is the number of years since 2010. You have been selected by the town council to help them plan for future growth.
		
		Explain what the function $p(t) = 1500(1.08)^{2t}$ means to the city council members.
		
		Example:
		Suppose a single bacterium lands on one of your teeth and starts reproducing by a factor of 2 every hour. If nothing is done to stop the growth of the bacteria, write a function for the number of bacteria as a function of the number of hours.
		
		Example:
		The expression $50(0.85)^x$ represents the amount of a drug in milligrams that remains in the bloodstream after x hours.
		   a. Describe how the amount of drug in milligrams changes over time.
		   b. What would the expression $50(0.85)^{12x}$ represent?
		   c. What new or different information is revealed by the changed expression?

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example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{(12t)}$, $y = (1.2)^{(t/10)}$, and classify them as representing exponential growth or decay.		

F.IF.C.9 ==Compare properties== of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.	MP.1 Make sense of problems and persevere in solving them. MP.4 Model with mathematics. MP.7 Look for and make use of structure. MP.8 Look for and express regularity in repeated reasoning.	**Example**: David and Fred each throw a baseball into the air. The velocity of David's baseball is given by $v(t) = 50 - 32t$ where v is in feet per second and t is in seconds. The velocity of Fred's baseball is given in the table. What is the difference in the initial velocity between the two throws? **Example**: Examine the two functions represented below. Compare the x-intercepts and find the difference between the minimum values. $f(x) = x^2 + 8x + 15$

x	$g(x)$
-7	4
-6	1.5
-5	0
-4	0.5
-3	0
-2	1.5
-1	4

Functions: Building Cluster

College and Career Readiness Cluster

Build a Function That Models a Relationship Between Two Quantities

Mathematically proficient students build appropriate equations using given information. The terms students should learn to use with increasing precision with this cluster are: **recursive, arithmetic sequences, geometric sequence, composition**

Enduring Understandings:

Functions can be used to represent and describe relationships between two quantities.

Essential Questions:

How can we use functions to model situations?

How do the domains of the original functions relate to the domain of the composed function?

College and Career Readiness Standards *Students are expected to:*	Mathematical Practices	Unpacking Explanations and Examples *What does this standard mean that a student will know and be able to do?*
F.BF.A.1 Write a function that describes a relationship between two quantities. **F.BF.A.1a** Determine an explicit expression, a recursive process, or steps for calculation from a context.	MP.1 Make sense of problems and persevere in solving them. MP.4 Model with mathematics.	**Example**: The height of a stack of cups is a function of the number of cups in the stack. If a 7.5" cup with a 1.5" lip is stacked vertically, determine a function that would provide you with the height based on any number of cups. Hint: Start with height of one cup and create a table, list, graph or description that describes the pattern of the stack as each additional cup is added.

Age	1	2	3	4	5	6	7
Value	$1575	$1200	$900	$650	$500	$400	$300

F.BF.A.1b
Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

F.BF.A.1c
Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time,*

Example:

The price of a new computer decreases with age. Examine the table above by analyzing the outputs.

   a.   Informally describe a recursive relationship.

   b.   Analyze the input and the output pairs to determine an explicit function that represents the value of the computer when the age is known.

then T(h(t)) is the temperature at the location of the weather balloon as a function of time.		

F.BF.A.2 <mark>Write arithmetic and geometric sequences</mark> both recursively and with an explicit formula, use them to model situations, and translate between the two forms.	MP.1 Make sense of problems and persevere in solving them.
	
	MP.4 Model with mathematics.
	
	MP.7 Look for and make use of structure.
		<u>**Example**</u>:
		Given the following sequences, write a recursive rule:
		
		   a. 2, 4, 6, 8, 10, 12, 14, …
		
		   b. 1000, 500, 250, 125, 62.5, …
		
		Devise a real world context which these series could represent.

Functions: Building Cluster

College and Career Readiness Cluster

Build new functions from existing functions.

Mathematically proficient students understand function notation and can compose functions from other functions. The terms students should learn to use with increasing precision with this cluster are: **inverse, even function, odd function, composition, exponential, logarithm, invertible, transformation, stretch, translation, reflection, parent function**

Enduring Understandings:
Functions can be used to build other functions.

Essential Questions:
How are transformed functions related to their parent functions?

How do we determine if a function has an inverse that is a function?

How are the domain and range of an inverse function related to the original function's domain and range?

How are logarithms and exponents related?

College and Career Readiness Standards *Students are expected to:*	Mathematical Practices	Unpacking Explanations and Examples *What does this standard mean that a student will know and be able to do?*
F.BF.C.3 Identify the effect on the graph of replacing *f(x) by f(x) + k, k f(x), f(kx), and f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with	MP.1 Make sense of problems and persevere in solving them. MP.4 Model with mathematics.	**Example**: Describe how $f(x) + 3$ compares to $f(x)$ represented in the graph. **Example**: Given the graph of $f(x)$ whose x-intercept is 3, find the value of k if $f(x + k)$ resulted in the graph having an x-intercept of -4. **Example**: Describe how the graph of $f(x) + k$ compares to $f(x)$ if k is positive. If k is negative.

cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*		**Example**: Given the graph of $f(x) = 3x$ and the graph of $g(x) = f(x) + k$. Find the value of k.
F.BF.C.4 <mark>Find inverse functions</mark>. **F.BF.C.4a** Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) =2(x^3) for x > 0 or f(x) = (x+1)/(x-1) for x ≠ 1*	MP.1 Make sense of problems and persevere in solving them. MP.4 Model with mathematics. MP.7 Look for and make use of structure.	**Example**: For the function $h(x) = (x - 2)^3$, defined on the domain of all real numbers, find the inverse function if it exists or explain why it doesn't exist. Graph $h(x)$ and $h^{-1}(x)$ and explain how they relate to each other graphically. What is x when h(x) = 10? **Example**: Find a domain for $f(x) = 3x^2 + 12x$ - 8 on which it has an inverse. Explain why it is necessary to restrict the domain of the function.

F.BF.C.4b Verify by composition that one function is the inverse of another. **F.BF.C.4c** Read values of an inverse function from a graph or a table, given that the function has an inverse.		
F.BF.C.5 <mark>Understand</mark> the <mark>inverse relationship</mark> between <mark>exponents</mark> and <mark>logarithms</mark> and use this relationship to solve problems involving logarithms and exponents.	MP.7 Look for and make use of structure. MP.8 Look for and express regularity in repeated reasoning.	**Example**: Find the inverse of $f(x) = 3(10)^{2x}$. **Example**: Solve: $5 \cdot 15^{7x} = 74$, justify your process.

Functions: Linear, Quadratic, and Exponential Models

College and Career Readiness Cluster

Construct and compare linear, quadratic, and exponential models and solve problems.

Mathematically proficient students can distinguish various types of mathematical relationships. The terms students should learn to use with increasing precision with this cluster are: **rate of change, constant rate, unit interval**

Enduring Understandings:

Different mathematical relationships have distinguishable features whether in equation, table, or graphed form.

Essential Questions:

How can I recognize the type of mathematical relationship being modeled?

How does the rate of change help you to determine the type of function being modeled?

College and Career Readiness Standards *Students are expected to:*	Mathematical Practices	Unpacking Explanations and Examples *What does this standard mean that a student will know and be able to do?*

F.LE.A.1 <mark>Distinguish</mark> between situations that can be modeled with <mark>linear functions</mark> and with <mark>exponential functions.</mark>
 	
F.LE.A.1a Prove that linear functions grow by equal differences over
	MP.1 Make sense of problems and persevere in solving them.
	
	MP.4 Model with mathematics.
		<u>Example</u>:
		
		Carbon-14 is a common form of carbon which decays exponentially over time. The half-life of Carbon-14, that is the amount of time it takes for half of any amount of Carbon-14 to decay, is approximately 5730 years. Suppose we have a plant fossil and that the plant, at the time it died, contained 10 micrograms of Carbon-14 (one microgram is equal to one millionth of a gram).
		
		   a. Using this information, make a table to calculate how much Carbon-14 remains in the fossilized plant after *n* number of half-lives.
		
		   b. How much carbon remains in the fossilized plant after 2865 years? Explain how you know.
		
		   c. When is there one microgram of Carbon-14 remaining in the fossil?

| equal intervals and that exponential functions grow by equal factors over equal intervals.

F.LE.A.1b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

F.LE.A.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.		

F.LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	MP.1 Make sense of problems and persevere in solving them. MP.4 Model with mathematics. MP.7 Look for and make use of structure.	**Example**: Albuquerque boasts one of the longest aerial trams in the world. The tram transports people up to Sandia Peak. The table shows the elevation of the tram at various times during a particular ride. _See table below._ a. Write an equation for a function that models the relationship between the elevation of the tram and the number of minutes into the ride. b. What was the elevation of the tram at the beginning of the ride? c. If the ride took 15 minutes, what was the elevation of the tram at the end of the ride? **Example**: After a typical winter storm, there are 10 inches of snow on the ground. Now that the sun is finally out, the snow is melting. At 7 am there were 10 inches and at 12 pm there were 6 inches of snow. a. Construct a linear function rule to model the amount of snow. b. Construct an exponential function rule to model the amount of snow. c. Which model best describes the amount of snow? Provide reasoning for your choice.

Minutes into ride	2	5	9	14
Elevation (_feet_)	7069	7834	8854	10129

F.LE.A.3 **Observe** using graphs and tables that a quantity increasing **exponentially** eventually exceeds a quantity increasing **linearly,** **quadratically**, or (more generally) as a polynomial function.	MP.8 Look for and express regularity in repeated reasoning.	**Example**: Kevin and Joseph each decide to invest $100. Kevin decides to invest in an account that will earn $5 every month. Joseph decided to invest in an account that will earn 3% interest every month. Create a table or graph to answer the following questions. a. Whose account will have more money in it after two years? b. After how many months will the accounts have the same amount of money in them? c. Describe what happens as the money is left in the accounts for longer periods of time.

F.LE.C.5 Interpret the parameters in a linear or exponential function in terms of a context.	MP.4 Model with mathematics.	**Example**: The fee of a plumber is $50 for a house call and $85 per hour. This can be expressed as the function $y = 85x + 50$. If the rate were raised to $90 per hour, how would the function change?
		Example: Lauren keeps records of the distances she travels in a taxi and what it costs:

Distance d in miles   3   5   11
Fare f in dollars   8.25   8.75   26.25

    a.   If you graph the ordered pairs (d, f) from the table, they lie on a line. How can this be determined without graphing them?

    b.   Show that the linear function in part a. has equation $f = 2.25d + 1.5$.

    c.   What do the 2.25 and the 1.5 in the equation represent in terms of taxi rides.

Example:
A function of the form $f(n) = P(1 + r)^n$ is used to model the amount of money in a savings account that earns 8% interest, compounded annually, where n is the number of years since the initial deposit.

    a.   What is the value of r? Interpret what r means in terms of the savings account?

    b.   What is the meaning of the constant P in terms of the savings account? Explain your reasoning.

    c.   Will n or $f(n)$ ever take on the value 0? Why or why not?

Example:
The equation $y = 8000(1.04)^x$ models the rising population of a city with 8,000 residents when the annual growth rate is 4%.

    a.   What would be the effect on the function if the city's initial population were 12,000?

    b.   What would happen to the population over 25 years if the growth rate were 6%?